Equivalence between Two Flowshop Problems
MaxPlus Approach

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Abstract: In this paper, a flowshop problem with minimal and maximal delays, setup and removal times is tackled. It is shown that this problem is equivalent to another flowshop problem with only minimal and maximal delays, which can be seen as a central problem. The proof is done using an algebraic way which allows to identify the role of each constraint, once the modeling is performed.

1 INTRODUCTION

In this paper, a flowshop problem with a large set of constraints is tackled, the objective is to minimize the makespan. It is shown that this problem can be modeled in an algebraic way and that, as a consequence, a kind of central problem can be highlighted.

The proposed model is based on MaxPlus algebra. This algebra is sometimes used in control system, particularly in relation with Petri Nets, but very few in scheduling theory. Nevertheless, one can cite some articles like (Giffler, 1963) on project scheduling, (Hanen and Munier, 1995) on cyclic parallel machine problems, (Cohen et al., 1985), (Gaubert, 1992) on cyclic flowshop scheduling problem and (Gaubert and Mairesse, 1999) on cyclic job-shop scheduling problems. The MaxPlus approach was applied in modeling and scheduling flowshop problem with minimal delays, setup and removal times in (Lenté, 2001), (Bouquard et al., 2006) and flowshop problems with minimal and maximal delays in (Bouquard and Lenté, 2006) for two machines flowshop problems and in (Augusto et al., 2006) for any number of machines. In each case, a square matrix was associated to each job and a similar matrix could be associated to each sequence of jobs. In the two last articles, it is shown how to derive lower bounds by constructing a no-wait instance from the job-matrices that can be solved by an efficient procedure designed for the traveling salesman problem (Carpaneto et al., 1995).

This paper presents a synthesis and a generalization of these studies, by considering simultaneously minimal and maximal delays and setup and removal times. The objective of this paper is to bring out equivalence between this general flowshop problem and the flowshop problem with only minimal and maximal delays. This equivalence will be put in evidence through a MaxPlus modeling.

Following this introduction, this general flowshop problem with associated constraints and also notations used in this paper will be described. The third part describes the algebraic approach, using MaxPlus algebra. The fourth one presents how to model the so called general problem and to calculate in detail job associated matrix in MaxPlus. The next part indicates how to transform a Job-matrix of this problem into a Job-matrix of a minimal - maximal delay flowshop problem that therefore will be highlighted as a central problem.

2 FLOWSHOP WITH MINIMAL - MAXIMAL DELAYS, SETUP AND REMOVAL TIME

Figure 1 illustrates the constraints that are applied to a job. The considered flowshop problem is composed of m machines numbered from M_1 to M_m and a set of n jobs. A job J_i is then composed of m operations with duration p_{i1}, p_{i2}, ..., p_{im}: the k^{th} operation has to be executed on the k^{th} machine and it can be done only when the $(k-1)^{\text{th}}$ operation of the same job has been completed on machine M_{k-1}. Before and after its execution, an operation requires a setup and a re-

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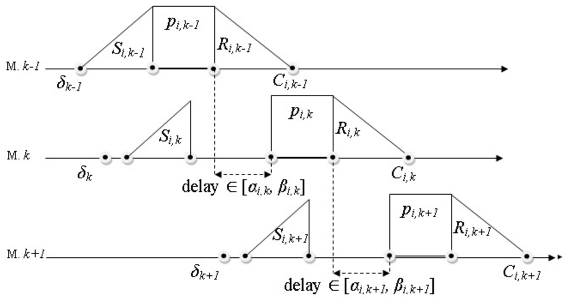

Figure 1: Schema of the system.

moval time, noted S_{ik} and R_{ik}. During these times the machine is considered as occupied. At any moment, each machine can handle at most one task.

Moreover, a waiting time between the execution of two consecutive operations of the same job must respect both a minimal and maximal delays that are noted α_{ik} and β_{ik} ($2 \leq k \leq m$).

The preemption is not allowed but only permutation schedules are considered, which means that all jobs are executed in the same order by all machines. For a given sequence, the end date C_{ik} of the operation k of the job J_i is measured by the end of its associated removal time. The makespan of a schedule is then defined by the maximum of dates C_{im} for all jobs J_i ($C_{max} = \max_{1 \leq i \leq n} C_{im}$). This problem can be denoted by $F_m—S_{nsd}, R_{nsd}, min-max\ delay, perm—C_{max}$ according to notation of Graham (Graham et al., 1979).

