

New 'Spider' Convex Hull Algorithm For an Unknown Polygon in Object Recognition

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Abstract: Object recognition in machine vision system and robotic applications has, and is still, an important aspect in automation applications of our everyday life. Although there are a lot of machine vision algorithms there are not always entirely clear and unified solutions for particular applications. This paper is concerned one particular step in image interpretation connected with the convex hull algorithm. This new approach to the process of convex hull step of object recognition offers a wide range of application and improves the accuracy of decision making on later steps. The challenging fundamental problem of computational geometry is offering the solution in this work to solve convex hull procedure for an unknown image polygon. The unique feature of the offered new approach is the flexible intersection of all convex set points of an object on a digital image. The convex combination points remains unknown and allow us to get the real vector space. The image segmentation algorithm and decision making procedure working in conjunction with this new convex hull algorithm will take robotic applications to a higher level of flexibility and automation. We present this unique procedure for automating and a new model of image understanding.

1 INTRODUCTION

The modern development in digital camera manufactures brings new potential for digital image processing. The fast growing image capturing CCD array development is capable of generating ever larger amounts of data for processing. Storage facilities have developed to nearly unlimited capacity, to the extent that a human is unable to process these volumes of data manually/visually. With this increased capability in image capture and storage all of industry and science will, in future, have to rely on stable and robust robotic potential to interpret the acquired data. Through this innovation Bioinformatic, visual navigation, quality control and medical diagnostics are coming to the new stage of automatic image recognition.

If we consider that each image consist of one or several objects each of which has certain features. The set of features that are recognisable to the human eye makes us to acknowledge the nature of the image that is in front of a camera. In computer memory there is fundamental connectivity problem for the compiling of the identified features into a recognisable image with the features contained within a defined border

line. This process of dividing an image into meaningful regions or segments called segmentation. Due to the nature of the image/features and/or the light condition under which the image is captured, a particular object's features could vary from one side of an object to another. This paper offers a new universal approach to the segmentation task and/or the selection of Regions Of Interest (ROI) for accurate border matching.

In absence of no complete and unique theoretical model available for simulating human object recognition, we consider a unified concept in this paper for convex hull algorithm as a component of image analysis which involves the use of digital image processing methods in attempt to provide a machine interpretation. The colour, morphological or pattern identifiers for automatic image recognition could vary and/or combine in this approach. This includes the possibility of combining the set of identifiers to provide a new level of stability and robustness in automatic object recognition and decision making procedures. The offered novel computational geometry roots has the appearance of a virtual spider net formation on an image space, so we are calling the new algorithm "Spider convex hull". The same approach could be easily

extended to multidimensional space, but for simplicity in this paper we present C++ code and graphical illustration for 2D application. This suggested solution is universal and can be used in wide range of machine vision applications. This new level of automation provides possibilities to improve the quality of people life in a multitude of potential applications.

2 IMAGE RECOGNITION

In this section, we consider the applied procedures that are necessary during object recognition. These procedures are adaptive and have no binding to a particular range of applications. A typical colour image consists of mixed RGB signals. A grey-tone image appears as a normal black and white photograph. However, on closer inspection it may be seen that it is composed of a large number of individual picture cells or pixels. In fact, a digital image is an $[x, y]$ array of pixels. One can get a better feel for the digital limitations of such a digitised image by zooming into a section of the picture that has been enlarged so that the pixels can be examined individually. It is then easy to appreciate that each pixel contains a grey level digitalisation. This level will include a certain amount of noise and so it is seldom worth digitising more accurately than 8 bits of information per pixel. The number of these levels depends on the signal-to-noise ratio of the image capture device and the analogue-to-digital converter. Modern digital cameras can store up to 24 bits per pixel. Note, that if the human eye can see an object in a digitised image of practical spatial and grey-scale resolution, then, it is in principle possible to devise a computer algorithm to do the same thing. In a human eye, image points are organised into a photosensitive matrix or array where each point can be enumerated in terms of coordinates x and y . The value of each isolated point can be represented by value of a function I with coordinates x and y . Here, x can be taken to represent the horizontal axis and y the vertical axis. Video information is stored in the same way, i.e. in terms of the function $I(x, y)$ (Rosenfeld, 1982; R.O. Duba, 1973). The colour content(s) of an image is very important and contributes significantly to the image processing operations required and the object recognition methodologies applied (Freeman, 1988). In the case of medical imaging in general, colour processing and colour interpretation is critical to the diagnosis of many conditions and the interpretation of the information content of an image by man and machine. Colour image processing is becoming more and more important in object analysis and pattern recognition. The numerous and non-

