A Fuzzy-stochastic Inventory Model without Backorder under Uncertainty in Customer Demand

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Abstract: In the current business scenario, a vital aspect of a realistic inventory model is to accurately estimate the customer demand especially in uncertain environment. Keeping this fact in mind recent trend of research includes uncertain demand, either random or fuzzy. In this paper, we amalgamate both random behavior and fuzzy perception into the optimization setting in modeling an inventory model without backorder. Treating customer demand as fuzzy random variable, we aim at providing an approach of modeling uncertainty that is closer to real situations. In addition, a distinct characteristic of this study is that the decision maker's degree of optimism is incorporated in this model using possibilistic mean value approach. The objective is to determine the optimal order quantity associated with cost minimization. An illustrative numerical example is presented to clarify the reality of the model.

1 INTRODUCTION

Decision-making environment of inventory management in retail supply chain is full of uncertainties especially when dealing with end customer demand of innovative and style goods. The uncertainty is mainly because of vague, uncertain and volatile demand for these products coupled with short selling season. Shrinking product lifecycles and intensifying competitive pressure add to the difficulty. Therefore, it is a real challenge for the retailers or the decision-makers to determine customer demand for a plan period T. Again, inventory problems in realistic situation are too complex to be represented in mathematical models. On this view, many researchers have developed fuzzy inventory models for situations where the customer demand is described linguistically like "demand is about d" ((Chen and Wang, 1996); (Hsieh, 2002); (Dutta et al., 2007a); (Dutta and Chakraborty, 2010); (Dutta et al., 2012)). Jing-Shing Yao and his group have presented several papers on fuzzy inventory without backorders. For instance, Lee and Yao (1999a, b) fuzzified the order quantity using triangular and trapezoidal fuzzy numbers and obtained the fuzzy total cost. They used centroid method to defuzzify the total cost to determine the economic order quantity. Again, Yao et al. (2000) fuzzified the order quantity and total demand with triangular fuzzy numbers and obtained total cost using extension principle and centroid method. Yao and Chiang (2003) presented a fuzzy inventory model without backorders where they fuzzified the total demand and storing cost and defuzzified the total inventory cost using centroid and signed distance method.

In all these papers, the authors have addressed the customer demand as a fuzzy number which is characterized by the phrase "demand is about d". But, problem arises when this linguistic information varies randomly. For example, the prediction of the future demand forecast varies from expert to expert and is described by the phrases "demand is about d_1", "demand is about d_2", etc. Stochastic variation is presented due to the difficulty to predict with precision and fuzzy sets enter into the figure because the above mentioned phrases are subjective and only partially quantifiable. In this case, fuzzy random variable (FRV) provides an appropriate mathematical tool to handle such situations. The concept and characteristics of FRV are available in Lopez-Diaz and Gil (1998), Feng et al. (2001) and Luhandjula (2004). An application of FRV in the field of inventory system can be found in Dutta et al. (2005, 2007b), Chang et al. (2006), Dutta and Roy (2007) and Nagare and Dutta (2012).

However, an inventory model *without backorder*

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under customer demand uncertainty arising out of both fuzziness and randomness under one roof is yet to receive attention. The purpose of this paper is to redefine the fuzzy inventory model without backorders (Yao and Chiang, 2003) in a mixed fuzzy-stochastic environment by incorporating customer demand as FRV. Moreover, in addition to the synergistic approach of demand, a decision-maker's (DM) attitudinal scale is employed in defuzzifying the cost function. Main thrust of this paper is to determine the optimal order quantity that minimizes associated total cost. The model is designed using possibilistic mean value of a fuzzy number proposed by Carlsson and Fuller (2001).

The rest of the paper is organized as follows. In Section 2, we present the mathematical model without backorder in presence of FRV. A detailed solution methodology is developed in Section 3. A sensible numerical example is provided in Section 4 to illustrate the model. Finally, Section5 summarizes the work done.

