

Shape from Multi-view Images based on Image Generation Consistency

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Abstract: There are various and a lot of depth recovery methods have been studied, but a discussion about an unification of individual methods is expected not to be enough yet. In this study, we argue that the importance and the necessity of an image generation consistency. Various clues including binocular disparity, motion parallax, texture, shading and so on can be effectively used for depth recovery for the case where each or some of those are completely performed. However, in general, those clues without shading cause ideal and simplified constraints, and for several cases those clues without shading are suitable for obtaining initial depth values for the unification algorithm based on an image generation consistency. On the other hand, shading indicates a strict characteristics for image generation and should be used for the key principle for the unification. Based on the above strategy, as a first step of our scheme, through a simple problem with two-views, the unification of binocular disparity and shading without explicit disparity detection is examined based on an image generation consistency, and simple evaluation results are shown by simulations.

1 INTRODUCTION

There are various clues for depth recovery, for example, stereo, motion, texture, blur and shading, and using each clue, a lot of methods have been proposed for recovering a three dimensional (3-D) shape from images. For each clue, there is a condition under which depth recovery is theoretically impossible, and Poggio (Poggio et al., 1988) asserted the unification of various modules, each of which recovers depth based on a specific clue respectively, using an edge map computed by preprocessing. Namely, multiple depth maps obtained by all modules are incorporated into a final result of a depth map. This strategy is effective for the case where whole region can be partitioned so that in the local regions the suitable clue for accurate depth recovery exists respectively. However, in general, there are the regions where an accurate depth can not be recovered by a single certain clue and hence, a superior unification of clues is required.

Most clues except shading are the constraints between specific features detected from images and depth. These constraints can be used simply and efficiently for depth recovery, but sometimes the constraints are required to be improved by complicated ways. For example, a simple binocular disparity constraint is inadequate for occlusions and intensity inconsistency of a corresponding image pair caused by a difference of appearance from two views includ-

ing a specular reflectance, and various studies have been carried out (Lazaros et al., 2008). On the other hand, shading constraint essentially depends on complete image information instead of specific image features, and hence, in addition to the depth, other 3-D quantities including albedo should be also considered, although general shape from shading algorithm assumes that albedo multiplied by an intensity of a light source is known. From the above discussion, the shading clue is fundamental and exact as compared with the other clues, and it should be applied to depth recovery in distinction from the other clues. The similar concept has been argued (Hayakawa et al., 1994), but in this research especially the unification of shading and edge information was discussed and the computational scheme to reduce computation costs is mainly examined. In the following, we call the constraints except shading "feature-based clues." As described above, since the shading constraint has many quantities to be determined as a 3-D recovery, the shading constraint for multi-view images becomes important. Therefore, the shading clue with respect to various 3-D quantities with multi-view images should be called "image generation consistency" preferably.

We propose a strategy, in which the feature-based clues and the image generation consistency are adopted hierarchically. At first, various feature-based clues are applied to the pixels or the regions where these clues are effective respectively, and all results

are unified so as to obtain a partial or sparse depth map. Subsequently, using it as an initial value the image generation consistency is imposed on all information of observed multiple images to obtain a whole depth map and other 3-D quantities accurately.

In this study, as a first step of our research, we take up a simple problem, "shape from two-view images," and confirm the effectiveness of the image generation consistency. We suppose that initial values of 3-D quantities including a depth map are obtained by various feature-based clues, and in the numerical evaluation below, good initial values are given heuristically and are used. In future, we are going to develop the system in which various feature-based clues for obtaining rough and sparse 3-D quantities are actually used and the unification scheme proposed in this study is effectively performed.

The intensity of images used in the numerical evaluation consists of a diffuse reflectance and a specular reflectance. The strength of a diffuse reflectance and a specular reflectance are unknown relative to the strength of a parallel light source, but those are constant on an object. Therefore, we recover both strength using the length of a light source as a unit. The direction of a light source is also unknown and recovered. We recover a depth and the other 3-D quantities by the image generation consistency with two images. The degree of the unknown variables is larger than the number of observations of one image, i.e. a pixel number, hence it is worried that a unique solution cannot be determined by an usual shading analysis using only one image. For the case where only the diffuse reflectance exists, it was clarified that a two-way ambiguity appears (Brooks and Horn, 1985). Additionally, since there is no clear texture, an accurate binocular disparity detection is difficult. Namely, our strategy is expected to be needed to solve this problem accurately in spite of the simpleness of this problem.

