

Sampling based Optimum Signal Detection in Concentration Encoded Molecular Communication

Receiver Architecture and Performance

Mohammad Upal Mahfuz, Dimitrios Makrakis and Hussein T. Mouftah

School of Electrical Engineering and Computer Science, University of Ottawa, Ontario, K1N6N5, Canada

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Abstract: In this paper for the first time ever a comprehensive analysis of the sampling-based optimum signal detection in diffusion-based binary concentration-encoded molecular communication (CEMC) system has been presented. A generalized amplitude shift keying (ASK) based CEMC system has been considered in diffusion-based noise and inter-symbol interference (ISI) conditions. We present an optimum receiver architecture of sampling-based signal detection, address the critical issues in signal detection, and evaluate its performance in terms of sampling number, communication range, and transmission data rate. ISI produced by the residual molecules deteriorates the performance of the CEMC system significantly, which is further deteriorated when the communication distance and/or the transmission data rate increase(s). The proposed receiver architecture can also be used to detect multilevel (M-ary) amplitude modulated signals by increasing the alphabet size and changing the modulation format.

1 INTRODUCTION

Nanotechnology has recently brought several research fields into a common ground in order to realize new and emerging communication paradigm of molecular communication (MC) (Nakano et al., 2012) through offering many potential applications involving nanonetworks, e.g. immune system support, bio-hybrid implants, targeted drug delivery in cancer treatment, health monitoring, and genetic engineering (Akyildiz et al., 2008). Nanomachines are tiny natural or engineered natural biological or artificial machines with dimensions in the nanometre to micrometre scale having at least one dimension in the range from 1 nm to 100 nm. 1 nm is a billion-th (i.e. 10^{-9}) of a metre. Concentration-encoded molecular communication (CEMC) system has been discussed in detail in several of our previous works (Mahfuz et al., 2010b). CEMC system uses only a single type of information molecules and the TN encodes information by modulating the amplitude of the transmission rate of the input signal. The molecules thus released by the TN undergo ideal diffusion-based propagation. The RN decodes the information by observing the concentration of the molecules available at the location of the RN. Signal

detection in CEMC is quite challenging because ISI plays a destructive role and increases the probability of incorrect decoding of the transmitted symbols. Earlier work reported in (Mahfuz et al., 2010b) first proposed the concept of sampling-based detection method for CEMC signaling. However, that did not present its receiver architecture or its range and rate dependent characteristics, which is the main focus of this paper. In this paper we have made two major contributions: first, a mathematical model of an optimum receiver architecture of sampling-based signal detection in diffusion-based CEMC system has been presented, and second, we develop the exact expressions of detection performance of the proposed receiver and evaluate that with the bit error rate (BER) characteristics when several influencing factors e.g. sampling number, communication range, and transmission data rate vary. The paper is organized as follows. Section 2 briefly discusses the system model. Section 3 describes the development of sampling-based receiver in detail and discusses the communication range and rate dependent characteristics. Finally, Section 4 concludes the paper with possible future research directions.

2 DIFFUSION BASED CEMC SIGNALING

As shown in Fig. 1 the RN has a number of receptors of the same kind that can bind with a single type of information molecules transmitted by the TN. Referring to Fig. 1 the concentration of molecules at the RN can be explained by Fick's laws of ideal diffusion (Berg, 1993) and so the molecules can become available to the RN multiple times. The TN and the RN are synchronized in time (Moore et al., 2009). The RN is assumed to be located at the centre of a small volume known as the virtual receive volume (VRV) (Atakan and Akan, 2010). Assuming a point source type TN located at (0,0,0) transmitting molecules as an impulsive fashion (i.e. $Q_m \delta(t)$ where $\delta(t)$ is Dirac delta function) the mean concentration of available molecules $U(r,t)$ in molecules per unit volume at a three-dimensional space $\vec{r} = \hat{i} \cdot x + \hat{j} \cdot y + \hat{k} \cdot z$ (where the RN is located) and at time t changes with time and space as below (Bossert and Wilson, 1963); (Berg, 1993).

$$U(\vec{r},t) = G(\vec{r},t) = \frac{Q_m}{(4\pi Dt)^{\frac{3}{2}}} \exp\left(-\frac{x^2 + y^2 + z^2}{4Dt}\right) \quad (1)$$

$$= \frac{Q_m}{(4\pi Dt)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4Dt}\right)$$

where \vec{r} is the vector representing distance between the TN and the RN, $r^2 = x^2 + y^2 + z^2$ when a Cartesian coordinate system is assumed, Q_m , $m = \{0,1\}$ is the number of the transmitted molecules, and D is the diffusion constant of information molecules in the homogenous medium. $G(\vec{r},t)$ is known as the impulse response of the CEMC channel.