2 MATHEMATICAL MODEL

The following notations have been adopted to develop the inventory model without backorder under fuzzy-stochastic environment.

T length of the finite planning period (in days) in which the inventory system operates
h the cost of storing one unit per day
A the cost of placing an order
d total customer demand over the planning time period $[0,T]$
t_Q length of the cycle
Q order quantity per cycle

The model is developed for a single item which is replenished, stored and consumed. Therefore, from Figure 1 it can have

$$\frac{Q}{t_Q}=\frac{d}{T} \text{ or } \frac{d}{Q}\times t_Q=T$$

The total cost in the planning time period $[0, T]$ consists of inventory holding cost, ordering cost and it can be written in the following form

$$TC(Q)=\left(ht_Q\frac{Q}{2}+A\right)\frac{d}{Q} \\ =\frac{1}{2}hTQ+\frac{Ad}{Q}, Q>0 \tag{1}$$

Using the classical approach, one can obtain the optimal order quantity $Q^0=\sqrt{\frac{2Ad}{hT}}$ and total minimum cost $TC(Q^0)=\sqrt{2hAdT}$. The Figure 1 shows the inventory without backorder.

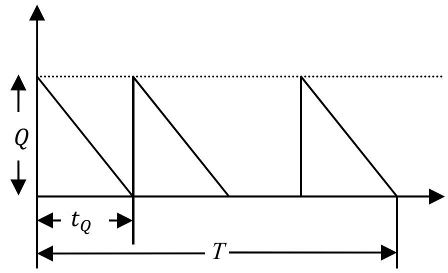

Figure 1: Inventory without backorder.

Many a times, due to uncertainty or vagueness, customer demand is prescribed in linguistic expressions like "demand is about d" and it can be characterized as a fuzzy number $\tilde{d}$(say). In such cases, optimization is attained using centroid or signed distance method as proposed by Yao and Chiang (2003). When subjective demand expressions vary from expert to expert randomly, demand can be aptly described as fuzzy random. Restated, the customer demand is treated as FRV. This paper assumes all the (fuzzy) observations of FRV as triangular fuzzy numbers. This consideration does not restrict the solution procedures for other fuzzy numbers.

The customer demand denoted as $\widetilde{D}$ assumes values on set of all triangular fuzzy numbers. Suppose $\tilde{d}_i$s $(i=1 \; to \; n)$ are the fuzzy observations of $\widetilde{D}$ with the given probability p_i, i.e.; $\{(\tilde{d}_1,p_1),(\tilde{d}_2,p_2),\dots,(\tilde{d}_n,p_n)\}$

Let $\mu_{\tilde{d}_i}$ denote the membership function of $\tilde{d}_i$ where

$$\mu_{\tilde{d}_i(x)}=\begin{cases} L_i(x)=\left(\dfrac{x-\underline{d}_i}{d_i-\underline{d}_i}\right), & \underline{d}_i\le x\le d_i, \\ R_i(x)=\left(\dfrac{\overline{d}_i-x}{\overline{d}_i-d_i}\right), & d_i\le x\le \overline{d}_i, \\ 0, & otherwise \end{cases}$$

with $[\underline{d}_i,\overline{d}_i]$ as the support of each $\tilde{d}_i$. Here d_i is the modal of fuzzy number $\tilde{d}_i$;$L_i,R_i:\Re\rightarrow[0,1]$ are the left and right shape continuous functions.

Thus, incorporating customer demand as FRV $\widetilde{D}$ in equation (1), the total cost in the fuzzy sense is given by

$$\widetilde{TC}(Q)=\frac{1}{2}hTQ+\frac{A\widetilde{D}}{Q} \tag{2}$$

Following proposition flows from this equation:

Proposition 1. *Total cost function defined in equation* (2) *itself is a FRV.*

Proof. It can be recalled that a FRV associated with a random experiment is an appropriate formulization of a process assessing a fuzzy value to each experimental outcome. Here the total cost is a function of uncontrollable variable $\widetilde{D}$. Again, the expert's opinions about $\widetilde{D}$ are linguistic, i.e., fuzzy. Therefore, for each values of $\tilde{d}_i$ of $\widetilde{D}$ there is a corresponding fuzzy value of $\widetilde{TC}$ with probability p_i. Consequently, $\widetilde{TC}$ becomes a fuzzy valued random variable. Restated, $\widetilde{TC}$ is also a FRV.

