

An Active Contour Model with Improved Shape Priors using Fourier Descriptors

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Abstract: Snakes or active contours are widely used for image segmentation. There are many different implementations of snakes. No matter which implementation is being employed, the segmentation results suffer greatly in presence of occlusions, noise, concavities or abnormal modification of shape. If some prior knowledge about the shape of the object is available, then its addition to an existing model can greatly improve the segmentation results. In this work inclusion of such shape constraints for explicit active contours is presented. These shape priors are introduced through the use of Fourier based descriptors which makes them invariant to the translation, scaling and rotation factors and enables the deformable model to converge towards the prior shape even in the presence of occlusion and context noise. These shape constraints have been computed in descriptor space so no reconstruction is required. Experimental results clearly indicate that the inclusion of these shape priors greatly improved the segmentation results in comparison with the original snake model.

1 INTRODUCTION

Active contours method was first introduced by (Kass et al., 1988). These methods can generally be bifurcated into explicit (or parametric) active contours (Kass et al., 1988) and implicit (or level set) methods (Caselles et al., 1995). In explicit methods a deformable model is represented by a set of connected points that evolve dynamically to finally settle on contours in the image. However, such models are unable to automatically adapt to sudden topological changes. On the other hand, implicit active contours intrinsically adapts to any topology changes.

So far, many improvements have been proposed for parametric active contours: (Cohen and Cohen, 1991), (Williams and Shah, 1992) and (Xu and Prince, 1998).

Even with these improvements the segmentation results become highly inaccurate in the presence of occlusions, noise, object overlapping and extrusions. To address these issues and improve the segmentation results, the need for inclusion of some shape prior information in the segmentation process becomes indispensable. Some efforts have already been made in this regard with diffusion snakes (Cremers et al., 2002) and Affine invariant eigen snakes (Xue et al., 2002). Such template matching schemes however, are sensitive to

the initialization of deformable template.

Fourier based shape descriptors provide quite an efficient and powerful way of contour representation. Although such a representation can be particularly useful in the context of explicit active contours with shape priors, not much work has been done in this context. One such study proposes the representation of shape priors by elliptical Fourier descriptors (Staib and Duncan, 1992). However, this method is too sensitive to parameters and initialization must be done very close to the goal shape to achieve the desired result. Most recently Fourier based geometric shape priors were used with the variational setup for snakes (Charmi et al., 2008). The limit of this method is that it is not invariant to the starting point and needs the reconstruction of template and deformable contours.

In this paper, an improved version of snakes with Fourier based shape priors is presented. A more stable greedy implementation of snakes as well as a robust set of invariant Fourier descriptors are used for representation of active contours. More importantly, unlike the previous work, the shape matching is performed directly in the Fourier descriptor space rather than in the spatial domain, so there is no need for reconstruction of contours. Moreover, a solution for automatic starting point invariance is presented to make the proposed method more robust and efficient.

This paper is organized as follows: section 2 provides an overview of the greedy approach. In section 3, the invariant Fourier descriptors are introduced. Section 4 presents our main contribution regarding the integration of Fourier based shape priors in active contour model. In section 5, experimental results are presented. Finally, in section 6, conclusions and perspectives are presented.

2 REVIEW OF GREEDY SNAKES

The *greedy algorithm* (Williams and Shah, 1992) was chosen because, inclusion of additional energy term is quite intuitive in such a setup. Similarly, curve representation is well suited for embedding shape based constraints. This method is also stable and being an iterative approach, the solution is guaranteed. In greedy approach an initial contour is defined as a discrete closed curve with n vertices. A discrete energy functional is then defined as the sum of energies associated with each vertex in a given window of fixed size w . The goal is to minimize this functional to achieve the target segmentation. The equation for this discrete energy functional at each pixel $\tilde{p}_i \in w$ is given by

$$E(\tilde{p}_i) = E_{\text{int}}(\tilde{p}_i) + E_{\text{ext}}(\tilde{p}_i) \quad (1)$$

where $E_{\text{int}}(\tilde{p}_i)$ and $E_{\text{ext}}(\tilde{p}_i)$ are the discrete energy functional for internal and external energies. In order to improve the convergence of active contour towards the desired boundary, a distance transform (Fabbri et al., 2008) based energy term has been used. Our Distance Transform based external energy term is given by $E_{\text{dt}}(\tilde{p}_i) = DT(\tilde{p}_i)$, where $DT(\cdot)$ is the distance transform matrix.

In this work, for internal energy continuity and curvature we have used the energy terms as proposed by (Williams and Shah, 1992), while for balloon energy a greedy derivation proposed by (Mille et al., 2006) has been used.