Assuming isotropic diffusion in homogenous case in three dimensions, hereafter we can drop the vector notation in $G(\vec{r},t)$ and write $G(r,t)$ only. Integrating $G(r,t)$ over the volume V of the RN we can get the mean number of the available molecules in the volume V as below.

$$G_{RN}(r,t) = \iiint_V G(\vec{r},t) dV$$

$$= \iiint_V \frac{Q_m}{(4\pi Dt)^{\frac{3}{2}}} \exp\left(-\frac{x^2 + y^2 + z^2}{4Dt}\right) dx dy dz \quad (2)$$

where V represents the volume of the RN sensing region and $dV = dx dy dz$ is the differential volume in the V . Expressing $G_{RN}(r,t)$ in energy-normalized quantity we can express the mean number of molecules available at the RN as

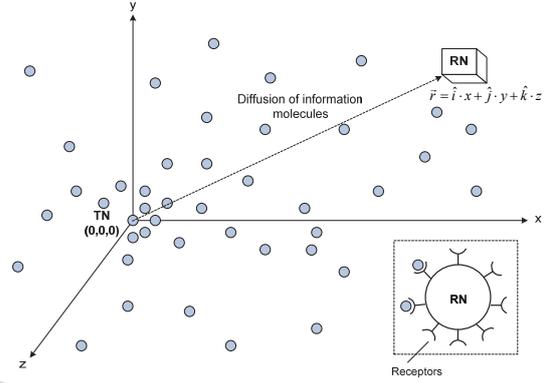


Figure 1: Ideal (free) diffusion of information molecules in the unbounded propagation medium. The receptors of the RN shown in inset bind with a single type of molecules.

$$s_m(r,t) = Q_m \frac{G_{RN}(r,t)}{\int_0^\infty G_{RN}(r,t) dt} = Q_m p$$

$$= \frac{1}{(4\pi Dt)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4Dt}\right) \quad (3)$$

where $p = \frac{G_{RN}(r,t)}{\int_0^\infty G_{RN}(r,t) dt} = \frac{1}{(4\pi Dt)^{\frac{3}{2}}} \exp\left(-\frac{r^2}{4Dt}\right)$

indicates the energy-normalized impulse response of the CEMC channel (Mahfuz et al., 2010a). $s_m(r,t)$ can be considered as mean concentration *signal intensity* of the available molecules at any TN-RN distance r at time t , and any integral of $s_m(r,t)$ over time is considered as mean *signal strength*. We assume that $D=10^{-6}$ cm²/sec. of small information molecules in water medium remains unchanged over the entire observation time and the effects of size of the information molecules on D is negligible.

3 SAMPLING BASED RECEIVER

3.1 Receiver Architecture

In a binary CEMC system the transmission of a symbol is done according to amplitude shift keying (ASK) modulation (Haykin, 2000) based on time-slotted manner as shown in Fig. 2, meaning that the TN transmits each bit at the beginning of the *bit interval* T_b . In binary ASK scheme the TN transmits $Q_0 \delta(t)$ molecules when it wants to send a bit "0" and it transmits $Q_1 \delta(t)$ molecules when it wants to send a bit "1." As a result, $\sum_{j=0}^{N_b} Q_m \delta(t - jT_b)$ is the transmitted signal and the TN transmits Q_m molecules, $m \in \{0,1\}$ and $Q_m \gg 1$, depending on the bit to be transmitted being $b_j \in \{0,1\}$ respectively, where $j = \{1,2,\dots,N_b\}$,

N_b being the total number of bits to be transmitted. The number of molecules that would be available and possibly received by the RN in the VRV would represent the deterministic amplitude $s_m(t)$ of the received molecular concentration signal following binomial distribution as shown below. When the TN sends Q_m , $m \in \{0,1\}$, molecules in the medium for each symbol, the probability of having k molecules in the RN out of the Q_m transmitted molecules during the i -th symbol interval (i.e. whether each of those k molecules arrives the RN during the i -th symbol interval or not) can be expressed by the binomial distribution function as below.

$$\Pr(k; Q_m, p) = \frac{Q_m!}{k!(Q_m - k)!} p^k (1-p)^{(Q_m - k)} \quad (4)$$

For a reasonably large value of $Q_m \gg 1$, when p is not close to 1 or 0 and p is finite such that as $n \rightarrow \infty$, $np \rightarrow \infty$, the binomial distribution on the right side of (4) can be approximated to a normal distribution $N(\mu_s, \sigma_s^2)$ where the mean (μ_s) and the variance (σ_s^2) can be expressed as

$$\mu_s = Q_m p = s_m(t), \quad \sigma_s^2 = Q_m p(1-p) = s_m(t)(1-p) \quad (5)$$

and so, $N(Q_m p, Q_m p(1-p)) \Rightarrow N(s_m(t), s_m(t)(1-p))$.