3 SOLUTION PROCEDURE

Corresponding to the crisp expected value of a positive classical random variable, the expectation of FRV is a unique fuzzy number. In other words, the fuzzy expected value summarizes central tendency of FRV. Therefore, total expected cost of $\widetilde{TC}(Q)$ becomes a fuzzy quantity on $\mathfrak{R}$. Let $\widetilde{ETC}$=$E(\widetilde{TC})$ be the fuzzy expected value of $\widetilde{TC}(Q)$.

The main problem is to determine the optimal policy under the synergetic approach of customer demand into the optimization setting. Decision policy is to find the optimal order quantity Q^* that minimizes the associated total inventory cost. Since the total expected cost is a fuzzy quantity, we first find out several α −level set of $\widetilde{ETC}$ and then using possibilistic mean value method the fuzzy quantity $\widetilde{ETC}$ is ranked.

Let us introduce the following lemma.

Lemma 1. *For $\widetilde{ETC} \in F$, let ETC_α^- and ETC_α^+ be the lower and upper endpoints of α −level set $[ETC]_\alpha$, respectively. Then*

i) ETC_α^- *is a left continuous nondecreasing function on* $(0,1]$ *and right continuous at* 0,

ii) ETC_α^+ *is a left continuous non-increasing function on* $(0,1]$ *and right continuous at* 0,

iii) $ETC_\alpha^- \leq ETC_\alpha^+$; $0 \leq \alpha \leq 1$.

Proof. Since the expected total cost $\widetilde{ETC}$ is a fuzzy number belonging to F (set of fuzzy numbers) and a fuzzy number is a fuzzy set with a normal, convex and continuous membership function of bounded support, the proof is straightforward.

Then, we get the following proposition.

Proposition 2. *If $\widetilde{ETC}$ be the fuzzy expected value of $\widetilde{TC}$ then for each values of $\alpha \in [0,1]$, $[ETC]_\alpha = [E(TC_\alpha^-), E(TC_\alpha^+)]$ is a closed bounded interval and we have*

i) *the lower end point of* $[ETC]_\alpha$ *as*

$$E(TC_\alpha^-) = \sum_{i=1}^n [\tfrac{1}{2}hTQ + \tfrac{A}{Q}\{\underline{d}_i + (d_i - \underline{d}_i)\alpha\}]p_i$$

ii) *the upper end point of* $[ETC]_\alpha$ *as*

$$E(TC_\alpha^+) = \sum_{i=1}^n [\tfrac{1}{2}hTQ + \tfrac{A}{Q}\{\overline{d}_i - (\overline{d}_i - d_i)\alpha\}]p_i$$

Proof. Since $\widetilde{ETC}$ is a fuzzy quantity, using Lemma 1, we obtain the α −level set of $\widetilde{ETC}$ as

$$\begin{aligned}[ETC]_\alpha &= E[TC_\alpha] \\ &= [ETC_\alpha^-, ETC_\alpha^+] \\ &= [E(TC_\alpha^-), E(TC_\alpha^+)]\ ; \quad 0 \leq \alpha \leq 1.\end{aligned}$$

At $\alpha = 0$, $[E(TC_0^-), E(TC_0^+)]$ is the support of fuzzy expected value $\widetilde{ETC}$