Finally, the equation for a discrete energy functional incorporating all these energy terms, along with their associated weights (α, β, γ and δ) is respectively given by

$$E = \alpha E_{\text{cont}} + \beta E_{\text{curv}} + \gamma E_{\text{ball}} + \delta E_{\text{dt}} \quad (2)$$

3 FOURIER BASED INVARIANTS

The Discrete Fourier Transform (DFT) of $z_i = x_i + j \cdot y_i$ is given by a set of Fourier coefficients as proposed by (Charmi et al., 2008) and (Bartolini et al., 2005)

$$Z_k = \sum_{i=0}^{n-1} z_i e^{-j \frac{2\pi i k}{n}} \quad \text{for } k = -\frac{n}{2}, \dots, \frac{n}{2} - 1 \quad (3)$$

The above DFT can also be represented as $Z_k = R_k e^{j\theta_k}$, where R_k is known as amplitude and θ_k as phase. The normalized descriptors

$$\hat{Z}_k = \hat{R}_k e^{j\hat{\theta}_k}, k = -\frac{n}{2}, \dots, \frac{n}{2} - 1 \quad (4)$$

having the following properties are translation, scale and rotation-invariant, as proved by (Bartolini et al., 2005):

$$\hat{Z}_0 = 0, \hat{R}_k = \frac{R_k}{R_1} \text{ and } \hat{\theta}_k = \theta_k - \theta_1 \quad (k \neq 0) \quad (5)$$

From this set of invariants, we can reconstruct the (normalized) shape using the Inverse Discrete Fourier Transform (iDFT):

$$\hat{z}_i = \frac{1}{n} \sum_{k=-n/2}^{n/2-1} \hat{Z}_k e^{j \frac{2\pi i k}{n}} \quad \text{for } i = 0, 1, \dots, n-1 \quad (6)$$

4 EMBEDDING SHAPE PRIORS

4.1 Shape Prior Energy

To introduce shape prior information, a prior energy term is added to the energy functional of the snake.

Let C_{ref} be a template contour describing a shape. This contour has n vertices $v_i^{\text{ref}} = (x_i^{\text{ref}}, y_i^{\text{ref}})$ ($i = 0, 1, \dots, n-1$) (practically, n is chosen depending on the complexity of the shape). Now for a contour C having the same size, $v_i = (x_i, y_i)$ ($i = 0, 1, \dots, n-1$), the prior term has to constrain it to evolve to a shape similar to the given template.

As described in the previous section, the reconstructed shapes \hat{C}_{ref} and \hat{C} can be obtained using the normalized Fourier descriptors. The prior energy can then be defined as the *distance* between \hat{C}_{ref} and \hat{C} . Let $D(a, b)$ denote the distance (in general) between a and b . Then, the shape prior energy of a snake C regarding a reference shape C_{ref} can be defined as

$$E_{\text{prior}}(C) = D(\hat{C}, \hat{C}_{\text{ref}}) \quad (7)$$

For integrating this energy into the discrete energy functional of the snake, we need its discrete form. For a neighboring pixel \tilde{p}_i of a vertex v_i ($i = 0, 1, \dots, n-1$), a new curve C_i is considered by replacing the vertex v_i by \tilde{p}_i . The shape prior energy of the pixel \tilde{p}_i is then defined as

$$E_{\text{prior}}(\tilde{p}_i) = D(\hat{C}_i, \hat{C}_{\text{ref}}) \quad (8)$$

Although the greedy algorithm is known for its rapidity, the integration of such distance calculation decreases its performance significantly. Indeed, at each iteration of the algorithm, for every vertex of the

snake, the DFT and iDFT are performed for all of the neighboring pixels.

Fortunately, by using the Euclidean distance metric, we can avoid performing the iDFT as proposed by (Bartolini et al., 2005). if \hat{Z}_k and \hat{Z}_k^{ref} denote the normalized descriptors of the snake and the reference shape, then, the shape prior energy term can be rewritten only in terms of DFTs and entirely calculated in the Fourier descriptor space as follows.

$$E_{\text{prior}}(C) = \sqrt{\sum_{k=-n/2}^{n/2-1} |\hat{Z}_k - \hat{Z}_k^{\text{ref}}|^2} \quad (9)$$

4.2 Snake's Evolution with Shape Prior

To evolve the snake under shape prior energy the following strategy is adopted:

In the beginning, the shape prior weight is kept very small as compared to the other weights, so that the snake can evolve freely from (or under little effect of) the shape prior constraint. This will allow it to capture parts of the object's contour.

Once the snake has some information about the object, the prior weight is then allowed to increase gradually. Thus, the weight parameters are remodeled as a function of time that may vary dynamically. In particular, it suffice to define the prior weight (i.e. ζ) as a strictly increasing function of time (t): $\zeta(t) = k \cdot t$ ($k > 0$) and the others as constants. A threshold value ζ_{max} is set as well:

$$\zeta(t) = \begin{cases} k \cdot t & \text{if } k \cdot t \leq \zeta_{\text{max}} \\ \zeta_{\text{max}} & \text{if } k \cdot t > \zeta_{\text{max}} \end{cases} \quad (10)$$

Finally, the energy functional can be rewritten as

$$E = \alpha E_{\text{cont}} + \beta E_{\text{curv}} + \gamma E_{\text{ball}} + \delta E_{\text{dt}} + \zeta(t) E_{\text{prior}} \quad (11)$$

There remains a problem with the Euclidean distance as it introduces a difference in phase between the constructed shape of the snake and the one of the reference shape.