3 ALGEBRAIC APPROACH

3.1 MaxPlus Algebra

As a short introduction of MaxPlus algebra, let's say that in this algebra we denote the maximum by $\oplus$ and the addition by $\otimes$. The first operator, $\oplus$, is idempotent, commutative, associative and has a neutral element ($-\infty$) denoted by $\mathbb{0}$. The second operator, $\otimes$, is associative, distributive on $\oplus$ and has a neutral element (0), denoted by $\mathbb{1}$. The element $\mathbb{0}$ is an absorbing element for the operator $\otimes$. These properties can be summarized by saying that $\mathbb{R}_{max} = (\mathbb{R} \cup \{-\infty\}, \oplus, \otimes)$ is a dioid. It is important to note that in MaxPlus algebra and more generally in dioids, the first operator does not allow simplification: $a \oplus b = a \oplus c \not\Rightarrow b = c$. Furthermore, in $\mathbb{R}_{max}$, the second operator $\otimes$ is commutative, and except for $\mathbb{0}$, every element is invertible: the inverse of a is denoted by a^{-1} or $\frac{1}{a}$. For more convenience we denote the ordinary subtraction by $\frac{a}{b}$ instead of $a \otimes b^{-1}$ and by ab the product $a \otimes b$.

Moreover, it is possible to extend these two operators to $m \times m$ matrices of elements of $\mathbb{R}_{max}$. Let A and B be two matrices of size $m \times m$ and let $[.]_{i,j}$ be the element at row i and column j of a matrix, operators $\oplus$ and $\otimes$ are defined by:

$$\forall (i,j) \in \{1,\ldots,m\}^2, [A \oplus B]_{i,j} = [A]_{i,j} \oplus [B]_{i,j}$$

$$\forall (i,j) \in \{1,\ldots,m\}^2, [A \otimes B]_{i,j} = \bigotimes_{k=1}^{m} [A]_{i,k} \otimes [B]_{k,j}$$

The set of $m \times m$ matrices in $\mathbb{R}_{max}$ endowed with these two operators is also a dioid. But the operator $\oplus$ is not commutative and matrices are generally not invertible. For more details, please see (Gaubert, 1992; Baccelli et al., 1992; Gunawardena, 1998).

3.2 Generalities about MaxPlus Modeling of Flowshop Problems

The MaxPlus modeling of a flowshop problem always follows the same principles, whatever the constraints are. A matrix T_i is associated to each job J_i. This matrix is computed from data linked to the job and defines entirely this job. This matrix allows also to compute the completion times $\vec{C_i}$ of the operations of the job knowing the dates of availability $\vec{\delta}$ of all the machines $\vec{C_i} = \vec{\delta} \otimes T_i$. As a consequence, considering a sequence σ of ν jobs, if machines are available at dates $\vec{\delta}$, the dates of liberation $\vec{C_\sigma}$ of the machines by the sequence are given by formula $\vec{C_\sigma} = \vec{\delta} \otimes \bigotimes_{i=1}^{\nu} T_{\sigma(i)}$. So it is possible to associate to a sequence σ the matrix $T_\sigma = \bigotimes_{i=1}^{\nu} T_{\sigma(i)}$. Then lower bounds can be derived from the algebraic modeling by extracting sub-matrices from matrices T_i (Lenté, 2001; Augusto et al., 2006). MaxPlus provides a formal framework for the study of flowshop problems. It allows to extend or adapt easily results from some kinds of flowshop problems to others.

3.3 Application to Flowshop with Minimal - Maximal Delays

In (Augusto et al., 2006), the authors have studied and solved a flowshop with minimual and maximal delays. The flowshop problem is the same that the one described in section 2 except that there is no setup nor removal time. They established the form of the matrix T_i associated to a job J_i (Equation (1) shows an example for three machines). Then they defined a lower bound following this way: from a matrix T_i, they built a lower matrix T_i^{NW} ($T_i^{NW} \leq T_i$) characteristic of a nowait flowshop job. The resolution of the corresponding nowait flowshop problem, which can be done using a fast TSP resolution procedure, gives a lower bound of the initial flowshop problem. Finally, they designed a Branch and Bound procedure to solve the $F_m|min-max\ delay, perm|C_{max}$.

$$T_i = \begin{pmatrix} p_{i1} & p_{i1}p_{i2}\alpha_{i2} & p_{i1}p_{i2}\alpha_{i2}p_{i3}\alpha_{i3} \\ \frac{\mathbb{1}}{\beta_{i2}} & p_{i2} & p_{i2}p_{i3}\alpha_{i3} \\ \frac{\mathbb{1}}{\beta_{i3}}\frac{\mathbb{1}}{p_{i2}\beta_{i2}} & \frac{\mathbb{1}}{\beta_{i3}} & p_{i3} \end{pmatrix} \quad (1)$$