Now, $\widetilde{D} = \{(D_\alpha^-, D_\alpha^+)\ ;\ 0 \leq \alpha \leq 1\}$ is a FRV, therefore, from the properties of FRV, TC_α^- and TC_α^+ are the crisp random variables (i.e.; measurable functions) for each $\alpha \in [0,1]$ and are respectively given by

$$TC_\alpha^- = \frac{1}{2}hTQ + \frac{AD_\alpha^-}{Q}$$

and

$$TC_\alpha^+ = \frac{1}{2}hTQ + \frac{AD_\alpha^+}{Q}$$

Thus according to the crisp probability theory, $E(TC_\alpha^-)$ and $E(TC_\alpha^+)$ can easily be found for different values of $\alpha\,(0 \leq \alpha \leq 1)$. The exact expressions obtained for $E(TC_\alpha^-)$ and $E(TC_\alpha^+)$, are given below:

$$\begin{aligned}E(TC_\alpha^-) &= \sum_{i=1}^n \left(\frac{1}{2}hTQ + \frac{A}{Q}d_{i,\alpha}^-\right)p_i \\ &= \sum_{i=1}^n \left(\frac{1}{2}hTQ + \frac{A}{Q}L_i^-(\alpha)\right)p_i \\ &= \sum_{i=1}^n [\frac{1}{2}hTQ + \frac{A}{Q}\{\underline{d}_i + (d_i - \underline{d}_i)\alpha\}]p_i\end{aligned} \tag{3}$$

and

$$\begin{aligned}E(TC_\alpha^+) &= \sum_{i=1}^n \left(\frac{1}{2}hTQ + \frac{A}{Q}d_{i,\alpha}^+\right)p_i \\ &= \sum_{i=1}^n [\tfrac{1}{2}hTQ + \tfrac{A}{Q}\{\overline{d}_i + (\overline{d}_i - d_i)\alpha\}]p_i\end{aligned} \tag{4}$$

This completes the proof.

Now, we calculate the possibilistic mean value of $\widetilde{ETC}$ as a function of order quantity Q, say $\overline{M}_\lambda(Q)$, and then optimize $\overline{M}_\lambda(Q)$ to determine the optimal order quantity Q^*.

Thus, using (3) the lower possibility-weighted average of the minima of the α −sets of $\widetilde{ETC}$ is given by

$$M_*(\widetilde{ETC}) = 2\int_0^1 \alpha E(TC_\alpha^-)d\alpha$$

$$= \frac{1}{2}hTQ + 2\frac{A}{Q}\sum_{i=1}^{n}[\frac{\underline{d}_i p_i}{6} + \frac{d_i p_i}{3}]$$

and using (4) the upper possibility-weighted average of the maxima of the α −sets of $\widetilde{ETC}$ is given by

$$M^*(\widetilde{ETC}) = 2\int_0^1 \alpha E(TC_\alpha^+)d\alpha$$

$$= \frac{1}{2}hTQ + 2\frac{A}{Q}\sum_{i=1}^{n}[\frac{\overline{d}_i p_i}{6} + \frac{d_i p_i}{3}]$$

Therefore, the interval-valued possibilistic mean of total expected annual cost is defined as $[M_*(\widetilde{ETC}), M^*(\widetilde{ETC})]$.

As $\widetilde{ETC}$ is a fuzzy quantity and its expected representative depends on the attitude (viz. optimism or pessimism) of the DM. In order to calculate a defuzzified value or mean value of fuzzy 'expected total cost', starting from a subjective assignation related to the relative importance of the lower and upper possibilistic mean of total expected cost, one could define a parameter $\lambda \in [0,1]$, which reflects DM's degree of optimism.