As a result, we can see that the mean of the number of molecules available for reception is actually the deterministic signal $s_m(t)$ that was found as the mean signal intensity in (3) using the macroscopic theory of the diffusion mechanism (Berg, 1993). Therefore, the total number of molecules $y(t)$ available for reception as a result of diffusion only can be expressed as a normal distributed random variable that is the sum of the deterministic part $s_m(t)$ and a zero-mean normal variable with variance $s_m(t)(1-p)$ as below (Kay, 1993).

$$y(t) = s_m(t) + n_s(t) \text{ where} \quad (6)$$

$$n_s(t) \sim N(0, s_m(t)(1-p)) = \sqrt{s_m(t)(1-p)} N(0,1)$$

During the i^{th} bit duration the RN would receive some of the molecules that were transmitted by the TN at the beginning of the i^{th} symbol interval, plus some of the molecules that were transmitted by the TN during the previous symbol durations i.e. from the first symbol duration up to the $(i-1)^{\text{th}}$ symbol duration. The former part constitutes the desired signal part and the latter constitutes the ISI part of the received signal. Fig. 3 shows the output signal $U(r,t)$ with 10 samples taken in each symbol duration.

The number of molecules available to the RN at any time during the symbol duration is a random

variable with signal-dependent mean and variance, and therefore, including the ISI the received signal intensity can be expressed as

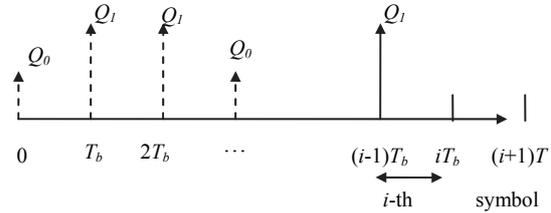


Figure 2: Binary ASK signaling at the beginning of each symbol duration.

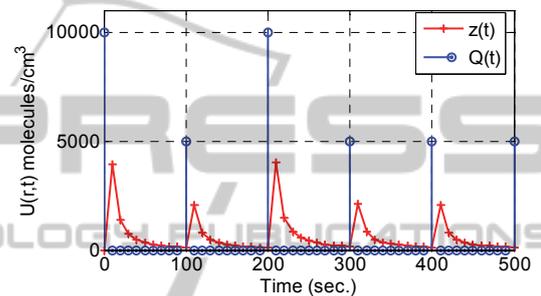


Figure 3: Input and output signals in binary ASK CEMC system. A “1” and a “0” are represented by sending 10,000 and 5,000 molecules respectively at the beginning of each symbol duration.

$$z(t) = s_m(t) + n_s(t) + n_{\text{ISI}}(t) \quad (7)$$

where $n_s(t)$ is as shown in (6) and $n_{\text{ISI}}(t)$ represents the residual molecules due to ISI and can be expressed as $n_{\text{ISI}}(t) \sim N(\mu_{\text{ISI}}, \sigma_{\text{ISI}}^2)$. Therefore, the binary signal detection problem in CEMC system can be formally written as below.

$$z(t) = \begin{cases} N(s_1(t) + \mu_{\text{ISI}}, s_1(t)(1-p) + \sigma_{\text{ISI}}^2); & H_1 \\ N(s_0(t) + \mu_{\text{ISI}}, s_0(t)(1-p) + \sigma_{\text{ISI}}^2); & H_0 \end{cases} \quad (8)$$

An optimum receiver is the one that gives the *minimum probability of error*. We consider the *minimum probability of error* criterion to derive a *test statistic* by calculating the logarithm of the likelihood ratio using Neyman-Pearson formula (Kay, 1993) with equal prior probabilities as below.

$$\frac{\ell(z | H_1)}{\ell(z | H_0)} > 1 \Rightarrow \ln \frac{\ell(z | H_1)}{\ell(z | H_0)} > 0. \quad (9)$$

The conditional probabilities can be expressed as shown in (10). Note that for any prior probability the optimum receiver is termed as the *maximum a*

posteriori probability (MAP) detector, which for equal prior probability $\Pr(H_0) = \Pr(H_1)$ reduces to maximum likelihood (ML) detector (Kay, 1993).