related algorithms for understanding two- and three-dimensional objects in a digital image have and continue to be researched in order to design systems that can provide reliable automatic segmentation, object detection, recognition and classification in an independent environment (e.g. (E.R.Davies, 1997), (Freeman, 1988), (Louis and Galbiati, 1990) and (Snyder and Qi, 2004)). In relation to an object's shape, size, morphological similarity, texture and continuity these tasks can be very challenging.

Several conditions need to be considered in the development of any machine vision system: 1) the target resolution or contrast 2) The structural algorithmic approach including object representation 3) What type of hardware would be suitable to reach speed and accuracy. For example, optical microscopy involves the use of image processing methods that are often designed in an attempt to provide a machine interpretation of a biological image, whereby some decision criterion to be applied, such that a pattern of biological significance can be recognised (Russ, 1990), (M.A.Hornish and R.A.Goulart, 2008).

Associations between the features and an object pattern attributes forms automatic learning context for knowledge data base (Dubovitskiy and Blackledge, 2009), (Dubovitskiy and Blackledge, 2008), whereby the representation of the object is assembled into the feature vector (Grimson, 1990), (Ripley, 1996). The knowledge data base depends on establishing equivalence relations that express a fit of evaluated objects to a class with independent semantic units. Whereby assigning a particular class to an object the pattern recognition task is accomplished.

2.1 Practical Image Recognition Implementation

Practical image recognition systems generally contain several stages in addition to the recognition engine itself. Image pre-processing is used for adjusting the artefacts after the operation of an image acquisition system. We consider the following sub-tasks.

Low brightness and Contrast. The correction of brightness and contrast is usually a pre-processing procedure, after which, the image looks clearer and more precise. Nevertheless, it is necessary to note, that such a correction does not provide any additional information of value to procedures such as feature extraction or boundary detection for example. The existence or otherwise of spatial frequencies is indifferent to whether the map of the image is contrast stretched or not. In current applications, the brightness and contrast of the images used is sufficiently good for the system to exclude this pre-processing procedure

although in other applications of the algorithm, this may be a necessary requirement. *Image graininess* Some types of images can have a grainy structure - often due to the nature or features of the image acquisition system. It is a typical problem in those cases where it is necessary to acquire an image with maximal resolution. The main problem with processing coarse-grained maps is related to the in-practicality of detecting the boundaries, i.e. boundaries are detected that are associated with grains instead of the contours of objects. A typical solution consists of smoothing the image using minor diffusion in which the boundaries of the grains become fuzzy and diffused with each other, while the contours of object remain (albeit over a larger spatial extent). A similar effect can be obtained using the median filter. However, use of the median filter includes an inevitable loss of information characterised by shallow details (i.e. low grey level variability). In this thesis, the Wiener filter (Wiener, 1949) is used which is computational efficient, robust and optimal with regard to grain diffusion and information preservation. This filter eliminates high-frequency noise and thus does not distort the edge of objects. Other solutions include preliminary de-zooming for the purpose decreasing grit size up to and including the size of a separate pixel. Such a method involves loss of shallow details however, and thus, the size of the map (and accordingly, the processing time) decreases. The other advantage of such a method concerns hardware implementation, e.g. application of a nozzle to an optical system. In situations where the methods described here are unacceptable, it is necessary to use a more complex quality detector for boundary estimation which is discussed below.

