Conformed Identification of the Fundamental Matrix in the Problem of a Scene Reconstruction, using Stereo Images

V. Fursov[1,2] and Ye. Goshin[1,2]

[1]S.P. Korolyov Samara State Aerospace University, Samara, Russia
[2]Image Processing Systems Institute of the RAS, Samara, Russia

Abstract. This paper deals with the problem of the fundamental matrix identification on the basis of corresponding points on stereo images. It is one of the main problems in a scene reconstruction, using stereo images. This problem is commonly solved by the error-adaptive algorithm RANSAC. In this research, this problem is approached in accordance with a conformed identification principle. The method we propose in this paper ensures higher accuracy of the 3D scene reconstruction.

1 Introduction

This paper is devoted to one of the solutions of the problem of a scene reconstruction. For the 3D scene reconstruction based on a set of pairs of corresponding points on stereo images, it is necessary to know the matrices of cameras. If the external parameters of the cameras are unknown, they can be estimated directly from the preset corresponding points on the images. The solution of this problem is a serious issue because the number of the test points is commonly small, thus the terms of statistical stability are not satisfied, and the prior uncertainty of probabilistic characteristics of noise models takes place.

If prior probabilistic models are absent, the least-squares method (LS) is commonly applied. It is known that LS-estimation is optimal when measurement errors have normal distribution [1, 2], but the LS method loses its efficiency in the presence of rough errors in the input data. The idea of the LS method improvement by searching for a noise-free subsystem has been introduced in the research [3]. Similar ideas of identification, based on different assumptions, are given in [4]. However, due to the lack of necessary computing power at that time, both of these approaches remained at the level of theoretical ideas.

Recent years, the estimation methods, in which the stability to rough errors such as failures is obtained at the cost of a significant increase in the computational complexity of algorithms, are gaining in popularity. RANSAC is the most widely used algorithm in the task of 3D-scenes reconstruction of this class [5, 6]. The conformed identification method proposed in [7] is still another approach to this issue.

The identification algorithm, which is based on the idea of conformed identification with the consecutive formation of a conformal set of estimates, is

Fursov V. and Goshin Y..
Conformed Identification of the Fundamental Matrix in the Problem of a Scene Reconstruction, using Stereo Images.
DOI: 10.5220/0004393200290037
In *Proceedings of the 4th International Workshop on Image Mining. Theory and Applications* (IMTA-4-2013), pages 29-37
ISBN: 978-989-8565-50-1

considered in the present work. This algorithm provides a significant reduction in computational complexity, at same time it maintains high accuracy and reliability. The results of quality metrics comparison of this algorithm and the RANSAC algorithm for solving the problem of determining fundamental matrix on test stereo images are given.

2 Problem Definition

Suppose there are two cameras with centers of projections at points O and O'. In the planes of projections P and P' of these cameras, it is possible to define e and e' points of intersection of OO' line with the planes P and P' (epipoles) (Fig. 1). For some point P of the scene there are epipolar lines l and l' on the planes of projections (lines of intersection of the plane OO'P with the planes P and P'), on which the corresponding points p and p' are projections of the point P on two images.

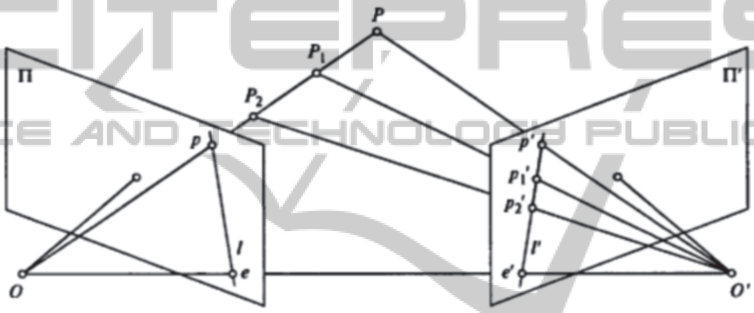


Fig. 1. The epipolar geometry model.

The corresponding points on the two projections are connected by the fundamental 3×3 -matrix F , in particular, for the corresponding points, which coordinates are set by 3×1-vectors x , x' : $x=[u,v,1]^T$, $x'=[u',v',1]^T$, the condition

$$(x')^T Fx = 0 \tag{1}$$

where $F=\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}$, is satisfied.