4 $F_M|S_{NSD}, R_{NSD}, MIN-MAX\ DELAYS, PERM|C_{MAX}$

4.1 Modeling

The description from section 2 leads to the sets of inequations (2), (3) and (4), written in MaxPlus notations. They summarize all the constraints applied to the k^{th} operation of a job J_i. The term δ_k represents the date of availability of machine M_k to perform the setup operation. Typically, in a schedule, δ_k could be the completion time of operation k of the job preceding J_i. These inequations are illustrated in figure 2, they will serve to determine a linear relation between the dates of availability δ_k of the machines and the end dates C_{ik} of job J_i.

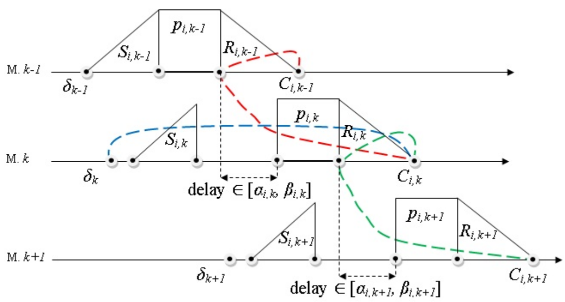

Figure 2: Model for calculation.

$$C_{ik} \geq C_{i(k-1)}\alpha_{ik}p_{ik}\frac{R_{ik}}{R_{i(k-1)}} \ (2 \leq k \leq m) \quad (2)$$

$$C_{ik} \geq C_{i(k+1)}\frac{\mathbb{1}}{p_{i(k+1)}\beta_{i(k+1)}}\frac{R_{ik}}{R_{i(k+1)}} \ (1 \leq k \leq m-1) \quad (3)$$

$$C_{ik} \geq \delta_k S_{ik} p_{ik} R_{ik} \ (1 \leq k \leq m) \quad (4)$$

4.2 Job Associated Matrix

Remembering section 3.2, an expected relation is of the form:

$$\vec{C_i} = \vec{\delta} T_i \quad (5)$$

where T_i is the matrix associated to the job J_i. Let introduce some new notations before pursuing calculus:

$$a_{ik} = \alpha_{i(k+1)}p_{i(k+1)}\frac{R_{i(k+1)}}{R_{ik}} \ (1 \leq k \leq m-1) \quad (6)$$

$$b_{ik} = \frac{\mathbb{1}}{\beta_{ik}p_{ik}}\frac{R_{i(k-1)}}{R_{ik}} \ (2 \leq k \leq m) \quad (7)$$

$$\vec{C_i} = (C_{i1}, C_{i2}, \ldots, C_{im}) \quad (8)$$

$$\vec{\delta} = (\delta_1, \delta_2, \ldots, \delta_m) \quad (9)$$

$$P_i = \begin{pmatrix} S_{i1}p_{i1}R_{i1} & 0 & \ldots & 0 \\ 0 & S_{i2}p_{i2}R_{i2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & S_{im}p_{im}R_{im} \end{pmatrix} \quad (10)$$

$$A_i = \begin{pmatrix} 0 & a_{i1} & 0 & \ldots & 0 \\ b_{i2} & 0 & a_{i2} & \ddots & \vdots \\ 0 & b_{i3} & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & a_{i(m-1)} \\ 0 & \ldots & 0 & b_{im} & 0 \end{pmatrix} \quad (11)$$

Basically, vector $\vec{C_i}$ represents end dates of operations of job J_i, matrix P_i regroups data defining an operation and matrix A_i the data linking two successive operations of job J_i, it can be seen as a "delay matrix". According to this system of inequations (2), (3) and (4), it can be inferred that: $\vec{C_i} \geq \vec{\delta}P_i \oplus \vec{C_i}A_i$. The smallest solution of this system is found using Kleene star: $\vec{C_i} = \vec{\delta}P_iA_i{}^*$ where $A_i^* = (\mathbb{1} \oplus A_i \oplus A_i^2 \oplus \ldots \oplus A_i^q \oplus \ldots)$. Therefore, to meet relation (5) we must define

$$T_i = P_iA_i{}^* \quad (12)$$

T_i is the matrix associated to job J_i. Note that this matrix is totally independent of position of the associated job in the sequence. The general form of matrix T_i is:

$$[T_i]_{\ell,c} = \begin{cases} S_{i\ell}p_{i\ell}R_{ic} \bigotimes_{k=\ell+1}^{c} p_{ik}\alpha_{ik} & \text{if} \quad \ell < c \\ S_{i\ell}p_{i\ell}R_{ic} & \text{if} \quad \ell = c \\ \frac{S_{i\ell}R_{ic}}{\beta_{i\ell}} \bigotimes_{k=c+1}^{\ell-1} \frac{\mathbb{1}}{p_{ik}\beta_{ik}} & \text{if} \quad c < \ell-1 \\ \frac{S_{i\ell}R_{ic}}{\beta_{i\ell}} & \text{if} \quad c = \ell-1 \end{cases} \quad (13)$$

As an example, in case of 3 machines, the matrix associated to the job J_i is:

$$T_i = \begin{pmatrix} S_{i1}p_{i1}R_{i1} & S_{i1}p_{i1}R_{i2}p_{i2}\alpha_{i2} & S_{i1}p_{i1}R_{i3}p_{i2}\alpha_{i2}p_{i3}\alpha_{i3} \\ \frac{S_{i2}R_{i1}}{\beta_{i2}} & S_{i2}p_{i2}R_{i2} & S_{i2}p_{i2}R_{i3}p_{i3}\alpha_{i3} \\ \frac{S_{i3}R_{i1}}{\beta_{i3}}\frac{\mathbb{1}}{p_{i2}\beta_{i2}} & \frac{S_{i3}R_{i2}}{\beta_{i3}} & S_{i3}p_{i3}R_{i3} \end{pmatrix} \quad (14)$$

It must be noted that this model can be generalized to additional constraints such as release dates (r_i), latency duration (q_i) or some batch constraints.

5 CENTRAL PROBLEM

In (Augusto et al., 2006) or (Fondrevelle et al., 2006), authors proposed a way to calculate lower bounds and optimal schedules for problem $Fm|min-max\ delays; perm|C_{max}$. This section represents now how to transform our studied problem into flowshop with only minimal and maximal delays. To do that, terms relating to setup and removal times must be integrated to the definition of new processing times and minimal and maximal delays. It will define a new flowshop problem whose characteristics are described below. In this new formulation, a job J_i is only defined by operations' processing times $\overline{p_{ik}}$ and minimal and maximal delays $\overline{\alpha}_{ik}$ and $\overline{\beta}_{ik}$. But its associated matrix T_i remains unchanged.

$$\begin{cases} \overline{p}_{ik} = S_{ik}p_{ik}R_{ik} & (1 \leq k \leq m) \\ \overline{\beta}_{ik} = \frac{\beta_{ik}}{S_{ik}R_{i(k-1)}} & (2 \leq k \leq m) \\ \overline{\alpha}_{ik} = \frac{\alpha_{ik}}{S_{ik}R_{i(k-1)}} & (2 \leq k \leq m) \end{cases} \quad (15)$$

For example, in case of three machines, the matrix associated to job J_i becomes

$$T_i = \begin{pmatrix} \overline{p}_{i1} & \overline{p}_{i1}\overline{\alpha}_{i2}\overline{p}_{i2} & \overline{p}_{i1}\overline{\alpha}_{i2}\overline{p}_{i2}\overline{\alpha}_{i3}\overline{p}_{i3} \\ \frac{\mathbb{1}}{\overline{\beta}_{i2}} & \overline{p}_{i2} & \overline{p}_{i2}\overline{\alpha}_{i3}\overline{p}_{i3} \\ \frac{\mathbb{1}}{\overline{\beta}_{i2}\overline{p}_{i2}\overline{\beta}_{i3}} & \frac{\mathbb{1}}{\overline{\beta}_{i3}} & \overline{p}_{i3} \end{pmatrix} \quad (16)$$

6 CONCLUSIONS

Thanks to MaxPlus approach, it is possible to transform a flowshop scheduling problem into a matrix problem. Some manipulations over these matrices allow us to exhibit a sort of central problem. Permutation flowshop with several classical constraints are equivalent to some permutation flowshop with minimal and maximal delays. Calculus have been presented with non sequence dependent setup and removal times, but the equivalence can also be stated for release dates or groups of jobs for example. This result means that one can focus our efforts to solve this central problem. It also means that it must be possible to adapt automatically a method developed for a type of constraints to another type of constraints.

Further research will concern modeling a greater set of constraints, like limited stocks between machines or blocking constraints and, if possible and if necessary, the definition of an other central problem. They will also concern improvements of solving methods presented in (Augusto et al., 2006).

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