The above simple algorithm evaluated in this study as a first step can be also regarded as a new unification method of the binocular disparity and the shading constraints. The most unification methods proposed recently adopt almost the same strategy that a stereo constraint is firstly used for specific image regions or points where disparity detection can be easily done to recover sparse depth map, and then a shading constraint is used for the other region where the shading constraint can be used suitably (Samaras et al., 2000). On the other hand, our algorithm does not use the binocular disparity constraint directly and the image generation consistency of two images is concerned to at most, although a disparity detection result can be used as an initial value. As the similar awareness of the issues, (Maki et al., 2002) proposed

a method based on the principle of the photometric stereo using known object motion, but in which only a shading and a motion are focused and a texture is not considered essentially. As against this, our strategy can deal with the distribution of albedo in principle, although, in this study, albedo is assumed to be constant.

2 SHADING CONSISTENCY FOR TWO-VIEWS

2.1 Formulation of Depth from Shading

Various shape from shading method have been examined (Zhang et al., 1999), (Szeliski, 1991), and almost are based on the image irradiance equation:

$$I(x,y) = R(\vec{n}(x,y)), \quad (1)$$

which represents that image intensity I at a image point (x,y) is formulated as a function R of a surface normal \vec{n} at the point (X,Y,Z) on a surface projecting to (x,y) in the image. General R contains other variables such as a view direction, a light source direction and albedo. These variables have to be determined in advance or simultaneously with the shape from images in general.

From the image irradiance equation, image intensity is uniquely determined by surface orientation not by surface depth. Most formulations of shape from shading problem have focused on determining surface orientation using the parameters (p,q) representing (Z_x, Z_y) , which is the first derivative of Z with respect to X and Y . Hence, we can express the shape from shading problem as solving for $p(x,y)$ and $q(x,y)$, with which the irradiance equation holds, by minimizing the following objective function.

$$J \equiv \int \{I(x,y) - R(p(x,y), q(x,y))\}^2 dx dy, \quad (2)$$

where $I(x,y)$ is an observed image intensity. However, this problem is highly under-constrained, and additional constraints are required to determine a particular solution, for example a smoothness constraint. Additionally, the solutions $p(x,y)$ and $q(x,y)$ will not correspond to orientations of a continuous and differential surface $Z(x,y)$ in general. Therefore, the post processing is required, which generates a surface approximately satisfying the constraint $p_y = q_x$, or (Horn, 1990) proposed the objective function including such a constraint implicitly.

To avoid these difficulties, we can represent $p(x,y)$ and $q(x,y)$ as a first derivative of $Z(x,y)$ explicitly and consider $R(p,q)$ as a function of $Z(x,y)$.

In addition, using the second derivatives $Z_{XX} = p_X$ and $Z_{YY} = q_Y$, Leclerc and Bobick (Leclerc and Bobick, 1991) proposed the following objective function for parallel projection,

$$J_{LB} \equiv (1 - \lambda) \int \{I(x, y) - R(Z_X(x, y), Z_Y(x, y))\}^2 dx dy + \lambda \int (Z_{XX}^2(x, y) + Z_{YY}^2(x, y)) dx dy, \quad (3)$$

and minimized it with a discrete representation of $Z(x, y)$ and its derivatives. The method in (Leclerc and Bobick, 1991) assumed only the Lambertian reflection as $R(Z_X, Z_Y)$. In the objective function, λ indicates a degree of smoothness required for $Z(x, y)$, and is initially set as 1 and is gradually decreased to near zero with a hierarchical coarse to fine technique using the multi-resolution image decomposition (Terzopoulos, 1983).

