Three Dimensional Packing Algorithm with Consideration of Loading and Unloading Order

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Abstract: A novel packing problem for truck or containership transportation is considered. A truck or a ship visits several accumulation places in a delivery tour, and items are loaded or unloaded at each accumulation place. In order to carry items as many as possible at one delivery tour, we often have to unpack and repack some items even at nondestination places if they blockade loading other items. Such packing and repacking, however, will make the transportation cost increase. Thus, a packing that requirs smaller number of unpacking and repacking is desired. In this paper, we extend the slicing-tree which is a method of a representation of packing and propose an algorithm to pack items into the container with minimum the number of unpacking and re-packing.

1 INTRODUCTION

From the viewpoint of reducing the cost of transportation by truck or containership, it is desired to carry as many items as possible at one delivery. Developing a plan of loading items into a container is often considered as "the rectangular solid packing problem," i.e., to allocate small rectangular solids without overlapping in a big rectangular empty box, and there have been published many papers on the rectangular solid packing problem (S. D. Allen et al., 2011), (G. Fuellerer et al., 2010), (H. Kawashima et al., 2010), (F. K. Miyazawa, and Y. Wakabayashi, 2009), and (H. Ohta et al., 2008) .

In practice, we often have to take care about the order of items loading and unloading when we develop a loading plan. For example, suppose that a ship or a truck visits several accumulation places in a delivery tour, and items are loaded or unloaded at each accumulation place. In such a case, the loading and unloading order of items is subject to the order of visiting the accumulation places, which is difficult to be changed without changing the delivery routing. There are, however, few preceding studies about the packing problem where the order of both loading and unloading is given.

Moreover, even if we consider only about either loading or unloading, the packing problem is still difficult. That is because, if we put each item in the rear side of the container in the order of loading (or the inverse order of unloading), all items may not be put in the container.

Therefore, we should permit to unload temporarily an item if it blockades other items loading (unloading), and after the loading (unloading), to reload the blockading item again into the container. Hereafter we call such unloading of blocking items *unpacking* and such reloading *repacking*. Since repacking causes increase of cost, we should pack the items in the container so that fewer items are to be repacked.

In this paper, we discuss the above packing problem where the order of both loading and unloading is given. An effective algorithm to put the items with the minimum number of repacking is proposed and its performance will be shown by computational experiments.

2 DEFINITION OF THE PROBLEM

In this paper, we assume that items are rectangular solids and the container is also rectangular placed along x-axis, y-axis and z-axis in three-dimensional Euclidean space. The direction from left [resp. front, top] to right [back, bottom] is viewed as the x[y, z]-

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direction. The door of the container is located at the right side. Figure 1 shows an aspect of the container and directions of loading and unloading of items. Centers of gravity of the items are out of consideration in this paper, i.e., any allocations of the items are permitted unless items overlap with other items or walls of the container.

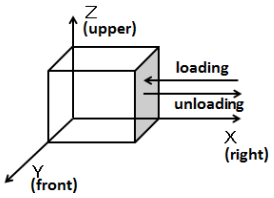


Figure 1: Container and direction of loading and unloading.

The loading and the unloading clock time are already given for each item. In accordance with the clock times, we obtain a partial order of loading and unloading. In the process of an item loading (unloading), other items which have been already located in the container may blockade the loading (unloading). In this case, we unload the items, which we call "blocking items," from the container. After loading (unloading) of the item, we re-pack the blocking items into the original position.

Now, we discuss the locations of the blocking items when an item is loaded to or unloaded from the container. When the item *a* is loaded into the container, the locations of the blocking items are on the left of or the upper of the place where the loading item *a* is to be located. In this paper, if the item *b* satisfies both of the following two conditions, then *b* is defined as the blocking item against the item *a* loading:

(i) After loading of the item *a,* the front face of the item *a* is located in the front side of the rear face of the item *b*, and the rear face of the item *a* is located in the rear side of the front face of the item *b*;

(ii) After loading of the item *a*, both of the following two conditions are satisfied:

(ii-1) The right face of the item *a* locates in left side of the left face of the item *b*, and the bottom face of the item *a* is lower than upper face of the item *b*:

(ii-2) The upper face of the item *a* is lower than the bottom face of the item *b*, and the left face of the item *a* locates in left side of the right face of the item *b*.

