Linear Plane Border
A Primitive for Range Images Combining Depth Edges and Surface Points

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Abstract: Detecting primitives, like lines and planes, is a popular first step for the interpretation of range images. Real scenes are, however, often cluttered and range measurements are noisy, such that the detection of pure lines and planes is unreliable. In this paper, we propose a new primitive that combines properties of planes and lines: Linear Plane Borders (LPB). These are planar stripes of a certain width that are delineated at one side by a linear edge (i.e. depth discontinuity). The design of this primitive is motivated by the contours of many man-made objects. We extend the J-Linkage algorithm to robustly detect multiple LPBs in range images from noisy sensors. We validated our method using qualitative and quantitative experiments with real scenes.

1 INTRODUCTION

A depth camera consists of a sensor array attached to an optical lens, that provides a measurement of distance or depth for each sensing element or pixel. Currently, depth cameras are experiencing an exponential growth, thanks to emerging mass market applications, that, without a doubt, have stimulated the industry to create cheap devices. At the moment, Time of Flight technology (Lange and Seitz, 2001) and Kinect (Smisek et al., 2011) sensor technology are leading the market. High resolution and high frame rates are becoming available, which make these sensors a good hardware alternative to many complex stereo video systems.

Depth sensors have had a remarkable impact in many computer vision tasks. Disciplines such as object recognition (Redondo-Cabrera et al., 2012), SLAM (May et al., 2009) and human pose estimation (Shotton et al., 2011) have received multiple, in many case outstanding, scientific contributions based on depth sensing.

As in other image sensing technologies, depth images contain rich and complex information. Therefore, a considerable number of methods and algorithms have focused on detecting basic geometric primitives in depth images. These methods are important and necessary as a building block for high level interpretation and recognition algorithms.

1.1 Related Work

Simplifying a scene as a grouping of multiple piecewise planar models is demonstrated to be an efficient and stable representation of most man-made structures (Bartoli, 2007). Depth sensors give powerful 3D information that can be used to find planar structures, providing accurate object and environment detection. However, in complex scenes, fitting planar structures can be challenging and time consuming (*i.e.* a table crowded with small objects). Moreover, depth sensors produce many errors, such as Multiple Path Interference in ToF cameras (Jiménez et al., 2012), that can affect full plane representations.

This paper defines a new geometric primitive called *Linear Plane Border* (LPB). It is defined as a planar stripe of a given width that can be detected in the vicinity of linear silhouette edges of geometric objects. LPBs give a compact but rich representation to detect regular objects in depth images. The main clue to find them is by looking at depth discontinuities. The benefit of using LPBs instead of full plane representations is demonstrated in this paper, in particular in complex scenes.

The use of 2D edge information in conjunction with 3D plane information used for the construction of these primitives provides a simplified but robust representation of planar objects in a scene. That such a combination of 2D edge information and 3D surface

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information is useful has been demonstrated recently by the design of 3D interest point detectors and descriptors (Steder et al., 2011; Fiolka et al., 2012) and by the design of pair features for voting-based object detection (Taguchi et al., 2012). Here, we utilize this insight for a primitive-based object representation.

Detecting multiple LPBs in depth images requires a method able to find multiple instances of a geometrical model from noisy data. Many such methods have been proposed in the literature with remarkable success. Among the most commonly used methods are Hough transforms (Xu et al., 1990), MeanShift (Comaniciu and Meer, 2002) and those based on random sampling such as RANSAC (Fischler and Bolles, 1981; Zuliani et al., 2005).

This paper finds multiple LPB hypotheses using the very recent J-Linkage (Toldo and Fusiello, 2008) algorithm, based on tailored agglomerative clustering, first proposed by Zhang and Kosecka (2007). This method is able to robustly detect multiple models without prior specification of its number and has shown remarkable detection performance. Recently, there exist some approaches that use J-Linkage to obtain piecewise-planar representation of point clouds. Fouhey et al. (2010) present a method for the detection and matching of multiple planes in pairs of images. They use J-Linkage to generate multiple local homography hypothesis. Feng et al. (2010) use J-Linkage to minimize the user input compared with traditional Single View Reconstruction approaches. Schwarz et al. (2011) suggest some adaptation of the original algorithm for the detection and tracking of multiple planes in sequences of ToF depth images.