In (Leclerc and Bobick, 1991), since parallel projection is adopted, we can use the relations $Z_X = \partial Z / \partial x$ and $Z_Y = \partial Z / \partial y$. However, when we assume perspective projection, the relations $x = X/Z$ and $y = Y/Z$ have to be considered, and hence, the following formulations are required to be used (Wakabayashi et al., 2012).

$$\frac{\partial Z}{\partial X} = \frac{1}{Z} \frac{\partial Z}{\partial x}, \quad \frac{\partial Z}{\partial Y} = \frac{1}{Z} \frac{\partial Z}{\partial y}, \quad (4)$$

$$\frac{\partial^2 Z}{\partial X^2} = \frac{1}{Z^2} \frac{\partial^2 Z}{\partial x^2} - \frac{1}{Z^3} \left(\frac{\partial Z}{\partial x} \right)^2, \quad (5)$$

$$\frac{\partial^2 Z}{\partial Y^2} = \frac{1}{Z^2} \frac{\partial^2 Z}{\partial y^2} - \frac{1}{Z^3} \left(\frac{\partial Z}{\partial y} \right)^2. \quad (6)$$

Equation 4 can be represented in a discrete manner as follows:

$$Z_{X_{i,j}} = \frac{1}{2Z_{i,j}\delta x} (Z_{i+1,j} - Z_{i-1,j}), \quad (7)$$

$$Z_{Y_{i,j}} = \frac{1}{2Z_{i,j}\delta y} (Z_{i,j+1} - Z_{i,j-1}), \quad (8)$$

where δx and δy are the sampling intervals in an image respectively along x and y directions.

For Eqs. 5 and 6, the second term in the both equations can be omitted as compared with the first term in those, and hence, the discrete formulations are introduced as follows:

$$Z_{XX_{i,j}} = \frac{1}{Z_{i,j}^2 \delta x^2} (Z_{i+1,j} - 2Z_{i,j} + Z_{i-1,j}), \quad (9)$$

$$Z_{YY_{i,j}} = \frac{1}{Z_{i,j}^2 \delta y^2} (Z_{i,j+1} - 2Z_{i,j} + Z_{i,j-1}). \quad (10)$$

By evaluating Eq. 3 and minimizing it with the use of a coarse to fine strategy, we can determine a depth map for perspective projection.

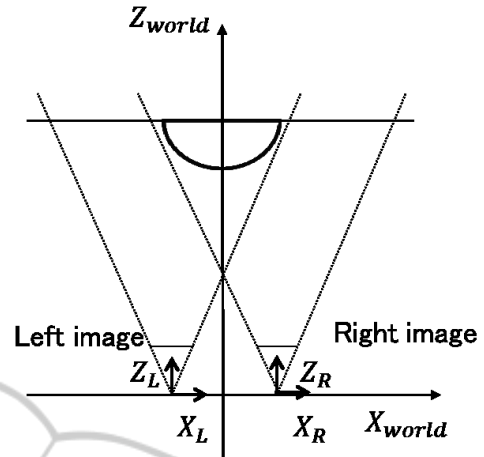


Figure 1: Two cameras coordinates and world coordinates used for imaging and recovering in evaluations.

2.2 Multi-view Consistency

We define an objective function J_{total} using two-view images corresponding to a left image and a right image,

$$J_{total} \equiv (1 - \lambda)(J_L + J_R) + \lambda J_{smooth}, \quad (11)$$

where each of J_L and J_R indicates the integrated value of the square errors of the image irradiance equations at the left camera and the right camera respectively, which corresponds to the integration part of the first term in the right-hand side of Eq. 3 with the perspective modifications Eqs. 7-10. J_{smooth} in this equation represents smoothness constraints, which corresponds to the second term without λ in the right-hand side of Eq. 3, but Z_{XX}^2 and Z_{YY}^2 are represented with the world coordinate system placed at the intermediate position of both camera's coordinates shown in Fig. 1. By using the world coordinates, we can deal with both of a left image and a right image equally to recover a depth map. This objective function should be minimized by varying the 3-D variables including a depth map and a direction of a light source and so on.