Therefore, the expected representative of $\widetilde{ETC}$ as a function of order quantity Q is obtained by

$$\overline{M}_\lambda(Q) = \lambda M_*(\widetilde{ETC}) + (1-\lambda)M^*(\widetilde{ETC}),$$

$$= \frac{1}{2}hTQ + \frac{2A}{3Q}\sum_{i=1}^{n} d_i p_i + \frac{A}{3Q}[\sum_{i=1}^{n}(\lambda\underline{d}_i + (1-\lambda)\overline{d}_i)p_i]$$

where the parameter λ is selected by a DM. Between the two extreme values of $\lambda = 0$ and $\lambda = 1$ there is an attitude scale for the uncertainty for each DM. Using the attitudinal values, the optimal Q^* is computed that minimizes the associated total cost in fuzzy sense. Equating the first order derivatives of $\overline{M}_\lambda(Q)$ with respect to Q to zero, we obtain

$$Q^* = \sqrt{A[\sum_{i=1}^{n}(2d_i + \lambda\underline{d}_i + (1-\lambda)\overline{d}_i)p_i] \Big/ \tfrac{3}{2}hT} \quad (5)$$

Since,

$$\frac{d^2}{dQ^2}\overline{M}_\lambda(Q) = \frac{2A}{3Q^3}[\sum_{i=1}^{n}(2d_i + \lambda\underline{d}_i + (1-\lambda)\overline{d}_i)p_i]$$

is positive, so $\overline{M}_\lambda(Q^*)$ is the minimum and is given by

$$\overline{M}_\lambda(Q^*) = \frac{1}{2}hTQ^* + \frac{2A}{3Q^*}\sum_{i=1}^{n} d_i p_i + \frac{A}{3Q^*}[\sum_{i=1}^{n}(\lambda\underline{d}_i + (1-\lambda)\overline{d}_i)p_i] \quad (6)$$

Equation (6) is the predicted total inventory cost in the possibilistic sense with an index of optimism $\lambda \in [0,1]$.

4 NUMERICAL EXAMPLE

The numerical example pertains to an Indian retailer of *ethnic fashion apparels* for women that combine the ethnic tastes with western styles making it attractive to educated Indian Women. The product is a style good-pair of salwar kameez with V-Neck and introduced every season.

The demand for this product is rather vague and uncertain for reason of its newness. Monetary unit is changed from rupee into Pound sterling with exchange rate of Rs 80/£. Required information is provided in Table 1.

Table 1: Input values of model parameters.

Parameter	Value
Selling Price (*P*)	100£
Inventory holding cost (*h*)	0.08 £/unit/day
Ordering cost (*A*)	125£/order
Planned season duration (*T*)	120 days
Average daily demand	20 units (approx.)

Demand estimation is obtained from thirty two experts. These estimates were in the form of linguistic expressions or in numerical form (minimum, mean and maximum). The linguistic expressions like "demand around 2400" were transformed into numerical data and segregated in five classes and probability is calculated. Table 2 provides the feature of $\widetilde{D}$.

Table 2: Input data of customer demand $\widetilde{D}$.

Demand	around 1800	around 2100	around 2400	around 2700	around 3000
Experts	5	6	11	7	3
Prob.	0.16	0.19	0.34	0.22	0.09

These fuzzy observations are transformed in to triangular fuzzy numbers as follows:

"around 1800" = $\tilde{d}_1$ = (1500,1800,2100)
"around 2100" = $\tilde{d}_2$ = (1800,2100,2400)
"around 2400" = $\tilde{d}_3$ = (2100,2400,2700)
"around 2700" = $\tilde{d}_4$ = (2400,2700,3000)
"around 3000" = $\tilde{d}_5$ = (2700,3000,3300)

In the following paragraphs, we first discuss variations of order quantity and expected cost as a function of the DM attitude parameter λ and then the optimal solutions as presented in Table 2.