$$\begin{aligned} \ell(z | H_1) &= \frac{1}{\sqrt{2\pi(s_1(t)(1-p) + \sigma_{\text{ISI}}^2)}} \times \\ &\exp\left[-\frac{\{z - (s_1(t) + \mu_{\text{ISI}})\}^2}{2(s_1(t)(1-p) + \sigma_{\text{ISI}}^2)}\right] \\ \ell(z | H_0) &= \frac{1}{\sqrt{2\pi(s_0(t)(1-p) + \sigma_{\text{ISI}}^2)}} \times \\ &\exp\left[-\frac{\{z - (s_0(t) + \mu_{\text{ISI}})\}^2}{2(s_0(t)(1-p) + \sigma_{\text{ISI}}^2)}\right] \end{aligned} \quad (10)$$

A sampling-based detector samples the received concentration signal at a number of points in each symbol duration. Each sample value represents one observation. Therefore, for a total of N observations, $n = [1, 2, \dots, N]$, combining (9) and (10) and simplifying yields the test statistic $T(z)$ as:

$$\begin{aligned} T(z) &= \sum_{n=1}^N \{a[n]z^2[n] + b[n]z[n]\} \underset{\text{Select } H_0}{>} \underset{\text{Select } H_1}{\gamma}, \text{ where} \\ a[n] &= \left\{ \frac{1}{2(s_0[n](1-p) + \sigma_{\text{ISI}}^2)} - \frac{1}{2(s_1[n](1-p) + \sigma_{\text{ISI}}^2)} \right\} \\ b[n] &= \left\{ \frac{(s_1[n] + \mu_{\text{ISI}})}{(s_1[n](1-p) + \sigma_{\text{ISI}}^2)} - \frac{(s_0[n] + \mu_{\text{ISI}})}{(s_0[n](1-p) + \sigma_{\text{ISI}}^2)} \right\} \\ -\gamma &= \sum_{n=1}^N \left\{ \frac{1}{2} \ln \frac{(s_0[n](1-p) + \sigma_{\text{ISI}}^2)}{(s_1[n](1-p) + \sigma_{\text{ISI}}^2)} - \frac{(s_1[n] + \mu_{\text{ISI}})^2}{2(s_1[n](1-p) + \sigma_{\text{ISI}}^2)} + \right. \\ &\quad \left. \frac{(s_0[n] + \mu_{\text{ISI}})^2}{2(s_0[n](1-p) + \sigma_{\text{ISI}}^2)} \right\} \end{aligned}$$

The resulting sampling-based receiver architecture is shown in Fig. 4.

Approximate (Closed Form) Expression. The exact performance in terms of probability of false alarm (P_{FA}) and probability of detection (P_D) is difficult to be determined analytically because the test statistic is a sum of a normal distributed random variable and a weighted sum of independent χ_1^2 random variables. However, in the following we assume an example scenario and try to derive the closed form expressions of P_{FA} and P_D under certain assumptions. Assuming that N is even i.e. $N=2L$ where L is a positive integer, and that the coefficients of $x^2[n]$ are all distinct and occur in pairs, P_{FA} and P_D can be expressed respectively as below, where γ' is the modified γ (Kay, 1993).

$$\begin{aligned} P_{FA} &= \sum_{n=1}^L A_n \exp\left(-\frac{\gamma'}{2a[n]\sigma_{z_0}^2}\right) \text{ where } A_n = \prod_{\substack{i=1 \\ i \neq n}}^L \frac{1}{1 - \frac{a[i]}{a[n]}} \\ P_D &= \sum_{n=1}^L B_n \exp\left(-\frac{\gamma'}{2a[n]\sigma_{z_1}^2}\right) \text{ where } B_n = \prod_{\substack{i=1 \\ i \neq n}}^L \frac{1}{1 - \frac{a[i]}{a[n]}} = A_n \cdot \end{aligned}$$

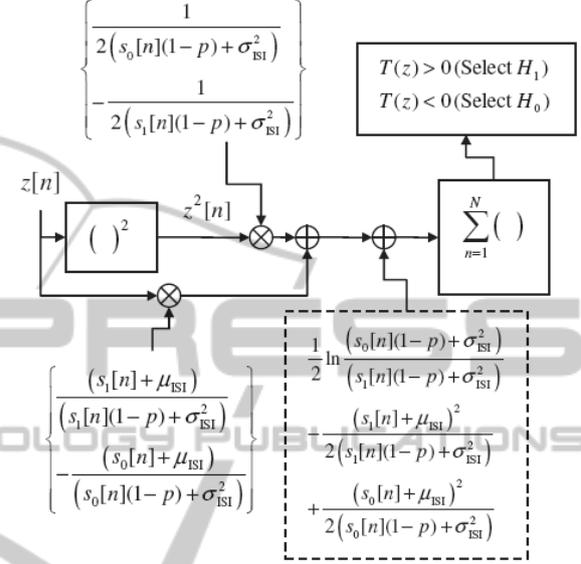


Figure 4: Sampling-based receiver architecture for binary CEMC system.