Taking as starting point the core of the fast J-Linkage implementation (Toldo and Fusiello, 2010), we extend in this work the original algorithm to the detection of LPBs in depth images of ToF cameras. The main contribution of the paper is to show that LPBs can be effectively detected in depth images using our modified version of J-Linkage. The experimental results show that this approach outperforms the most common case of searching for planar structures in terms of computational time and accuracy. We strongly believe that LPBs can be very useful in object recognition tasks and object pose computation using depth images.

The paper is organized as follows: Section 2 presents a general diagram of the proposal. Section 3 details the individual steps required to detect Line Plane Borders. We evaluate our method qualitatively and quantitatively using real scenes. These results are summarized in Sec. 4.

2 DETECTION OF LINEAR PLANE BORDERS

We represent a depth image as a scalar function $\mathcal{D}$, where $\mathcal{D}(\mathbf{p})$ represents the depth measured at pixel position $\mathbf{p} = (u, v)^\top$. Due to optics, depth values are computed along optical rays. We suppose that the camera optics are properly calibrated so that metric 3-dimensional coordinates are available for each image position $\mathbf{p}$. We denote as $Q(\mathbf{p}) = (Q_x, Q_y, Q_z)^\top$ the three-dimensional coordinates of the point $\mathbf{p}$.

Given a depth image $\mathcal{D}$, the main problem to solve is how to find all LPB candidates, grouping together all pixels belonging to the same LPB. This paper proposes a pipeline consisting of three stages, illustrated in Fig. 1.

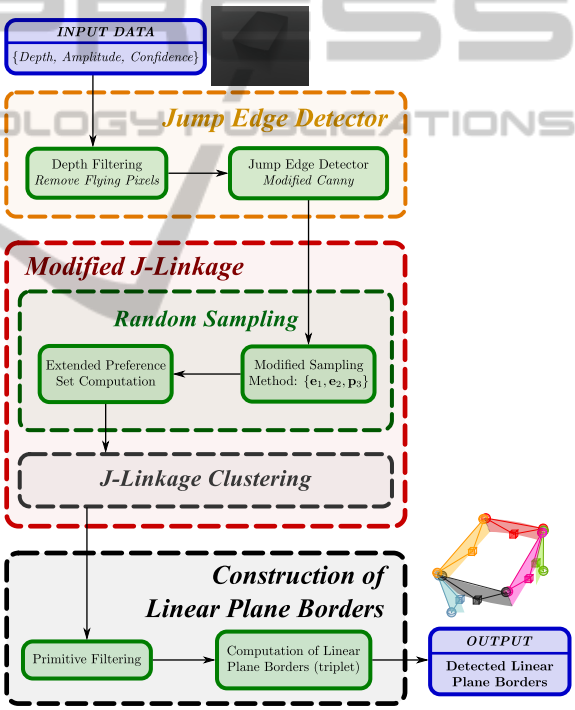

Figure 1: General diagram of the proposed detection of Linear Plane Borders. $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{p}_3\}$ represent the sampled points needed to calculate a LPB hypothesis: two edge points and one point in the correct face of the linear edge.

1. **Jump Edge Detector.** This stage detects depth discontinuities which are the clue of finding edges of geometrical objects. The process is carefully designed to deal with noise and depth artifacts that usually appear around such discontinuities.
2. **Modified J-Linkage.** J-Linkage is an algorithm that groups together points based on the consistency with randomly sampled model

hypotheses. In our case, each LPB hypothesis is created by sampling two jump edge points likely belonging to the same plane border and a third point that is far enough from the edge but at a distance limited by the LPB width.

3. **Construction of LPBs.** After running J-Linkage, points in the depth image are grouped in different LPBs hypotheses. This stage refines the LPBs using linear least squares fit produce an accurate estimate of the three points used to parameterize each LPB.

3 PROPOSED METHOD

Each of the three aforementioned stages of our method are detailed next.

3.1 Jump Edge Detector

In this paper, a *jump edge* is defined as a depth discontinuity that produces an edge in the depth image. Detecting jump edges properly in depth images is not trivial. In most of depth-image technologies (specially ToF cameras), depth measurements are highly unstable around jump edges, producing artifacts commonly known as "flying pixels" or "veil points" (see Fig. 2). The detection of jump edges is performed in two steps:

1. **Depth Filtering:** this step implements a very simple statistical filter, described by Rusu et al. (2008), to detect "flying pixels". It is based on comparing the statistical distribution of depths around each pixel. Once detected, they are filtered out, using the depths of their neighboring pixels.
2. **Canny-based Edge Detector:** Lejeune et al. (2011) proposed a modified magnitude of the depth gradient for use in the Canny edge detector which considers the uncertainty of depth measurements, characterized for the physical device used to compute depth. We use their version that is specifically designed for ToF cameras. If depth filtering is not performed before the Canny detector, "flying pixels" will be highlighted by the filter, giving erroneous depths in the edges.