In our study, the dichromatic reflection model (S.A.Shafer, 1985) in which image intensity is defined by a linear sum of diffuse and specular reflections is adopted.

$$R = R_{diffuse} + R_{specular}. \quad (12)$$

To simplify the problem, we assume that the diffuse and the specular reflections are constant on the object, and the strength of both reflections are measured using the intensity of a light source as a unit. $R_{diffuse}$ indicates the diffuse reflection component. Using the strength of the diffuse reflectance K_d , a surface normal vector \vec{n} and a unit vector \vec{l} indicating the direc-

tion of a parallel light source, $R_{diffuse}$ can be formulated as follows:

$$R_{diffuse\ i,j} = K_d \vec{n}_{i,j} \cdot \vec{l}. \quad (13)$$

$R_{specular}$ indicates the specular reflection component. In this study, we apply the Phong's reflection model (Phong, 1975). In the same way, $R_{specular}$ is formulated with the strength of the specular reflectance K_s , a unit vector \vec{r} denoting the direction of the reflected light, a unit vector \vec{v} denoting the direction of a view point and α indicating a highlight factor as follows:

$$R_{specular\ i,j} = K_s (\vec{r}_{i,j} \cdot \vec{v}_{i,j})^\alpha. \quad (14)$$

In Eqs. 13 and 14, $(\vec{a} \cdot \vec{b})$ represents an inner product of \vec{a} and \vec{b} . In the formulation of $R_{i,j}$, $\{Z_{i,j}\}$, \vec{l} , K_d , K_s and α are concerned as unknown variables in this study. However, in general, $R_{i,j}$ can have various models and can include many unknown variables, for example, the albedo distribution of the diffuse reflection and the intensity of a light source. Such an extension is an indispensable future work in the framework of our strategy.

We discretize the X and Y axes of the world coordinates, and represent the depth as the Z value at the discretized (X, Y) position. To evaluate the value of J_{total} , the updated depth map defined in the world coordinate as the above way is projected to both camera images, and the depth values at all pixels in both images are calculated using an interpolation technique. We use Z_{cam} as the depth value corresponding to a certain pixel in the image of the camera. At each iteration, Z_{cam} is obtained with an adaptive interpolation as follows:

$$Z_{cam} = \sum_{i=1}^4 f_i Z_{world}^i, \quad (15)$$

where $\{Z_{world}^i\}_{i=1,\dots,4}$ means the depth values of the neighboring four 3-D points in the world coordinates, and f_i indicates the interpolation weight holding $\sum_{i=1}^4 f_i = 1$. In this study, we simply adopt a linear interpolation. Using also the updated other 3-D variables, subsequently the values J_L and J_R are computed respectively and those are summed up with a same weight.

We minimize J_{total} with decreasing λ from 1.0 to 0.0 using a coarse to fine technique based on a hierarchical multi-resolution decomposition of images. Minimization for each λ is performed by the conjugate gradient method. In our minimization, the variables to be recovered, i.e. $\{Z_{i,j}\}$, \vec{l} , K_d , K_s and α , are updated reciprocally and individually. This minimization procedure is repeated until convergence. To lower J_{total} repeatedly, we need derivatives with respect to the each variable. Note that the exact differentiation with respect to the depth is complicated,

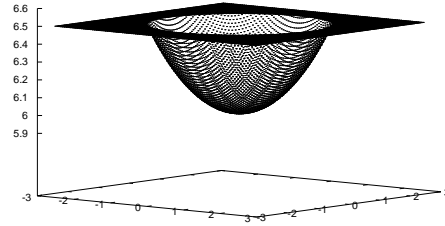


Figure 2: True depth map using Z values represented with world coordinates.

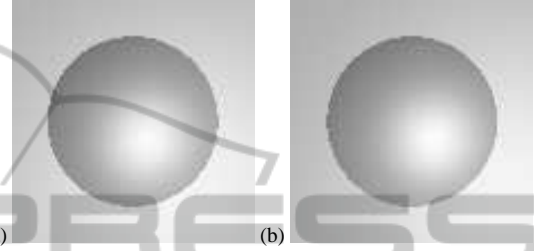


Figure 3: Artificially generated image using the imaging model shown in Fig. 1: (a) left camera image; (b) right camera image.

since the depth values explicitly appearing in J_L and J_R are the functions of the each camera's coordinates. To differentiate J_{total} with respect to Z_{world}^i , the following computation is required.