(1) Optimistic Scenario ($\lambda = 1.0$): In this situation, the DM is absolutely optimistic for the estimation of expected total cost, which reflects the least possible cost $\overline{M}_{\lambda=1.0}(Q^*)$ and using the results of equations (5) and (6), we get the optimal decision as follows:

$$\overline{M}_{\lambda=0.5}(Q^*) = \frac{1}{2}hTQ^* + \frac{A}{3Q^*}\sum_{i=1}^{n}(2d_i + \underline{d}_i)p_i$$

along with

$$Q^* = \sqrt{A\left[\sum_{i=1}^{n}(2d_i + \underline{d}_i)p_i\right]/\tfrac{3}{2}hT}.$$

(2) Moderate Scenario ($\lambda = 0.5$): In this situation, the DM is moderately optimistic for the estimation of expected total cost reflecting in a crisp representative of $\widetilde{ETC}$ provided by

$$\overline{M}_{\lambda=0.5}(Q^*) = \frac{1}{2}hTQ^* + \frac{A}{6Q^*}\sum_{i=1}^{n}(4d_i + \underline{d}_i + \overline{d}_i)p_i$$

along with

$$Q^* = \sqrt{A\left[\sum_{i=1}^{n}(4d_i + \underline{d}_i + \overline{d}_i)p_i\right]/3hT}.$$

(3) Pessimistic Scenario ($\lambda = 0.0$): This situation provides an absolutely pessimistic decision viewpoint. In this case, the choice of the expected total cost $\overline{M}_{\lambda=0.0}(Q^*)$ can be put forth as

$$\overline{M}_{\lambda=0.0}(Q^*) = \frac{1}{2}hTQ^* + \frac{A}{3Q^*}\sum_{i=1}^{n}(2d_i + \underline{d}_i)p_i$$

along with

$$Q^* = \sqrt{A\left[\sum_{i=1}^{n}(2d_i + \overline{d}_i)p_i\right]/\tfrac{3}{2}hT}.$$

From above results, it is clear that for an absolutely optimistic ($\lambda = 1$) DM, the expected total cost is the lowest and hence the optimistic scenario should be selected. In this case, the optimal order quantity works to Q^* = 242.97 and minimum total expected cost $\overline{M}_{\lambda=1.0}$ = 2332.55. On the other hand, for a pessimistic DM ($\lambda = 0$), the optimal order quantity is Q^*= 253.46 along with expected cost of $\overline{M}_{\lambda=0.0}$= 2433.27.

For a DM with moderate attitudinal scale, optimal order quantity and expected cost are 248.27 and 2383.44 respectively. Results for different values of λ are given in Table 3.

Table 3: Optimal solutions for different values of λ

λ	Q^*	Expected cost
0.0	253.46	2433.26
0.1	252.43	2423.38
0.2	251.40	2413.46
0.3	250.36	2403.49
0.4	249.32	2393.49
0.5	248.27	2383.44
0.6	247.22	2373.35
0.7	246.17	2363.21
0.8	245.10	2353.04
0.9	244.04	2342.81
1.0	242.97	2332.55

The percentage change in expected total cost (PCEC) with extreme values of λ as compared with the moderate scenario is computed as

$$PCEC = 100 \times [\overline{M}_{\lambda\in[0,1]} - \overline{M}_{\lambda=0.5}]/\overline{M}_{\lambda=0.5}$$

It yields -2.13% changes at $\lambda = 1$ and +2.09% changes at $\lambda = 0$.

5 CONCLUSIONS

The current paper has considered a common inventory model without backorder and presented a fuzzy-stochastic inventory model where both fuzziness and randomness are considered under one roof. Since the expected value of a FRV is a fuzzy quantity, a method of ranking fuzzy numbers using their possibilistic mean values is adopted to find the optimal order quantity that minimizes associated cost. It is observed that results obtained in comparable situations are numerically closer to the well-known classical results. The paper consider all fuzzy observations of customer demand as triangular fuzzy numbers and on that basis the model is developed as an interactive decision making problem. It is important to note that the proposed methodology is capable of providing optimal solution even for fuzzy observations represented by trapezoidal fuzzy numbers or by *s*-curves. Moreover, incorporating the attitudinal parameter $\lambda \in [0,1]$ reflecting DM's degree of optimism offers more flexibility in decision making to a DM, required in a real world.

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