

Color Quantization via Spatial Resolution Reduction

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Abstract: A color quantization algorithm is presented, which is based on the reduction of the spatial resolution of the input image. The maximum number of colors n_f desired for the output image is used to fix the proper spatial resolution reduction factor. This is used to build a lower resolution version of the input image with size n_f . Colors found in the lower resolution image constitute the palette for the output image. The three components of each color of the palette are interpreted as the coordinates of a voxel in the 3D discrete space. The Voronoi Diagram of the set of voxels corresponding to the colors of the palette is computed and is used for color mapping of the input image.

1 INTRODUCTION

Color quantization is a process that reduces the number of distinct colors present in an input image in such a way to originate a transformed image with a noticeably smaller number of colors and at the same time still visually similar to the input image. One of the main applications of color quantization is for image compression, especially when dealing with the transmission of multimedia data files. These files may have rather large size, so that their transmission may result difficult due to the bandwidth restrictions of computer networks. Other applications of color quantization are for image display, and for color based indexing and retrieval from image databases.

Color quantization methods can be roughly divided in image independent and image dependent methods, see (Brun and Trémeau, 2002), (Celebi, 2011). The former methods, e. g., (Paeth, 1990), (Mojsilovic and Soljanin, 2001), are generally efficient, but produce poor results since the distribution of colors in the input image is not taken into account. Image dependent methods generally provide good results, but are a bit more expensive. Image dependent methods can be divided in pre-clustering methods, e.g., (Heckbert, 1982), (Gervautz and Purgathofer, 1990), (Wu, 1992), (Kanjawanishkul and Uyyanonvara, 2005), and post-clustering methods, e.g., (Ozdemir and Akarun, 2002), (Bing et al., 2004), (Kim and Kehtarnavaz, 2005), (Atsalakis and Papamarkos, 2006), (Chen et

al., 2008), (Ramella and Sanniti di Baja, 2010), (Rasti et al., 2011), (Celebi 2011). Pre-clustering methods determine only once the color palette by using features derived from the image at hand. Post-clustering methods define an initial palette and then improve it by resorting to an iterative process.

Color quantization can be interpreted as a clustering problem in the 3D space, where the three axes are the three color channels and the points are the colors of the input image. To perform quantization, points are suitably grouped into clusters. Then, for each cluster a representative color is selected, which is obtained e.g., as the average of the points in the cluster. The number of clusters, i.e., the number of quantized colors, is generally fixed a priori.

In principle, for a 24-bit true color image I the number of possible colors may reach 16 millions. However, the maximum number of colors actually present in an image consisting of r rows and c columns is $r \times c$. For example, consider an image I with size 1024×1024 . For such an image, at most 1.048.576 different colors are possible. In general, a considerably smaller number of colors exists, since the same color is likely to appear more than just once in the image. On the other hand, if the spatial resolution of I is reduced, say to 256×256 , a maximum of 65.536 colors will be possible for its pixels.

By taking into account the above considerations, we here present a new image dependent technique for color quantization that can be framed among the

pre-clustering methods. The method consists of two processes, respectively dealing with the detection the representative colors and with color mapping.

To find the representative colors, we resort to spatial resolution reduction. This can be achieved by any scaling down method. We use a classical interpolation method (Pratt, 2001) and compute the proper reduction factor that will allow us to build the lower resolution image having the desired size. The reduction of the spatial resolution of the input image automatically implies an upper limit to the number of colors in the palette.

During the second process, the Voronoi Diagram is computed and is then used for color mapping.

2 NOTIONS

Let I be an RGB color image. We interpret colors as three-dimensional vectors, with each vector element having an 8-bit dynamic range. We represent the RGB color space as a 3D cube, where the three coordinates of each point are the red, green and blue components of that point in the color space, see Figure 1. The edges of the cube have length 256, since the values for the color components are in the range $[0, 255]$.

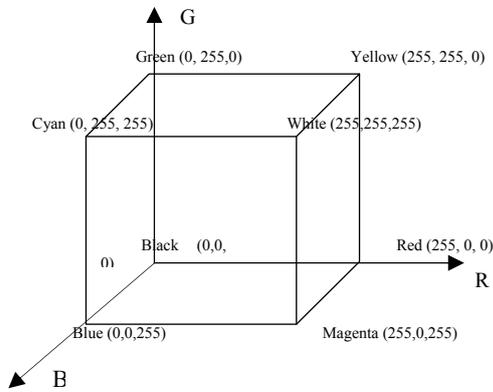


Figure 1: The 3D cube representing the RGB color space.

The color histogram of I can be built by reporting in position (x, y, z) of the 3D cube the number of pixels of I whose three color components have values $x, y,$ and $z,$ respectively. The 3D histogram of colors generally consists of a large and sparse set of points. Since $x, y,$ and $z,$ as well as the value stored in position (x, y, z) are integer numbers, the cube is a discrete cube and we can refer to its points as to voxels. Actually, we use a binary version of the 3D

histogram, where each voxel corresponding to an existing color is set to 1, while all other voxels are set to 0.