$$\frac{\partial J_{total}}{\partial Z_{world}^i} = (1 - \lambda) \left\{ \sum_{k=1}^{K_L} f_{L_i}^k \frac{\partial J_L}{\partial Z_{Lcam}^k} + \sum_{k=1}^{K_R} f_{R_i}^k \frac{\partial J_R}{\partial Z_{Rcam}^k} \right\} + \lambda \frac{\partial J_{smooth}}{\partial Z_{world}^i}, \quad (16)$$

where Z_{Lcam}^k is the depth corresponding to the pixel in the left camera image and is interpolated using Z_{world}^i , K_L indicates the number of Z_{Lcam}^k , and $f_{L_i}^k$ is the interpolation weight of Z_{world}^i for Z_{Lcam}^k . Z_{Rcam}^k , K_R and $f_{R_i}^k$ are defined in the same way. It is noted that, in this study, there are no rotation between both camera coordinates, and hence the light source direction \vec{l} is common to both images. Therefore, the derivative of J_{total} with respect to \vec{l} needs no special techniques. The other variables also can be derivative straightforwardly.

3 NUMERICAL EVALUATIONS

3.1 Evaluation Methods

We used a very simple target object, i.e. a hemisphere on the flat board placed perpendicular to an optical axis. The coordinate systems including two cameras

and the the world coordinates are shown in Fig. 1. The true depth map used in the evaluations is shown in Fig. 2. The parallel light source with the direction $\vec{l} = (0.236, 0.236, -0.943)$ irradiates the object and cameras are assumed as pin-hole cameras. The strengths of both reflectances are defined as $K_d = xx$ and $K_s = xx$, and the highlight factor $\alpha = xx$.

In Fig. 3(a), an artificially generated image for the left camera under this condition. When this image is only watched, an optical illusion occurs generally, and human can recognize also spurious shape and light direction. On the other hand, in Fig. 3(b) shows the image generated for the right camera. To generate these test figures, the surface normals $\vec{n}_{i,j}$ s are calculated analytically and used in Eq. 13, but when we recover the 3-D variables, the first and the second differentials are calculated in a discrete manner using Eqs. 7-10. This discrepancy causes the error for the image radiance equation defined by Eq. 1, although no intensity noise is added explicitly. The spatial distance between two cameras are set as 1 using a focal length of the cameras as a unit. Since the both image include binocular disparity information, using the two images we can recover the shape, and hence using the recovered shape and images the light source direction can be also determined. However, it is noted that explicit disparity matching between these images is difficult because of existing specular reflection, and hence only the poor depth recovery is performed. This means that, in the evaluations, we can examine the case where only the poor initial values are used.

Using these two figures, we define J_{total} in Eq. 11 and minimize it to recover 3-D information, specifically a depth map, a light source direction, a highlight factor related to a specular reflectance and strengths of a diffuse reflectance and a specular reflectance using an intensity of a light source as a unit. In the recovery processing, as initial values, we use a plane depth $Z = 6.5$, a light source direction parallel to optical axis $\vec{l} = (0.0, 0.0, -0.1)$, strengths of both reflectances $K_d = 0.5$, $K_s = 0.5$ and highlight factor $\alpha = 2.0$, and adopt 4 layer multi-resolution decomposition of images. Those initial values are determined heuristically.

3.2 Evaluation Results

The depth maps for each hierarchical stage recovered from two views are shown in Fig. 4. The RMSE of the recovered depth at 4th layer is 9.85×10^{-3} , the inner product of recovered \vec{l} and true value is 1.00, the relative errors of the other parameters are respectively $K_d = 2.47 \times 10^{-2}$, $K_s = 3.89 \times 10^{-2}$ and $\alpha = 6.43 \times 10^{-2}$. In Fig. 5, the re-generated images