Geometrical Distortions. In practice, the most important geometrical distortions are directly related to character of an image acquisition. In the majority of cases is possible to use a standard video camera as the image sensor. However, the majority of industrial production specifications for video systems use an interlaced scan technique for image capture. This leads to 'captured lines' in the image of both even and odd types which leads to a time delay between neighbouring lines (equal to half the acquisition time frame). If there is a moving object in the field of view, then its position on even and odd lines will be different - the picture of the object will be 'washed' in a horizontal direction. This is a particularly important problem in the extraction of edges. In this case, it is impossible to bleed the verticals. The elementary solution to this problem is to simply skip the even or odd frames (preferably the even frames as the odd frames consist of later information). Another way is to handle even and odd frames separately providing the processing

speed allows for practical implementation. If this is not possible, it is necessary to use a video system with non interlaced scanning. Over the past few years, with the development of digital video and engineering the capability has emerged to use digital video cameras with high resolution. A singular advantage of this is the uniformity of the picture without the distortions discussed above. However, the video RGB of matrices need to be analysed to avoid inter-colour distortions. These distortions are connected to the geometrical distribution of the RGB cells on the surface of a CCD matrix and can be seen when increases in the size of the digital are introduced. Special filters need to be designed that can be used in the prevention of this kind of distortion

Edge Detection. has gone through an evolution spanning more than 20 years. Two main methods of edge detection have been apparent over this period, the first of these being template matching and the second, being the differential gradient approach. In either case, the aim is to find where the intensity gradient magnitude g is sufficiently large to be taken as a reliable indicator of the edge of an object. Then, g can be thresholded in a similar way to that in which the intensity is thresholded in binary image estimation. Both of these methods differ mainly in that they proceed to estimate g locally. However, there are also important differences in how they determine local edge orientation, which is an important variable in certain object detection schemes.

Each operator estimates the local intensity gradients with the aid of suitable convolution masks. In a template matching case, it is usual to employ up to 12 convolution masks capable of estimating local components of the gradient in the different directions. Common edge operators used are due to Sobel (J.M.S.Perwitt, 1970), Roberts (L.G.Roberts, 1965), Kirsch (R.A.Kirsh, 1971), Marr and Hildreth (Marr and E.Hildreth, 1977), Haralick (R.M.Haralick, 1980; R.M.Haralick, 1984), Nalwa and Binford (Nalwa and Binford, 1986) and Abdou and Pratt (Abdou and W.K.Pratt, 1979). In the approach considered here, the local edge gradient magnitude or the edge magnitude is approximated by taking the maximum of the responses for the component mask:

$$g = \max(g_i : i = 1 \text{ to } n)$$

where n is usually 8 or 12. The orientation of the boundary is evaluated in terms of the number of a mask giving maximal value of amplitude of a gradient.

The integration of local operands into convex hull algorithm is the way forward to isolate ROI or in particular cases accuracy identify object location. The

texture analysis is the next step of image recognition. The combination of edge detection and texture analysis into convex hull offer the new accurate and reliable stage of image processing tool.

3 TECHNOLOGY OVERVIEW

In this section, we briefly review the currently available convex hull algorithms and components associated with the application. The common practice in segmentation and image recognition technics is to use procedure call binarisation. The principal question is what does comes comes first segmentation or recognition? The answer is a combination of both through the use of the convex hull approach.

The first task in common solution is to remove points with small amplitude of a local gradient of brightness with the purpose of separating points of a contour from textures, shallow details and noise. There are two cases to the segmentation algorithm:

- (i) Pixels similarities based approach
- (i) Surface discontinuities based approach

The first way is to select some value of a threshold binarisation TR and to remove points with amplitude of a gradient $|g| < TR$. Some of the thresholding processing needs to be considered *a priori*.