Fig. 2 shows the result of the "jump edge" detector in a real scene. After this step, "jump edge" gradient magnitude $\nabla\mathcal{D}(\mathbf{p})$ and gradient direction $\nabla_\phi(\mathbf{p})$ are obtained. The Canny detector performs binary segmentation of $\nabla\mathcal{D}$ so that jump edge candidates can be easily found.

3.2 Modified J-Linkage

The core of our method is a modified version of the J-Linkage algorithm, originally proposed by Toldo and Fusiello (2008), to group points in the depth image into different LPB hypotheses.

In a nutshell, J-Linkage can be explained in three steps: first, n hypotheses (models) from the input data (*e.g.* depth image) are selected randomly using minimal set solutions (two points for a line, three points for a plane, ...). Second, each point in the input set is classified according to its distance to each model. All models compatible with a point form the so-called preference set of that point. Each point forms its own cluster. Third, the clusters of points with similar preference sets are merged together, keeping only the models in the preference set of a cluster that are compatible with all of its points. The algorithm stops when all clusters have disjunct preference sets.

We propose modifications to the two first steps of J-Linkage so that LPBs are efficiently detected given an input depth image and the resulting jump edge detection detailed in the previous section. Instead of randomly selecting three points to construct plane hypotheses, our approach makes use of a semi-deterministic procedure to sample these three points leading to a reduction of the n hypothesis that have to be generated. Preference sets for every point are computed by applying an extended test to adjust planar models to the proposed LPBs.

3.2.1 Random Sampling Stage

We propose a new random sampling strategy to create n LPB hypotheses from the input depth image. Although LPBs are indeed plane structures (minimal set of three points), they must be attached to linear jump edges. We sample sets of only two points, in image coordinates, belonging to jump edges, namely $(\mathbf{e}_1, \mathbf{e}_2)$. These two points must be close enough to belong to the same edge, and far enough to produce a stable LPB hypothesis. The required third point is determined from the other two in a deterministic way that is explained below. In this way, the original method proposed by Toldo and Fusiello (2010) for detecting planes with J-Linkage is sped up.

The detailed steps to obtain a single random LPB hypothesis from the input data are:

1. Sample uniformly a jump edge point ($\mathbf{e}_1$), where $\mathbf{e}_1$ is a two-dimensional point in image coordinates.
2. Sample an edge point ($\mathbf{e}_2$) with the following conditional probability function:

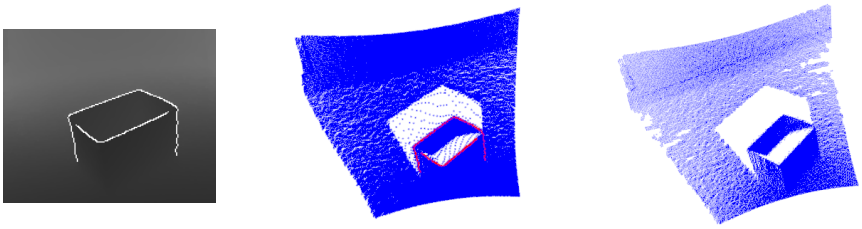

Figure 2: Detection of jump edges and flying-pixel removal. From left to right, we show the detected jump edges (white points) in the depth image, the corresponding point cloud before the flying-pixels removal (jump edges are tagged as red points) and the point cloud after removing flying-pixels.

$$p(\mathbf{e}_2|\mathbf{e}_1) = \begin{cases} p_{close} & \text{if } \|\mathbf{e}_1-\mathbf{e}_2\| \in (\delta_1,\delta_2) \text{ and } |\nabla_\phi(\mathbf{e}_1)-\nabla_\phi(\mathbf{e}_2)| < \delta_\phi \\ p_{med} & \text{if } \|\mathbf{e}_1-\mathbf{e}_2\| \in (\delta_1,\delta_2) \text{ and } |\nabla_\phi(\mathbf{e}_1)-\nabla_\phi(\mathbf{e}_2)| > \delta_\phi \\ p_{far} & \text{if } \|\mathbf{e}_1-\mathbf{e}_2\| \notin (\delta_1,\delta_2) \end{cases}$$

where $p_{far} < p_{med} << p_{close}$ and the interval (δ_1,δ_2) establishes the lower and upper Euclidean distances between $\mathbf{e}_1$ and $\mathbf{e}_2$ so that they can be considered compatible neighbors. The values $\nabla_\phi(\mathbf{e}_1)$ and $\nabla_\phi(\mathbf{e}_2)$ represent the local jump edge orientation at points $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively.