Figure 2 (a) shows that the item *d* blocks when the item *b* is loaded, where the space in which the item *b* is to be located is represented by broken lines. Item *d* has to be unloaded first and then be repacked after the loading of the item *b*. Thus, the number of repacking is one.

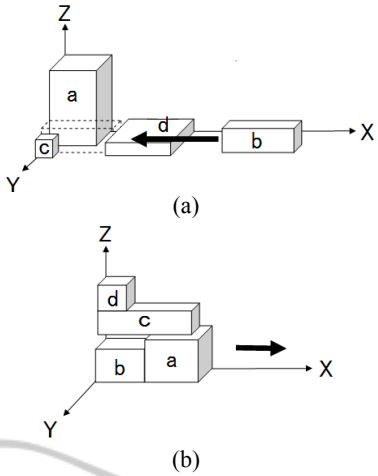


Figure 2: Blocking items against loading and unloading.

The blocking item against the item unloading is also defined similarly. Figure 2 (b) shows that the item *c* blocks when the item *a* is unloaded. Item *c* has to be unloaded, but the item *d* blocks unloading of the item *c*. Therefore the item *c* has to be also unloaded and then to be repacked after the unloading of the item *a*. Thus, the number of repacking is two.

The purpose of this paper is to propose an algorithm of packing items into a container so that the number of re-packing is the minimum.

3 THREE-DIMENSIONAL SLICING-TREE

A rectangular solid dissection, which is a dissection of a rectangular solid region into smaller rectangular solids (rooms) by places is often used to represent a packing. That is, items are assigned to the distinct rooms (any two items do not share a single room). In particular, a structure of a rectangular solid dissection which is obtained by recursively cutting by planes perpendicular to *x, y* or *z*-axis is called *three-dimensional slicing-structure*. Figure 3(a) shows a three-dimensional slicing-structure.

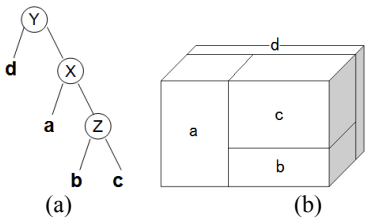


Figure 3: Three-dimensional slicing structure and the corresponding slicing-tree.

A three-dimensional slicing-structure can be represented by a binary tree, where internal nodes and external nodes correspond to planes and rooms respectively. In particular, the internal nodes are classified into “X-nodes”, “Y-nodes” and “Z-nodes”. X-nodes [resp. Y-nodes, Z-nodes] correspond to the planes perpendicular to *x*-axis [*y*-axis, *z*-axis] in the rectangular solid dissection. The left subtree of the X-node [resp. Y-node, Z-node] corresponds to the leftward [frontward, upward] region of the corresponding plane, and the right subtree of the X-node [resp. Y-node, Z-node] corresponds to the rightward [backward, downward] region of the corresponding plane. The slicing tree does not allow the representation of all packings, However the near optimal packing can be modeled. Figure 3(b) shows the slicing-tree which is the representation of the slicing-structure shown in figure 3(a).

It is obvious that a region which corresponds to an external node must be larger than the item. In addition, the shape of each region which corresponds to a subtree is a rectangular solid. Now we express the size of rectangular solid A as $(X_A \times Y_A \times Z_A)$, where X_A [resp. Y_A, Z_A] is the x[y, z]-length of the rectangular solid *A*. Assume the size of the rectangular solid which corresponds to the left subtree of a X-node P is $(X_L \times Y_L \times Z_L)$, the size of the rectangular solid which corresponds to the right subtree of P is $(X_R \times Y_R \times Z_R)$ and the size of the rectangular solid which corresponds to the subtree which is merged the trees and whose root is P is $(X_P \times Y_P \times Z_P)$ respectively. Then we have the following inequalities.

$$X_P \geq X_L + X_R,$$
$$Y_P \geq \max(Y_L, Y_R),$$
$$Z_P \geq \max(Z_L, Z_R)$$

In the case where P is Y-node or Z-node, similar inequalities are also satisfied. The properties mean that we can decode the slicing-tree into the corresponding packing by the post order traverse of the tree in $O(n)$ time, where n is the number of items (L. Cheng et al., 2004).