To overcome this problem, we propose to minimize this phase difference. As the descriptors used are not starting point-invariant, it is obvious that any change in the starting point of the snake, will produce a change in the phase of the constructed shape. This leads us to the idea of shifting the starting point to the point for which the phase difference is minimal. This step is called the *starting point correction*. We use $ED(\hat{C}, \hat{C}_{\text{ref}})$ for calculating the phase difference.

To sum up, below is the greedy algorithm of our method. \hat{C} and \hat{C}_{ref} are the constructed shape of the snake and the reference shape, respectively.

1. For each vertex v_i of the snake, search its neighborhood to find the location that minimizes the energy functional. Move v_i to that location.
2. Once 1. has finished with all the n vertices (i.e. 1 iteration), shift the starting point of the snake to the one that minimizes the phase difference between the constructed shape of the snake and the one of the reference shape, i.e. \hat{C} and \hat{C}_{ref} . Update the value of $\zeta(t)$.
3. Repeat steps 1 and 2 until only a very small fraction of snake points move in an iteration.

5 EXPERIMENTAL RESULTS

In all experiments presented below, we fixed parameters: $\alpha = 1, \beta = \gamma = 0, \delta = 1$ while, for the weight of the prior energy $\zeta(t)$ as defined as in the equation (10) the values of k and ζ_{max} may vary for different shapes and/or different images.

Experiments on both synthetic and real images are performed. The results are compared with the classic snakes without any prior term and also with another state of the art method using prior shape information. Moreover, to evaluate the quality of the segmentation results by our method Pratt Criterion (Pratt et al., 1978) has been used. This gives us a quantitative measure of segmentation quality with respect to a ground truth segmentation. The higher values of Pratt Criterion indicates more accurate segmentation with respect to ground truth and vice versa.

In the following section the segmentations are carried out on some synthetic and real images with and without using our proposed prior based energy term. The quality of these segmentation result, in terms of Pratt criterion, along with prior energy parameters k and ζ_{max} are presented in Table 1.

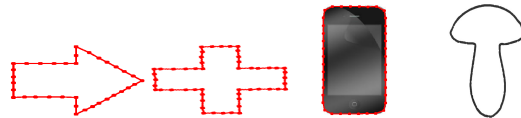


Figure 1: Reference shapes.



Figure 2: Initial snake (Red outline) along with ground truth (Blue outline).

It is clear that the results are more precise with our proposed shape prior based energy term. Next,

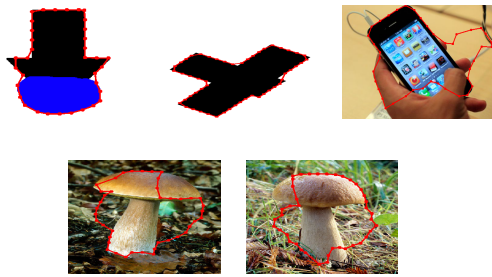


Figure 3: Results without shape priors.

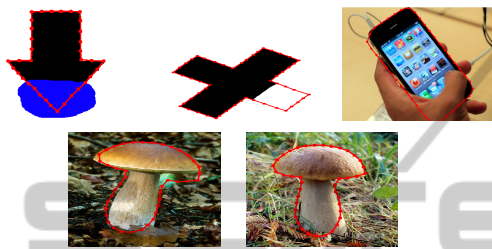


Figure 4: Results with our Fourier based shape priors.

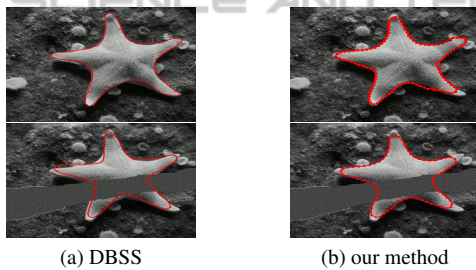


Figure 5: Results by (DBSS) method (a) and by our method (b).

we compare our method with another state-of-the-art method which uses the distance-based shape statistics (DBSS) (Charpiat et al., 2007) as shape priors.

Results presented in Table 2, clearly indicates that our method is more accurate as compared to (DBSS).

6 CONCLUSIONS

In this paper, a new model with translation, scale, rotation and starting point invariant shape priors for explicit active contour has been presented. Calculation of shape based energy was entirely performed in the descriptors space, i.e. there is no need to reconstruct the prior shape during evolution of active contours, which is a gain in terms of computation time. Visual and numerical evaluations on both synthetic and real images have shown that our method greatly improves the segmentation results, even in presence of occlusion and incomplete shapes. In the near future, we

Table 1: Segmentation results using Pratt criterion with and without shape priors (SP).

Image	k	ζ_{\max}	Without SP	With SP
Arrow	0.03	1.5	28.09	82.86
Cross	0.05	2.5	20.64	85.03
Phone	0.03	1.5	16.09	46.85
Boletus1	0.03	1.5	10.08	12.40
Boletus2	0.02	1.2	13.69	15.96

Table 2: Comparison of segmentation results.

Image	DBSS	Our method
Non-occluded	23.53	29.00
Occluded	12.57	13.62

would also like to test our method on medical images and video data. The implementation for the level set method will be considered as well.

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