Hence, $\mathbf{e}_2$ is chosen with the highest probability (p_{close}) if its distance with respect $\mathbf{e}_1$ is inside the interval (δ_1,δ_2) and the difference between their edge orientations is smaller than δ_ϕ. If only the distance condition is met, $\mathbf{e}_2$ is chosen with a smaller probability (p_{med}). If none of the conditions is met, then point $\mathbf{e}_2$ is chosen with the smallest possible probability (p_{far}).

In order to speed up searching for candidates of $\mathbf{e}_2$, a k-Nearest Neighbor tree is built.

3 The selection of the third point ($\mathbf{p}_3$) is computed in a deterministic way. We find $\mathbf{p}_3$ by selecting the point that is at a distance d from the middle point of the line connecting $\mathbf{e}_1$ and $\mathbf{e}_2$, where d is half the length of the line. This leaves two possibilities for point $\mathbf{p}_3$. We use the depth to select the point closest to the camera as the solution (see Fig. 3).

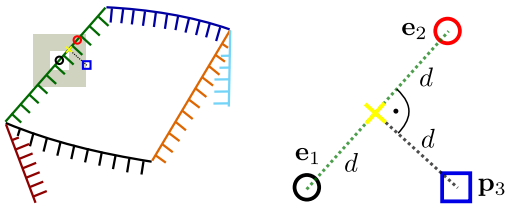

Figure 3: Overview of the modified ramdom sampling stage.

4 Compute the normal plane equations, denoted as Π, of the LPB using the 3D points $(Q(\mathbf{e}_1), Q(\mathbf{e}_2), Q(\mathbf{p}_3))$.

As it is shown in Section 4 the new sampling method speeds up the random sampling stage and provides sufficient flexibility for the generation of the primitives that could support the correct LPBs.

3.2.2 Preference Set Computation

From the previous step, a set of n LPB models has been obtained:

$$\mathcal{M} = \{\mathbf{m}_1, \cdots, \mathbf{m}_n\} \quad \text{where} \quad \mathbf{m}_i = (\Pi^i, \mathbf{e}_1^i, \mathbf{e}_2^i, \mathbf{p}_3^i) \tag{1}$$

Given a point $\mathbf{p}$ from the input data, the preference set of $\mathbf{p}$, namely $PS(\mathbf{p})$ consists of the set of sample models $\mathbf{p}$ is compatible with. It can be computed as the intersection of the following three sets (see Fig. 4 for a graphical representation):

- $\mathcal{A} = \{\mathbf{m}_i \quad \text{s.t.} \quad d_{plane}\left(Q(\mathbf{p}), \Pi^i\right)\} < \varepsilon$, where d_{plane} is the distance from the point $Q(\mathbf{p})$ to the plane Π^i,
- $\mathcal{B} = \{\mathbf{m}_i \quad \text{s.t.} \; d_{edge}\left(Q(\mathbf{p})_{\Pi^i}, Q(\mathbf{e}_1^i), Q(\mathbf{e}_2^i)\right) < L\}$, where d_{edge} is the distance from the projection of $Q(\mathbf{p})$ in the plane Π^i to the edge formed by $Q(\mathbf{e}_1^i)$ and $Q(\mathbf{e}_2^i)$, and
- $\mathcal{C} = \{\mathbf{m}_i \quad \text{s.t.} \quad \mathcal{D}(\mathbf{p}) < \mathcal{D}(\mathbf{p}_{\Pi^i})\}$.

Finally, the preference set of point $\mathbf{p}$ is given as:

$$PS(\mathbf{p}) = \mathcal{A} \cap \mathcal{B} \cap \mathcal{C}. \tag{2}$$

3.3 Construction of Linear Plane Borders

After J-Linkage terminates, each cluster of points corresponds to an LPB hypothesis. In this section we detailed how these clusters are actually parameterized with a refined LPB of a proper length and width. A LPB is mathematically defined as the triplet created by two points corresponding to the bounds in the edge segment and one point corresponding to the maximum width of the "stripe" (see Fig. 5).