For a given pair of the corresponding points, the ratio (*) is a homogeneous linear equation for the coefficients $F_{i,j}$, $i,j=\overline{1,3}$ of the fundamental matrix. For N pairs ($N\geq 8$) of the corresponding points with $F_{33}=1$, it is possible to write the system of N non-homogeneous linear equations [8]:

$$\mathbf{y} = \mathbf{Xc} + \boldsymbol{\xi} \tag{2}$$

where **c** is the vector with the required characteristics composed from the coefficients of the fundamental matrix F:

$$\mathbf{c} = \begin{bmatrix} c_1, & c_2, & \cdots, & c_8 \end{bmatrix}^T = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{21} & F_{22} & F_{23} & F_{31} & F_{32} \end{bmatrix}^T \quad (3)$$

$N \times 8$ -matrix $\mathbf{X}$ and $N \times 8$ -vectors $\mathbf{y}$ and $\boldsymbol{\xi}$ are defined as

$$\mathbf{X} = \begin{bmatrix} u_1^{'}u_1 & u_1^{'}v_1 & u_1^{'} & v_1^{'}u_1 & v_1^{'}v_1 & v_1^{'} & u_1 & v_1 \\ u_2^{'}u_2 & u_2^{'}v_2 & u_2^{'} & v_2^{'}u_2 & v_2^{'}v_2 & v_2^{'} & u_2 & v_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_N^{'}u_N & u_N^{'}v_N & u_N^{'} & v_N^{'}u_N & v_N^{'}v_N & v_N^{'} & u_N & v_N \end{bmatrix}, \ \mathbf{y} = \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}, \ \boldsymbol{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_N \end{bmatrix} \quad (4)$$

The $\xi_1, \xi_2, \cdots, \xi_N$ are errors, connected with an imprecise coordinate assignment of the corresponding points.

The problem consists in the estimation $\hat{\mathbf{c}}$ of the vector of parameters $\mathbf{c}$ in case of an unknown $N \times 1$ -error vector ξ, using $N \times M$ -matrix $\mathbf{X}$ and $N \times 1$ -vector $\mathbf{y}$ ($N > M$). To solve the tasks of this kind, it is offered to apply the so-called conformed identification method. The idea of this method can be briefly described as follows.

The set of the *higher-level* subsystems, the dimension of which is $S < P < N$, is created of the initial system (1). On each of the subsystems, in turn, the set of the *lower-level* subsystems, the dimension of which is $M \le S < N$, is created. It is supposed that the matrix X and all possible matrices, composed of its lines, are nonsingular (if it is not so, the procedure of a preliminary monitoring of the conditionality and the selection of such subsystems can be provided).

Let

$$\hat{\mathbf{c}}_k = [\mathbf{X}_k^T \mathbf{X}_k]^{-1} \mathbf{X}_k^T \mathbf{G}_k \mathbf{y}_k \quad (5)$$

is the LS method estimation, calculated on the k^{th} lower-level system, and $\Theta_l = \{\hat{\mathbf{c}}_{l,k} : k = \overline{1, C_P^S}\}$ is a set of those estimations, belonging to one of $l = \overline{1, C_N^P}$ higher-level subsystems. To describe these sets Θ_l , *the mutual closeness criterion* of the estimates on the corresponding lower-level subsystems is introduced:

$$W\left[\Theta_l\right] = \sum_{i,j=1}^{|\Theta_l|} \left(\hat{\mathbf{c}}_{l,i} - \hat{\mathbf{c}}_{l,j}\right)^2, \quad l = \overline{1, C_N^P}, \quad i, j = \overline{1, C_P^S} \quad (6)$$

The problem lies in estimation building on the most *conformal set of estimates* $\hat{\Theta}$, for which the criterion (3) takes its minimum value. This problem essentially leads to the determination of the index $\hat{l}$:

$$W(\hat{l}) = \min_l W(\Theta_l) \quad (7)$$

Finding the most consistent set of estimates in accordance with (3), (4) is a computationally time-consuming task. Therefore, these algorithms are usually implemented on multiprocessor systems, even with a relatively small dimension of the system (1). Nevertheless, there are some problems associated with the storage of a large number of estimates, calculated on the lower-level subsystems on every node.