The main problem in defining the value of a threshold TR say, is that it should be different for different images. Moreover, if the objects on the image have different brightness, the value TR should be different for different areas of the image. The solution is usually employed through a method of adaptive binarization, based on calculating the value of a threshold TR for small areas of the image (size 88.. 1212) - so-called block binarisation. For each area, the average value I_0 of brightness amplitude is evaluated and then the value of a threshold is calculated as follows:

$$TR = k * I_0$$

where $k = 1.2.. 1.8$ - binarisation coefficient. Change in the value of k are invalid when considerable changes in the quality of an extracted contour occur (as against the level of a threshold TR), and the value $k = 1.5$ can be adopted for the overwhelming majority of the images.

Another modification of this method involves the calculation of the level of a threshold separately for different boundary orientations. This prevents the deletion of important details close to brighter objects. However, this method requires more computing cost (as it is necessary to compute the local histograms but allow for an increased value of a threshold TR without

loss of essential details and, as a corollary, reduction in the quantity of false points.

In addition to the thresholding methods one can employ multi-region based segmentation. Regions, which are continuous, are simply connected clusters of pixels which are mutually exclusive and exhaustive (i.e. a pixel can only belong to a single region and all pixels have to belong to some region). A region may support a set of predicates; however, an adjacent region cannot support the same set of predicates. The advantages of using a region-based segmentation are: (1) there are far fewer regions than pixels in an image, thus allowing data compression; and (2) regions are connected and unique. The disadvantages of the method are: (1) assumptions are made about the uniformity of image features; (2) a region could be erroneously considered to be a single surface; (3) surface properties or such as reflection can produce regions of noise.

There are two principal approaches to region-based segmentation which are discussed below

1. **Region Growing.** Initially each pixel can be considered to be a separate region. Adjacent regions are merged if they have similar properties (such as grey-level). This merging process continues until no two adjacent regions are similar. The similarity between two regions is often based upon simple statistics such as the variance measure or the range of grey-levels within the regions. Region-based segmentations are described in (Brice and Fenema, 1970), (Yakimovsky and Feldman, 1973), (Feldman and Yakimovsky, 1974), (C.A.Harlow and S.A.Eisenbeis, 1973).
2. **Region Splitting.** Initially the image is regarded as being a single region. Each region is recursively subdivided into subregions if the region is not homogeneous. The measure for homogeneity is similar to that for region growing. Robertson et al. subdivided a region either horizontally or vertically if the pixel variance in the region is large (P.W.Swain and Fu, 1973). Others - e.g. (J.M.S.Perwitt, 1970) - use the bimodality of histograms to split regions. This is referred to as the *mode method* and has been extended (Ohlander, 1975; Price and Reddy, 1978) to use multiple thresholds from the histogram of the region. However, as discussed above, a histogram gives global information about a region. Using this method, some pixels can be assigned a wrong label and therefore pre- and post-filtering is required (Ohlander, 1975).

Growing regions is a more difficult task than region splitting. However, region splitting, using the method described in (P.W.Swain and Fu, 1973)

(J.M.S.Perwitt, 1970), can lead to the generation of more regions than required. Some region merging at the end of the splitting phase is required. A large group of segmentation techniques is used texture analysis. In the previous section, we described methods for image segmentation based on the grey-level properties of objects. These methods generally work well for man-made objects which usually have a smooth grey-level surface. We observe a textured region as being *homogeneous*, although the intensity across the region may be non-uniform. This leads to the intensity-based segmentation methods to produce results which do not match with our perception of the scene.

Texture is important not only for distinguishing different objects but also because the texture gradient contains information describing the objects depth and orientation. Texture can be described by its statistical or structural properties (B.Lipkin and A.Rosenfeld, 1970). A texture surface having no definite pattern is said to be *stochastic*, while texture with a definite array of sub-patterns is said to be *deterministic*. These textured surfaces can then be described by some placement rule for the pattern primitives. In reality, deterministic texture is corrupted by noise so that it is no longer ideal; this is referred to as the *observable* texture. If the pattern making up the deterministic texture itself has subpatterns, then these are called *microtextures* and the larger patterns are called *macrotextures*. One of most powerful texture measures is the fractal geometry. The main idea is that fractal properties can be used like individual features of an image or part of an image. One of the ways to achieve adaptive thresholding has been patented in a previous publication (Dubovitskiy and Blackledge, 2012).