If the size of the rectangular solid which corresponds to the entire slicing-tree is smaller than the size of the container, every item can be located into the container and the packing is feasible.

4 PROPOSED ALGORITHM

We extend the three-dimensional slicing-tree to represent placement, loading and unloading of the items. Using the extended representation, we search for a packing that minimizes the number of re-packing with simulated annealing. The search of item placement consists of two phases. In Phase 1, every item are put into the container. In Phase 2, the number of re-packing is minimized.

4.1 Extension of Slicing-Tree

For any two items, if one is unloaded earlier than the loading of the other, they never overlap with each other. So these two items can be located into the same place in the container. To represent the above placement of two items, we introduce "T nodes" as internal nodes of the slicing-tree. The meaning is that any two items, which correspond to the external nodes on the left and right subtree of the T-node respectively, do not exist in a container at the same time. Figure 4(a) shows an original three-dimensional slicing-tree, and Figure 4(b) shows a proposed slicing-tree which is obtained by introducing T node into the slicing-tree shown in Figure 4(a). Note that the clock time of loading and unloading are set and each external node stores the times. Figure 5 (a) and (b) show the placement of items which is represented by the slice tree shown in Figure 4(b).

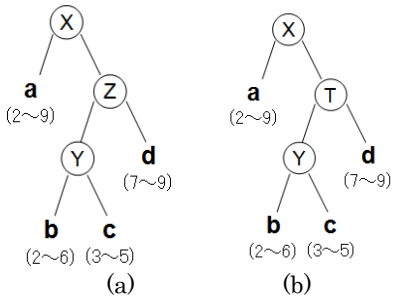
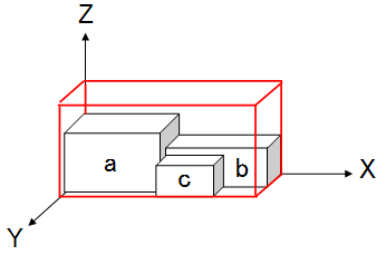
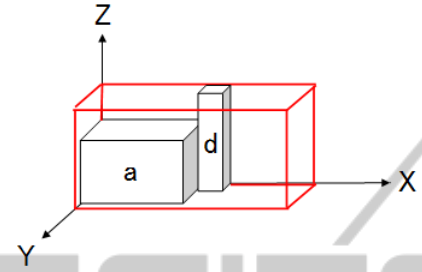


Figure 4: Introduction of T node to slicing-tree.

We can check whether any X(Y, Z) node in the original slicing-tree can be changed to T-node by comparing "clock times of loading of items in the right subtree" and "clock times of unloading of items in the left subtree". Therefore, we obtain the extended slicing-tree by the original slicing-tree in $O(n)$ time.



(a) Placement of items when time clock is 4



(b) Placement of items when time clock is 8

Figure 5: Placement of items corresponding to slicing-tree shown in Figure 4 (b).

If the size of the rectangular solid which corresponds to the left subtree of a T-node P is $(X_L \times Y_L \times Z_L)$, the size of the rectangular solid which corresponds to the right subtree of P is $(X_R \times Y_R \times Z_R)$, and the size of rectangular solid which corresponds to the subtree whose root is P is $(X_P \times Y_P \times Z_P)$ respectively, then the following inequalities are satisfied.

$$X_P \geq \max(X_L, X_R),$$
$$Y_P \geq \max(Y_L, Y_R),$$
$$Z_P \geq \max(Z_L, Z_R).$$

4.1.1 Counting the Number of Re-Packings

Using an extended slicing-tree, we can count the number of re-packing of the placement of items. For any pair of items *a* and *b,* the following necessary conditions are satisfied if *b* will block to *a* loading:
(i) The type of a least common ancestor of *a* and *b* is X-node or Z-node;
(ii) *a* is contained in the left subtree of the least common ancestor ;
(iii) When *a* is loaded into the container, *b* is located in the container.