In this paper, a conformed identification algorithm modification, which provides a significant reduction in computational complexity, is considered in more detail in the next section. The results of the experimental tests of accuracy and reliability of the proposed modified algorithm in comparison with the algorithm of RANSAC are also provided. The results are illustrated by solving the problem of weak camera calibration, which is part of technology of 3D scenes reconstruction based on stereo images.

3 Consecutive Conformed Identification Algorithm

Let us consider the conformed identification algorithm, in which all generated models are processed sequentially. We assume that the dimension of the subsystems of the lower-level is equal to the dimension of the vector of the estimated parameters: $S = M$.

Suppose $k_1, k_2, ..., k_N$ are the numbers of the initial system rows (2). Let us consider an arbitrary the lower-level basic subsystem, which consists of M rows ($M < N$), further designated as $S(k_1, k_2, ..., k_M)$. Let us define the set Θ_l of the lower-level subsystems, which are generated from the above mentioned basic subsystem by replacement of one of the rows by any other row of the system (2) which is not included in $S(k_1, k_2, ..., k_M)$:

$$\Theta = \left\{ S(k_1,k_2,...,k_M): \quad \begin{cases} S\left(k_{a_1},k_{a_2},...,k_{a_{M-1}},l\right), & \\ a_i=\overline{1,M}, & a_i \neq a_j, \quad i,j=\overline{1,M-1}, \quad i \neq j \\ l \neq k_p, & p=\overline{1,M}. \end{cases} \right\} \qquad (8)$$

Finding the most conformed set of estimates $\hat{\Theta}$ and the corresponding estimate $S(k_1, k_2, ..., k_M)$ leads to the minimization of the measure of conformity among all the subsystems

$$c = \hat{c}(k_1,k_2,...,k_M): \ (k_1,k_2,...,k_M) = \underset{\substack{k_1,k_2,...,k_M \in 1,N \\ k_i \neq k_j}}{\arg\min} W(S(k_1,k_2,...,k_M)),$$

where $W(S(k_1,k_2,...,k_M))$ is the measure of conformity for the subsystem $S(k_1,k_2,...,k_M)$:

$$W\left[S(k_1,k_2,...,k_M)\right] = \sum_{S\left(k_{a_1},k_{a_2},...,k_{a_{M-1}},l\right)\in\Phi} \left[\hat{c}(k_1,k_2,...,k_M) - \hat{c}\left(k_{a_1},k_{a_2},...,k_{a_{M-1}},l\right)\right]^2,$$

and $\hat{c}(k_1,k_2,...,k_M)$ is the estimate on the subsystem $S(k_1,k_2,...,k_M)$.

The described method of evaluation spares us storage of estimates on all the square lower-level subsystems and reduces the number of comparisons, but this method requires that the same solutions are repeatedly calculated. . Since any of M rows of the subsystem can be replaced by one of $(N-M)$ rows, which are not in this subsystem, $M\left(N-M\right)$ subsystems are formed. Thus, each subsystem is calculated $M\left(N-M\right)+1$ times that is redundant.

For the problem considered, the replacement of one row in a subsystem by a new one comes to finding the intersection of the line corresponding to the remaining M-1 rows, and a hyperplane corresponding to added rows. As a result, the main computational cost is connected with basic subsystems solving.

The number of inefficient computations can be reduced due to the use of a small set of basic subsystems, which include every row of the initial system (2) only once.

The last variant of the solution is suitable for parallel implementation on multiprocessor systems. In this case, the most conformed subset of the estimates can be studied for on each set of the lower-level subsystems associated with one of the basic subsystems. Then, by comparing the functions of mutual closeness of the estimates obtained for different subsets and comparing their corresponding line numbers, the most conformed set of the subsystems can be found.

4 Outcome of the Experiment

Comparative experimental studies of the conformed identification method and the RANSAC algorithm were conducted in order to compare their accuracy in the same operating conditions. The experiments were carried out on the data sets that were modeled as follows.

In various systems (2) M = 8 and N = 12 or N = 16. The components of the parameter vector **c** were specified as uniformly distributed random numbers in the range of 1 to 10. The elements of matrix X were calculated in accordance with (2) the corresponding points coordinates, which are modeled as random sequences with given variances.