Segmentation should be invariant to indexing algorithms. The indexing phase has until recently been almost entirely ignored. In practice, emphasis of the recognition procedure has been placed on producing reliable correspondence algorithms (Grimson, 1990). The results of all these methods combined are not sufficient to automatically segment complex image structures such as medical images e.g. a cervical smear. This can be achieved by using a local estimation and a special suite of algorithm(s) developed with convex hull to allow us to produce a successful result

Currently available convex hull algorithms could be found (Andrew, 1979), (Brown, 1979). Some Quickhull algorithms and a randomised incremental program is available (C. B. Barber and Huhdanpaa, 1993). Let consider a problem of the object form, where Q_k are finite sets and $G_k(x,y)$ represents a linear objective function. Then the optimal solution for

convex hull obtains the value of the following:

$$Q_k = \max_k \sum G_k(x,y)$$

The decomposition optimisation is possible and it is equivalent to filter primal objective of the convex hulls of the individual feasible sets. Most of them are based on the decomposition of linear cost function. However, due to either the lighting of the object, exposure condition or the object positioning the object properties can vary, making the application of the algorithm not possible. Therefore there is a strong need for local micro decision making about the object border.

4 CONVEX HULL ALGORITHM 'SPIDER'

Suppose we have an image which is given by a function $f(x,y)$ and contains some object described by a set (a feature vector that may be composed of integer, floating point and strings) $S = \{s_1, s_2, \dots, s_n\}$. We consider the case when it is necessary to define a sample which is somewhat 'close' to this object. As in the previous algorithm described, after binarisation of image $I(r,c)$ we acquire a two dimensional binary representation of an object on the index map $I_{bin}(r,c)$ which has the same dimension as the initial image (1 corresponds to the presence of an edge or body of the object and 0 corresponds to the background of the image). Let us consider the task of obtaining the co-ordinates of a convex polygon. This task is given in the MATLAB function 'Qhull'. The algorithm applied in this paper differs from that of MathWorks Inc one in terms of its simplicity, reliability and fast computation. The reason is that a number of cycles performed is limited and equal to the total border length of the object.

The main idea can be presented in terms of a 'Spider', which creeps on a wall of the map and pulling behind itself a thread. This thread is attached to the object. At the 'point of curvature' the thread stores the co-ordinates of the outer polygonal point. Each thread could be considered as a line on an image. We can propagate along a line of any local function. Thus, the path on the perimeter around the object provides the co-ordinates of all the outer polygonal points as shown in Figure 1

Let us consider the algorithmic solution of this task. For initial conditions we should select a position of a thread without bends. Clearly, this will be along one of four boundaries of the image. The direction of a detour and the selection of the initial conditions

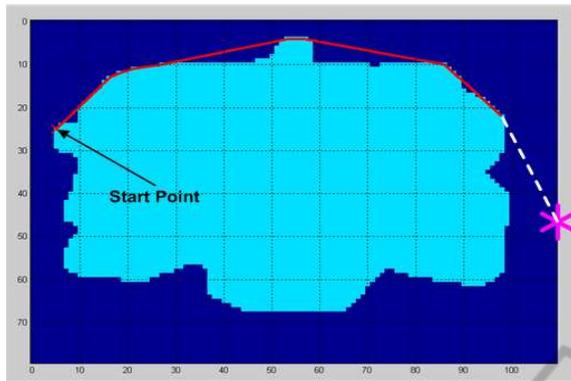


Figure 1: Obtaining co-ordinates for Convex hull.