We can count the number of re-packing by using above conditions in $O(n^2)$ time. However, since the conditions are not a sufficient condition, we may overestimate, if we count only by using the conditions. Such overestimate can be detected by considering the positional relation between *a* and *b*. Thus, we present an algorithm to count the number of re-packing by the proposed slicing-tree. The running time of the algorithm is $O(n^2t)$, where t is the number of clocking time when items are loaded or unloaded.

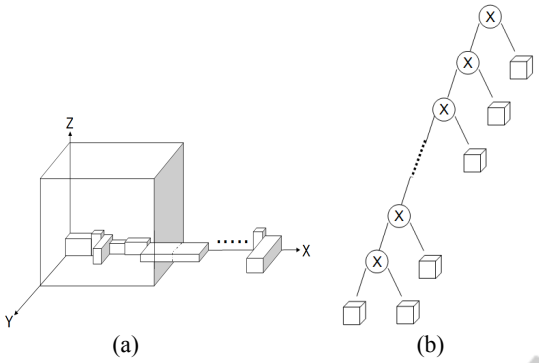
```
Input: extended slicing-tree
Output: N_r  (the number of re-packing
                                  for items loading)

Count_repacking  (extended slicing-tree)
{
N_r = 0;
for (each clock time  t) {
   for (each node n in post-order traverse of the
                              extended slicing-tree) {
     calculate num_of_blocked(n, t) ;
     }
   N_r = N_r + num_of_blocked(root of the extended
                                    slicing-tree, t) ;
   }
return N_r;
}


num_of_blocked(node n, clock time t)
{
if (node n is an external node){
   check the positional relation between n and each item
   if (when the clock time is t, there exists at least
       one item whose loading is blocked by n) return 1;
   else return 0;
   }
else
   return( num_of_blocked(left child of n,  t)
           + num_of_blocked(right child of n,  t)) ;
}
```

4.2 Phase 1

In Phase 1, we don't evaluate the number of re-packings but evaluate the size of a rectangular solid corresponding to the entire extended slice-tree to see whether all items can be located into the container or not.

An objective function of simulated annealing search is the *x*-length of a rectangular solid corresponding to the extended slice-tree. In addition, if the *y*-length or *z*-length is larger than that of the container, a penalty proportional to the surplus is added to the cost function.

We search by using the original slicing-tree (The tree does not include the T-node), and obtain the extended slicing-tree for the evaluation. The initial solution is that the items are laid out on the line. Figure 6 (a) shows the placement of items in the initial solution, Figure 6 (b) shows the corresponding slicing-tree.



(a) (b)

Figure 6: Initial placement of items and corresponding slicing-tree.

The neighbourhood solution is obtained by one of the following four operations:
(a)Rotation of the item: One item is chosen at random, and the direction of the item is changed;
(b)Exchange of subtrees: Two subtrees are chosen at random, and exchanged with each other, if the sub-trees do not contain each other;
(c)Exchange of items: Two external nodes are chosen at random and are exchanged with each other;
(d)Transposition of placement: an internal node is chosen at random and its type (X-node, Y-node or Z-node) is changed.

4.3 Phase 2

In Phase 2, we evaluate the number of re-packings and minimize it. During the search, if a neighbourhood solution is infeasible, the solution is rejected and new neighbourhood solution is created. The initial solution of the search is the result of searching in Phase 1. The neighbourhood solution is obtained by using the above four operations.

5 COMPUTER EXPERIMENTS

We carried out an experiment on a computer to evaluate the performance of the proposed method. The computer environment is an Intel Core i7-200 3.40 GHz CPU with 2 GB of memory. The programming language used is C.

The instances for the experiment were made at random. The number of items is 20, 30 or 50. The length of each side of the items is 10 through 30. The loading clock time and unloading clock time of each item are 1 through 10. Several sets of containers of different size are made. The ratio of length along x, y, and z axis of a container is 5:2:3, which reflects the side ratio of real trucks.

The results of the experiments are shown in table 1, 2 and 3. These results show that we obtain small number of re-packings except in one instance.

Time_Phase1 is the average of the time until all items be located into the container. We almost obtained feasible solutions in a practical time. But the increase of the packing ratio or the number of items makes it difficult to gain the feasible solutions, and we failed loading in some instances. To solve them, we need more flexible re-packing.