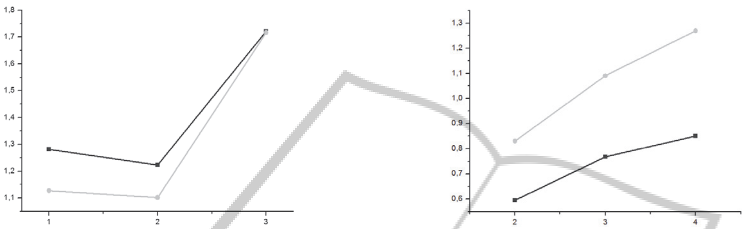
Components of the error vector were formed in such a manner that the normal error signal-to-noise ratio was in the range of 40-60 dB. For rough errors signal-to-noise ratio was specified in the range of 0-10 dB.

In the most models described in the papers dedicated to the effectiveness of RANSAC, the number of observations N significantly exceeds (by 2-3 orders of magnitude) the number of the estimated parameters M. If the intensity of the noise is reasonable, traditional statistical processing schemes give good results. Therefore, if the number of degrees of freedom is great, the goal of most studies is to show the advantages of the RANSAC in case when the number of rough errors reaches 80-90% of the total number of observations.

In this paper, we consider the case when the number of observations N is usually slightly higher than M (not more than 2-3 times). In this case, the statistical schemes are not suitable. Also, the frequency of the anomalous error was specified more realistically: 50-60% of the number of degrees of freedom (N-M) of the resulting system.

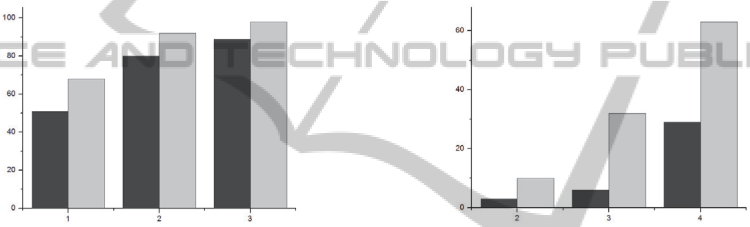
For comparative evaluation of the accuracy and reliability of the algorithm, we used the following parameters: the identification is correct if the ratio of the error vector norm to the parameter vector norm does not exceed 0.3.

Figure 2 shows a plot of the mean values of the error on the number of correct identifications of anomalous errors for different values of the number of estimated parameters (light - for the method of RANSAC, dark - for the conformed identification method). Figure 3 shows graphs illustrating the number of false identifications to 100 experiments on the number of rough errors for the same values of the estimated parameters. It is obvious that in all implementations, the conformed identification method shows better results both in reliability and accuracy.



a) b)

Fig. 2. Mean error of the correct identifications: a) N=12, M=8; b) N=16, M=8.



a) b)

Fig. 3. The number of false identifications for 100 experiments: a) N=12, M=8; b) N=16, M=8.

3D scene stereo images from two cameras were obtained by using the POV-Ray program. The cameras' internal (camera matrices $\mathbf{K}_1, \mathbf{K}_2$) and external (rotation matrices $\mathbf{R}_1, \mathbf{R}_2$ and translation vectors $\mathbf{C}_1, \mathbf{C}_2$) parameters [8] take the following values :

$$\mathbf{K}_1 = \mathbf{K}_2 = \begin{bmatrix} 400 & 0 & 400 \\ 0 & 400 & 300 \\ 0 & 0 & 1 \end{bmatrix},\ \mathbf{R}_1 = \begin{bmatrix} 0{,}999048 & 0 & -0{,}0436194 \\ 0{,}0308436 & 0{,}707107 & 0{,}706434 \\ 0{,}0308436 & -0{,}707107 & 0{,}706434 \end{bmatrix},$$

$$\mathbf{R}_2 = \begin{bmatrix} 0{,}999048 & 0 & 0{,}0436194 \\ -0{,}0308436 & 0{,}707107 & 0{,}706434 \\ -0{,}0308436 & -0{,}707107 & 0{,}706434 \end{bmatrix},\ \mathbf{C}_1 = \begin{bmatrix} -0{,}5 \\ 5 \\ -5 \end{bmatrix},\ \mathbf{C}_2 = \begin{bmatrix} 0{,}5 \\ 5 \\ -5 \end{bmatrix}.$$