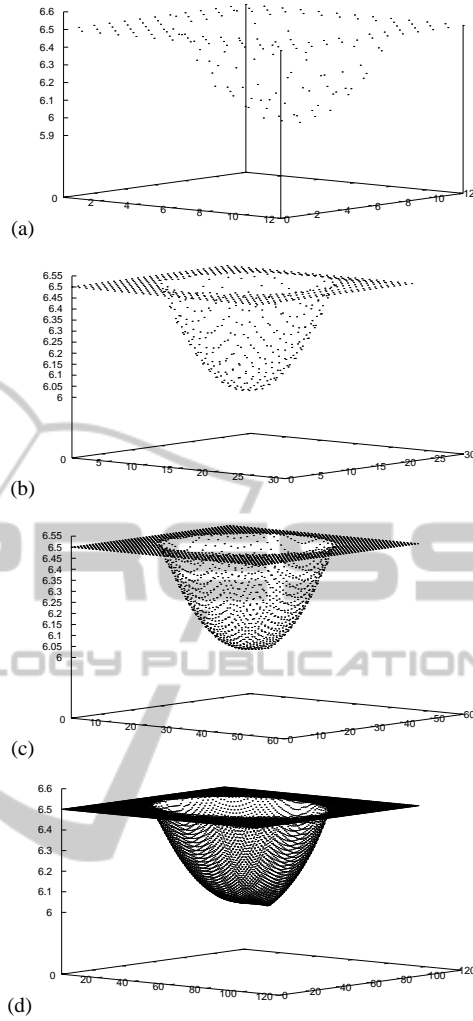


Figure 4: Recovered depth from two-view images: (a) at first layer with 13×13 pixels; (b) at 2nd layer with 27×27 pixels; (c) at 3rd layer with 54×54 pixels; (d) at 4th layer with 108×108 pixels.

using the recovered 3-D quantities for the both camera are shown, and the RMSEs of those and input images are 6.80×10^{-3} for the left camera and 6.51×10^{-3} for the right camera. From these results, unknown quantities related to an image generation, such as a depth, can be recovered using the image generation consistency from multi-views despite the difficulty of disparity detection caused by specular reflection.

Subsequently, we confirm usefulness of using multi-view images. Fig. 6 shows the depth map recovered by using only the right camera image and defining objective function with $(1 - \lambda)J_R + \lambda J_{smooth}$. The RMSE of the recovered depth is 4.54×10^{-2} , the inner product of recovered \vec{l} and true value is 0.990, the relative errors of the other parameters are respectively $K_d : 8.65 \times 10^{-2}$, $K_s : 7.11 \times 10^{-1}$ and $\alpha : 6.28 \times 10^{-1}$.

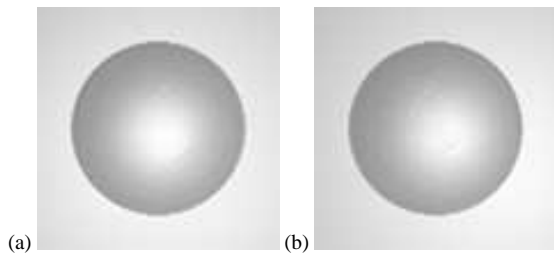


Figure 5: Re-generated image using 3-D quantities recovered from two-view images: (a) left camera image; (b) right camera image.

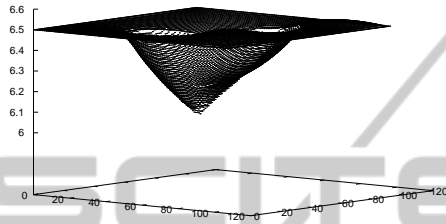


Figure 6: Recovered depth from only right camera image.

The errors are larger for the case of using two-view images. By comparing the minimized value of the objective function, J_{total} corresponding to the result in Fig. 4(d) divided by 2 is 2.65×10^{-4} and J_R is 2.53×10^{-4} . It is noted that $\lambda = 0$ when the objective function is minimized finally. This means that the objective function for only the right camera is minimized to the the same level with the objective function for the both camera. The RMSE of the re-generated image for the right camera and input image is 9.19×10^{-3} , hence it is shown in Fig. 7 (b) that the recovered 3-D quantities generate the image sufficiently close to the input image of the right camera. Fig. 7 (a) shows the left camera image generated by the recovered 3-D quantities from only the right camera image, and the RMSE of that and the original left camera image is 4.16×10^{-2} . This result denotes that the image generation constraint from only one-view point does not have enough information, and the image generation consistency from multi-view prevents falling an erroneous solution.