Time_Phase2 is the average of the time until the convergence of the search.

Table 1: Experiment results (20 items).

20 items				
total packing ratio(%)	maximum packing ratio(%)	time_Phase1 (sec.)	time_Phase2 (sec.)	number of re-packings
64.80	34.15	0	2310	0
87.50	46.10	1	1167	2
122.16	64.37	1	1167	2
146.52	77.21	1546	113	21
177.82	93.70	loading failure		

Table 2: Experiment results (30 items).

30 items				
total packing ratio(%)	maximum packing ratio(%)	time_Phase1 (sec.)	time_Phase2 (sec.)	number of re-packings
79.46	43.31	2	23542	0
91.99	50.13	2	24161	3
107.29	58.47	3	23877	4
126.18	68.77	134	48683	10
149.79	81.63	loading failure		

Table 3: Experiment results (50 items).

50 items				
total packing ratio(%)	maximum packing ratio(%)	time_Phase1 (sec.)	time_Phase2 (sec.)	number of re-packings
86.89	44.26	53	11318	4
100.91	51.40	48	58699	4
118.17	60.19	491	550445	4
150.14	76.48	loading failure		

In particular, we will show the detail of the result of one of our instances. The instance has 30 items whose length of each side is 10 through 30. The size of the container is $(90 \times 36 \times 54)$. The maximum packing ratio is 68.77%, and the ratio of the sum of volume of all items to the volume of the container (total packing ratio) is 126.18%.

Figure 7 shows the result in the Phase 1, that is the relation between the length of the rectangular solid corresponding to the extended slicing-tree and the searching time.

Figure 8 shows the relation between the number of re-packings and the searching time.

Figure 9 shows the result of placement of items when the packing ration is the maximum (68.77%). The number of repacking of the placement is 10.

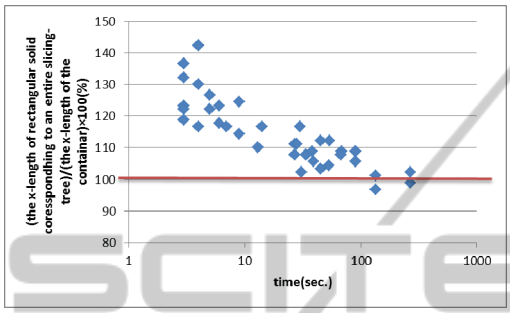


Figure 7: Relation between the length of the rectangular solid corresponding to the extended slicing-tree and the searching time.

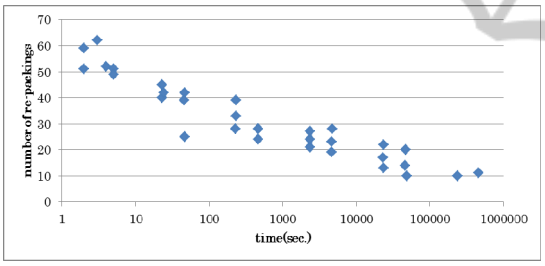


Figure 8: Relation between the number of re-packing and the searching time.

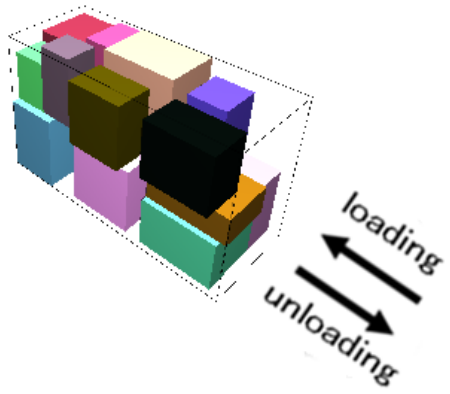


Figure 9: Placement of items when the packing ratio is the maximum.

6 CONCLUSIONS

In this paper, we described a novel packing problem which is the order of loading and unloading of items is given and proposed an algorithm that minimizes the number of repacking. Further works are as follows:

(i) More flexible re-packing: when an item is repacked, we would like to allow that the item is put on arbitrary position in the container, i.e., the position different from the position which the item was unloaded from;

(ii) Application to a more practical case: we should consider about a center of gravity of the items and packaging materials.

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