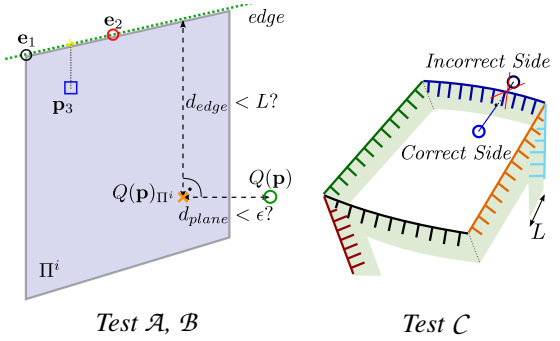

Figure 4: Tests applied to compute the preference set of every point.

3.3.1 Primitive Filtering

Extending the initial J-Linkage proposal, we consider that outliers to the new primitive not only emerge as small cluster in the number of points but also in the width of the stripe that they support. In order to avoid non desirable LPBs, depending on the application, we apply rejection thresholds in:

- minimum number of points, and
- minimum width.

By definition, a LPB must be supported by some edge points, so that if there is any clustering that do not hold any edge point it will also be discarded as a correct primitive.

3.3.2 Primitive Definition

Let m be the number of primitives after the filtering method, then LPBs are constructed as follows:

- For LPB^m, compute the two boundary edge points $(Q(\mathbf{E}_1^m), Q(\mathbf{E}_2^m))$:
  1. Consider all the edge points that belong to LPB^m.
  2. Compute the plane that best fits all the points belonging to LPB^m using linear least squares. As edge points are noisy measurements, other methods like jointly fitting the edge line and the plane could degenerate.
  3. Project all the edge points onto the best fitted plane, previously computed.
  4. Apply Principal Component Analysis (PCA) to compute the first two principal directions.
  5. Calculate the minimum and maximum values along the principal components and extract the boundary points.
  6. Reproject the boundary points onto the original coordinate frame to get the edge limits of LPB^m: $(Q(\mathbf{E}_1^m), Q(\mathbf{E}_2^m))$
- Compute the representative plane point $(Q(\mathbf{P}_3^m))$:
  1. Obtain the equation of the 3D line created by $\{Q(\mathbf{E}_1^m), Q(\mathbf{E}_2^m)\}$.
  2. Project all the points belonging to LPB^m into the best fitted plane, previously computed.
  3. Calculate the distance of every projected point to the 3D edge line and select the maximum distance (wd_{max}).
  4. Compute the two points at distance wd_{max} from the middle of the segment $\{Q(\mathbf{E}_1^m), Q(\mathbf{E}_2^m)\}$ in the perpendicular direction.
  5. Select the point in the correct side by checking for the one that has a nearer point in LPB^m: $(Q(\mathbf{P}_3^m))$.

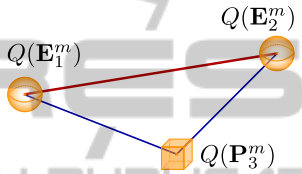

Figure 5: Conceptual representation of a LPB.

4 EXPERIMENTS AND RESULTS

This section tests the detection of LPBs in real scenes captured with a commercial ToF camera. We compare LPB detection, as it is proposed in this paper against a state-of-art plane fitting using J-Linkage.

In Fig. 6, we show 7 different scenes composed of planar structures and the result of each step of the algorithm. We also add the result of detecting planes with J-Linkage where input data have been re-sampled in a **1:4 ratio** (one of each two columns and rows), so that the number of input data is comparable in both approaches. In these experiments, the number of J-Linkage random samples are around $\{5000-7000\}$ for LPB and almost ten times more for planes.

As quantitative results we show the accuracy of LPB detection in Table 1. Ground-truth data are obtained by manual annotation over the depth images so that the different planes it is composed of are clearly identified. Error is measured as the mean of the Euclidean distance between points projected onto the ground-truth plane and the projection onto the plane obtained in LPBs detection.

It is clearly shown that LPB detection is very accurate, capturing very useful geometric information. It is also faster than detecting only planes as it is shown in Table 2.