Then with the help of OpenCV library, 16 corresponding points were found. On basis of these points the fundamental matrix was formed. From the received fundamental matrix, projective transformation matrices for each of the images were generated, and stereo rectification was done. The same procedure was carried out for the exact fundamental matrix, obtained from the given internal and external camera parameters. The exact ($\mathbf{F}$) and estimated ($\hat{\mathbf{F}}$) matrices are given below:

$$\mathbf{F} = \begin{bmatrix} 0 & -1{,}25\cdot 10^{-5} & -0{,}00125 \\ -1{,}25\cdot 10^{-5} & 0 & 0{,}166954 \\ -0{,}00125 & -0{,}156954 & 1 \end{bmatrix},$$

$$\hat{\mathbf{F}} = \begin{bmatrix} 2{,}8918\cdot 10^{-6} & 3{,}16982\cdot 10^{-5} & -0{,}0164114 \\ -5{,}51261\cdot 10^{-5} & 5{,}70682\cdot 10^{-6} & 0{,}170538 \\ 0{,}012072 & -0{,}162094 & 1 \end{bmatrix}.$$

Figures 4 and 5 show the rectified stereo pairs with the epipolar lines obtained from the experiment for the exact and the estimated fundamental matrices, respectively.

Fig. 4. Stereo pair rectified from the exact matrix.

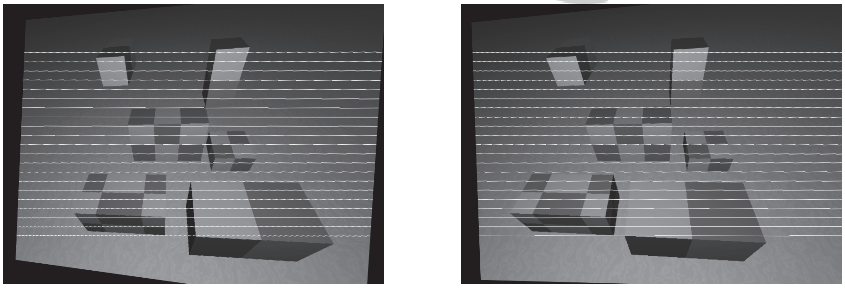


Fig. 5. Stereo pair rectified from the estimated fundamental matrix.

Disparity maps were formed from the transformed stereograms by means of OpenCV. Figure 6 shows the maps for the exact and estimated fundamental matrices, respectively.

5 Conclusions

The application of the conformed identification method allowed us to come to certain conclusions. It is hence shown that:

a) b)

Fig. 6. Disparity map for: a) exact fundamental matrix, b) estimated fundamental matrix.

1. The accuracy and reliability of fundamental matrix identification significantly affects the accuracy and reliability of the reconstruction of the object.
2. It is possible to achieve much higher accuracy and reliability of 3D-scene reconstruction by using the conformed identification method in comparison with the RANSAC algorithm.
3. The computational complexity of the considered modification of the conformed identification method still remains higher than that of the RANSAC method. Although a high computational complexity of this method causes dissatisfaction, in practice there are situations when the problem of determining the parameters of the model has to be solved as accurately as possible on the single, though perhaps small and very noisy, data set. Fortunately, as it shown in this paper, when implemented on a multiprocessor system, it is not a serious problem because this algorithm has a high degree of parallelism.

Acknowledgements

This work was financially supported by the RFBR (grants # 11-07-12051, #12-07-00581, #13-07-97000).

References

1. Graupe, D. (1978). *Identification of systems*. New York: Robert E. Kriger Publishing Company Huntington.
2. Ljung, L. (1987). *System Identification – Theory for the User*. Prentice Hall, Englewood. Cliffs, N.J.
3. Kalman, R.E. (1985). Noised systems identification. *Advances of Mathematical Sciences*, v. 40, issue 4(244).
4. Allen, D.M. (1971). *The prediction sum of squares as a criterion for selecting predictor variables*. Technical Report 23, University of Kentucky.
5. Fischler, M. & Bolles R. (1981). Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. *Comm. Of the ACM 24*, 381–395.

6. Torr, P.H.S. & Murray, D.W. (1997). The Development and Comparison of Robust Methods for Estimating the Fundamental Matrix. *International Journal of Computer Vision,* 24, 271-300.
7. Fursov, V. & Gavrilov, A. (2004). Conforming Identification of the Controlled Object. *Proceeding International Conference on Computing, Communications and Control Technologies: CCCT'2004*, 326-330.
8. Forsyth, D. & Ponce J. (2004). *Computer Vision: A Modern Approach.* Moscow: "Williams" Publisher.