3.2 Communication Range and Rate Dependent Characteristics

We explain the proposed sampling-based receiver architecture in terms of three factors, namely, N , r , and f . The receiver model developed analytically has been evaluated numerically for average BER using 100,000 randomly generated bits at each simulation scenario. As shown in Fig. 5 when more number of samples are used in the receiver the average BER decreases. For instance, when 4 samples per bit are used in the receiver, we observe a high BER of approximately 0.5 meaning that approximately half of the bits are decoded incorrectly. However, we found that when $N=20$ or more (data not shown) the receiver can decode all the bits correctly i.e. BER=0. The more the N is the better the receiver performs because the receiver gets more information from more samples that it can use in decoding the bit correctly. On the other hand, Fig. 6 shows the effects of communication range (r) on BER such that when r increases BER increases. This is due to the temporal spreading the signal experiences as r increases, owing to the nature of the diffusion-based CEMC channel (Mahfuz et al., 2010a). The communication

range investigated is in between 400 nm and 100 μm , which covers a wide range of TN-RN distances for water medium as reported in (Mahfuz et al., 2010b). Finally, the effects of data rate on BER are shown in Fig. 7 such that BER increases as f increases. This is also due to the ISI caused by the temporal spreading of the channel when the input symbol changes at a higher rate. When f increases the symbol duration decreases and as a result the receiver cannot cope up with the input signal to decode the transmitted bits correctly, and in addition, suffers from the ISI. The effects of ISI become more severe when f increases further giving rise to BER of $\sim 6\%$ at $f=0.01$ bits per second (bps) to $\sim 7\%$ at $f=1$ bps when r and N are kept fixed at 800 nm and 10 samples per symbol respectively.

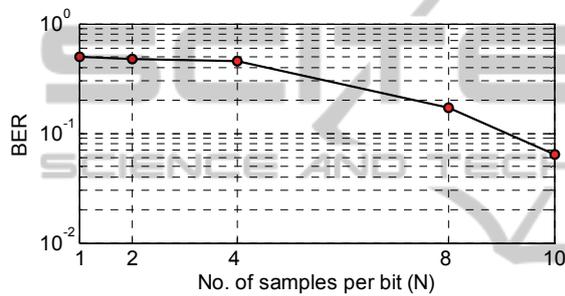


Figure 5: Effects of number of samples per symbol (N) on BER when $r=800$ nm and $f=0.01$ bps.

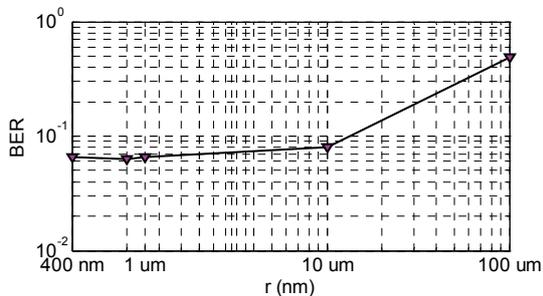


Figure 6: Effects of communication range on BER when $N=10$, $f=0.01$ bps.

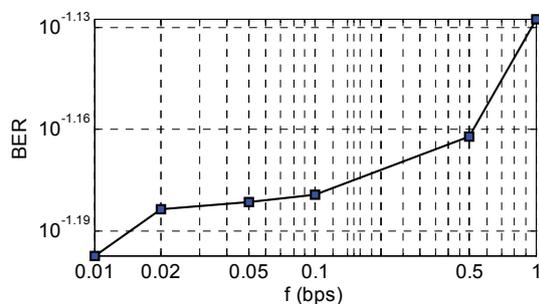


Figure 7: Effects of transmission data rate on BER when $r=800$ nm and $N=10$.

4 CONCLUSIONS

In this paper we have developed and evaluated the performance of sampling-based optimum receiver architecture of CEMC system. The proposed receiver model should be valid for any type of input signal transmission with any modulation format, e.g. pulse amplitude modulation (PAM) transmission, and can also be extended to detect signals with multilevel (M-ary) amplitude modulation in CEMC system. Bionanomachines existing in the nature can sense the concentration of molecules at their receptors, which may help implement sampling-based receivers through engineering of bionanomachines. Finally, the results presented in this paper will surely help a molecular communication engineer to evaluate the performance of a CEMC system in greater details.

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