In a qualitative manner we show in Fig. 7 the

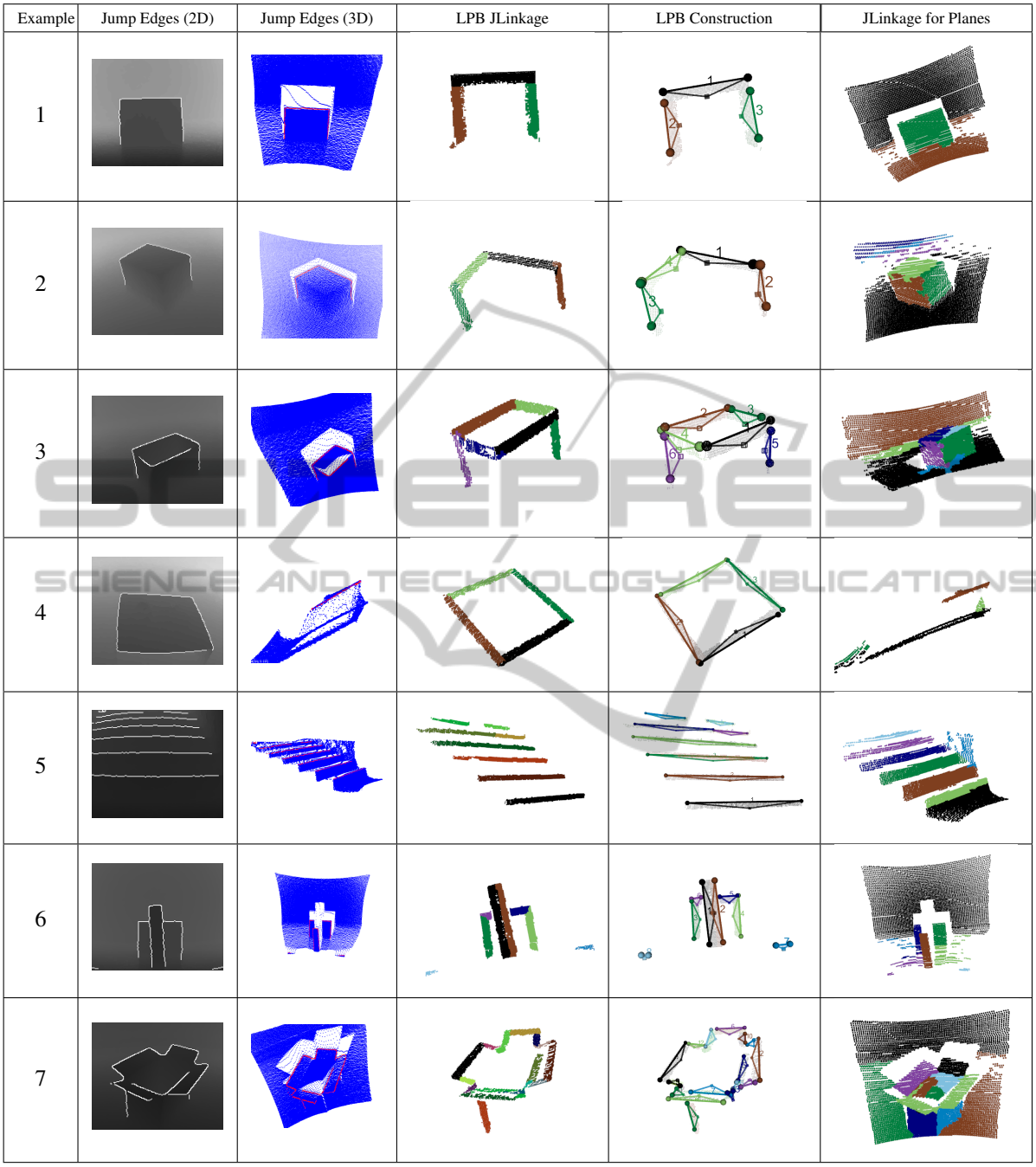

Figure 6: Comparison between the detection of planes using the original JLinkage method and the main stages of the LPB algorithm presented in this paper.

results with curved objects (non planar objects can also be approximated as an arrangement of LPBs). While these scenes are not piece-wise planar, LPB detection is giving useful and stable information.

5 CONCLUSIONS

This paper shows that LPBs can be efficiently detected in depth images, taken with a comercial depth sensor of real man-made scenes composed of planar structures.