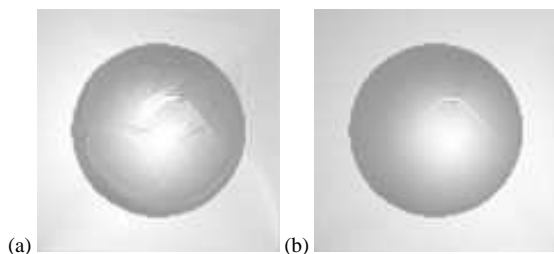


Figure 7: Re-generated image using 3-D quantities recovered only from right camera image: (a) left camera image; (b) right camera image.

4 CONCLUSIONS

In this paper, we discussed the unification strategy of various clues for shape from X. The feature-based clues are desirable for mainly obtaining the initial values for the unification processing based on an image generation constraint. The unification strategy proposed in this paper mainly consists of forward computations, i.e. computer graphics computations. On the other hand, the methods using the feature-based clues can be considered to be used for solving the inverse problems. Hence, it is necessary that a wide and profound discussion about the way for combining a feature-based method and an image generation constraint method. To minimize the objective function using an image generation consistency, which is strongly asserted in this study, effectively and/or stably, the feature-based clues may be used powerfully to update the values to be determined.

In this paper, we showed only that the image generation consistency can be used for the problem which can not be solved uniquely or accurately by each of the usual shading method and the disparity detection individually. Our strategy is expected to be powerful especially if there are the vague distributions of a diffuse and a specular reflectance, in which a binocular disparity detection is drastically difficult and hence, the unification based on the image generation consistency is increasingly effective to update the rough and bad initial depth. Now, we are examining and generating the algorithm for this situation based on the image generation constraint.

Additionally, we can expand our multi-view strategy toward a temporal processing. By using image sequence, the previously recovered 3-D quantities can be known and only the change between frames should be computed with small computation costs. The Kalman filter technique can also be applied and hence, it is expected that the reliability of the recovered 3-D quantities increases over time.

REFERENCES

- Brooks, M. J. and Horn, B. K. P. (1985). Shape and source from shading. In *proc. Int. Joint Conf. Art. Intell.*, pages 18–23.
- Hayakawa, H., Nishida, S., Wada, Y., and Kawato, M. (1994). A computational model for shape estimation by integration of shading and edge information. *Neural Networks*, 7(8):1193–1209.
- Horn, B. K. P. (1990). Height and gradient from shading. *Int. J. Computer Vision*, 5(1):37–75.
- Lazaros, N., Sirakoulis, G. C., and Gasteratos, A. (2008).

- Review of stereo vision algorithm: from software to hardware. *Int. J. Optomechatronics*, 5(4):435–462.
- Leclerc, Y. G. and Bobick, A. F. (1991). The direct computation of height from shading. In *proc. CVPR '91*, pages 552–558.
- Maki, A., Watanabe, M., and Wiles, C. (2002). Geotensity: combination motion and lighting for 3d surface reconstruction. *Int. Journal of Comput. Vision*, 48(2):75–90.
- Phong, B. T. (1975). Illumination for computer generated pictures. *Communication of the ACM*, 18(6):311–317.
- Poggio, T., Gamble, E. B., and Little, J. J. (1988). Parallel integration of vision modules. *Science*, 242:43–440.
- Samaras, D., Metaxas, D., Fua, P., and Leclerc, Y. G. (2000). Variable albedo surface reconstruction from stereo and shape from shading. In *proc. Int. Conf. CVPR*, volume 1, pages 480–487.
- S.A.Shafer (1985). Using color to separate reflection components. *Color Research and Application*, 10(4):210–218.
- Szeliski, R. (1991). Fast shape from shading. *CVGIP: Image Understanding*, 53(2):129–153.
- Terzopoulos, D. (1983). Multilevel computational processes for visual surface reconstruction. *Computer Vision, Graphics, and Image Processing*, 24:52–96.
- Wakabayashi, K., Tagawa, N., and Okubo, K. (2012). Direct computation of depth from shading for perspective projection. In *proc. Int. Conf. Comput. Vision, Theory and Applications*, pages 445–448.
- Zhang, R., Tsai, P.-S., Cryer, J. E., and Shah, M. (1999). Shape from shading: A survey. *IEEE Trans. PAMI*, 21(8):690–706.