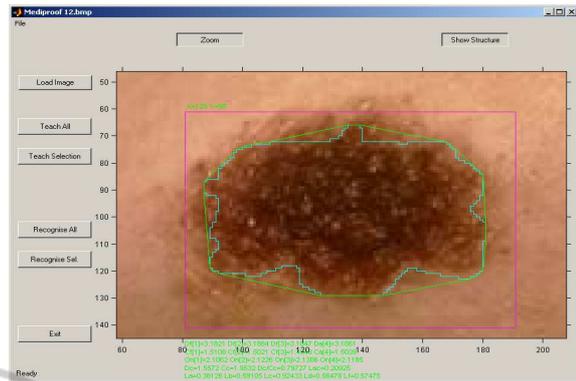


Figure 3: Object with Contour and Convex Hull.

```

{NListDotsX[02*((maxX-minX)+(maxY-minY))] //Object's
NListDotsY[02*((maxX-minX)+(maxY-minY))]} //dots list
ListDotsX[0]=StartX; // Sets the initial co-ordinates
ListDotsY[0]=StartY; // for other end of a thread
int nc=0,x4,y4,Mx4,My4;
double fi,cs,sn,step,r,RR,bz,sz;
for(nt=0;nt<(2*((maxX-minX)+(maxY-minY)));nt++){//Begin
fi=atan2(NListDotsY[nt]-StartY,...// creeps around
NListDotsX[nt]-StartX); // object
RR=sqrt(pow((NListDotsX[nt]-StartX),2)+...
+pow((NListDotsY[nt]-StartY),2));
cs=cos(fi);
sn=sin(fi);
if (fabs(sn)>fabs(cs)){ //Calculation
bz=fabs(sn); //the step length
sz=fabs(cs);
}else{
bz=fabs(cs);
sz=fabs(sn);
}
step=sqrt(pow(((sz*(1-bz))/bz),2)+pow((1-bz),2))+1;
for (r=0;r<=RR;r+=step){ // Searching for objects
x4=round((double)StartX + r*cs); //in way of thread
y4=round((double)StartY + r*sn);
if (*(pgg + x4*h + y4) == 1){
Mx4=x4; // saving last coordinate
My4=y4; // in temporary variables
}
}
if ((Mx4!=StartX)&&(My4!=StartY)) || //Stop check
((Mx4==StartX)&&(Mx4==NListDotsX[nt])&&
(Mx4!=NListDotsX[nt+1])) || ((My4==StartY)&&
(My4==NListDotsY[nt])&&(My4!=NListDotsY[nt+1]))){
StartX=Mx4; // Assign new start co-ordinates
StartY=My4;
nc=nc++;
ListDotsX[nc]=StartX; // Saving list Convex hull
ListDotsY[nc]=StartY; // coordinates
}
}
    
```

Figure 2: C++ algorithm for Convex Hull.

do not depend on to the aforementioned conditions. In the example considered here, the detour is clockwise and starts along the left vertical boundary of the image. The solution is shown in Figure 2.

The presented algorithm is also useful for defining the geometrical location of separated points or objects. The algorithm has been successfully used in the the developed computer recognition system. The example of computing of the outer co-ordinates of a polygon and a detour over the object contours are presented in Figure 3.

5 CONCLUSIONS

The work reported in this paper is part of a wider investigation into automating the application of image recognition algorithms. Authors have been concerned with the creation of the unified approach to the Convex hull algorithm for digital image processing. The novel algorithm has been explained in section IV. The two main tasks concerned in this paper: (1) the partial image analysis in terms of textural and morphological properties that characterise an object (2) the possibility of using a recursive segmentation approach for object recognition based on local features. Both of these tasks are fulfilled by the geometrical properties of the suggested solution. Novel in this approach is that analysis line is not along cartesian or polar co-ordinates but along the object border. Focussing the analysis line by the expected object's border allows researchers achievable scanning time and will optimise accurate edge detection in the correct image region. This considered convex hull algorithm is generic and could be used in various applications. That is why we have not included any particular application area in this paper. One potential application could be in medical imaging, e.g. cancer screening, others include surface inspection system, visual navigation and many more.

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