Table 2: Achieved speed-up for the main consuming processes.

Achieved Speed-Up							
Example	1	2	3	4	5	6	7
Random Sampling	x124.5	x103.5	x101.2	x78.3	x48.2	x85.6	x74.5
Clustering	x8.8	x23.1	x6.1	x4.7	x1.8	x9.1	x5.4

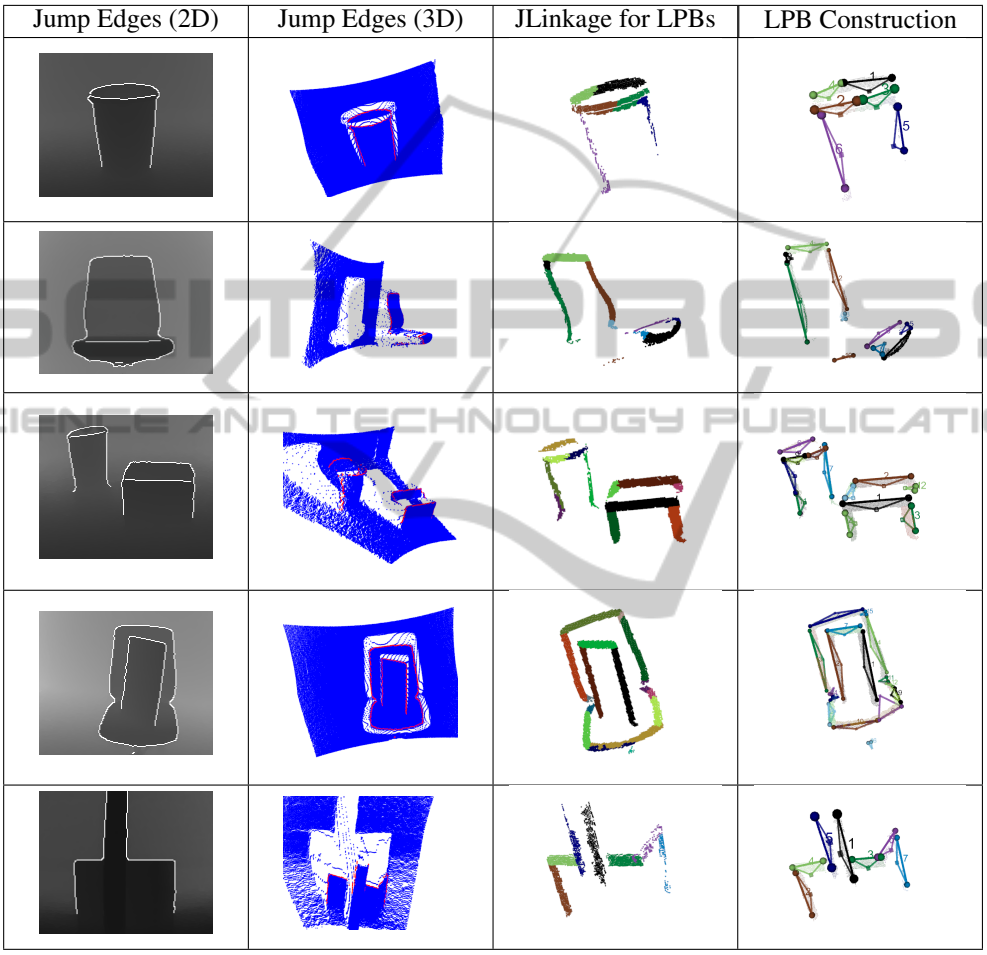

Figure 7: Qualitative results for curved objects in more complex scenarios.

Table 1: Plane detection accuracy using LPB.

Plane Detection Accuracy	
Example	Proposal
1	0.00123253
2	0.00176433
3	0.00192681
4	0.00280092
5	0.00795743
6	0.00384587
7	0.00461953

The proposed semi-deterministic sampling method for LPBs allows to perform J-Linkage with ten times less hypothesis than general plane fitting methods. This reduction has a critical impact on the algorithm's speed.

As it can be seen in the results, LPBs are able to capture rich information about the objects in the scene compared to looking only for planar structures. It also behaves reasonably well in curved objects.

In future works, edges due to changes in plane orientations that are not detected now as jump edges will be included. These additional edges will increase

the number of LPBs that can be obtained from a geometric object. We also believe that LPBs can be extended to curved objects, using curved stripes attached to jump edges.

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