To evaluate the performance of our color quantization algorithm, we use the Peak Signal to Noise Ratio $PSNR,$ the Structural SIMilarity $SSIM,$ the Colorloss $CL,$ and the Compression Ratio $CR.$

For gray-level images, $PSNR$ is computed as:

$$PSNR = 20 \times \log_{10} \left(\frac{255}{\sqrt{MSE}} \right) \quad (1)$$

where

$$MSE = \frac{1}{H \times K} \sum_{i=1}^H \sum_{j=1}^K (v_{i,j} - w_{i,j})^2 \quad (2)$$

and $v_{i,j}$ and $w_{i,j}$ respectively belong to the input image and to the output image of size $H \times K.$

For RGB images, the definition of $PSNR$ is still the same, but MSE is the sum over all squared value differences divided by image size and by three.

The $SSIM$ index for two gray-level images v and w is computed as:

$$SSIM(v, w) = \frac{(2\mu_v\mu_w + c_1)(2cov_{vw} + c_2)}{(\mu_v^2 + \mu_w^2 + c_1)(\sigma_v^2 + \sigma_w^2 + c_2)} \quad (3)$$

where μ_v is the average of $v;$ μ_w is the average of $w;$ σ_v^2 is the variance of $v;$ σ_w^2 is the variance of $w;$ cov_{vw} is the covariance of v and $w;$ $c_1=(k_1L)^2$ and $c_2=(k_2L)^2$ are two variables to stabilize the division with weak denominator; L is the dynamic range of the pixel values (255 for 8-bit image); and $k_1=0,01$ and $k_2=0,03$ are default values.

The $SSIM$ index is computed within an 8×8 sliding window, which moves pixel-by-pixel from top-left to bottom-right. As a result, an $SSIM$ index map of the image is obtained, and the overall quality value is defined as the average of the $SSIM$ index map, i.e., the mean $SSIM$ index. The value of $SSIM$ is in the range $[0,1],$ where higher values denote better structural similarity. For RGB images, the $SSIM$ index is computed for the three channel components independently and the quality value is obtained by the average of the three indexes.

The Colorloss CL is used to measure the loss of color information caused by quantization. CL is computed as the Euclidean color distance between a pixel in the original image and the corresponding

pixel in the quantized image. The loss in color information increases with the colorloss. Let I consist of N pixels, and let the RGB values of a pixel p be (r_p, g_p, b_p) . Let I' be the quantized image, where q is the pixel corresponding to p with RGB values (r_q, g_q, b_q) . Then, the average colorloss of a pixel between these two images is defined as follows:

$$CL(I, I') = \frac{\sum_1^N \sqrt{(r_p - r_q)^2 + (g_p - g_q)^2 + (b_p - b_q)^2}}{N} \quad (4)$$

The compression ratio CR measures the percentage of the original size of the image resulting after compression. CR is computed as the ratio between the size of the output stream and the input stream expressed in bit per pixel (Salomon and Motta, 2010).

3 QUANTIZATION METHOD

Let I be the input color image with r_i rows, c_i columns and n_i colors. Let n_f be the maximum number of colors desired for the quantized image. To identify at most n_f colors constituting the palette, we reduce the spatial resolution of I so that the lower resolution image is characterized by size equal to n_f . To this aim, we need to compute the proper value for the reduction factor f .

The reduction factor is the ratio between the number of rows r_f (columns c_f) that will characterize the quantized image and the number of rows r_i (columns c_i) in the input image. Since it is:

$$r_f \times c_f = n_f \quad (5)$$

$$\frac{r_f}{c_f} = \frac{r_i}{c_i} \quad (6)$$

from which it results:

$$r_f = \sqrt{\frac{n_f \times r_i}{c_i}} \quad (7)$$

we compute the reduction factor f that will originate a lower resolution version of I including at most n_f pixels and, hence, characterized by at most n_f different colors.

Once the lower resolution image is available, it includes at most n_f different colors that constitute the palette. The three components of the colors of the palette are interpreted as coordinates of the only

voxels in a $256 \times 256 \times 256$ cube, BH , that are set to 1. Then, we perform connected component labeling, so as to assign a different identity label to each connected component of non zero voxels in BH .

Since some colors of the lower resolution image may be very similar to each other, their corresponding voxels in BH may be connected to each other. Thus, the number of connected components (and, hence, the number of final colors) is likely to be smaller than the number of colors existing in the lower resolution image. If connected components including more than one voxel exist, for any such a component the centroid is computed and is used as representative color for the component.

The 3D Voronoi Diagram, where seeds consist of connected components of voxels instead of consisting of single voxels, is computed to divide BH into as many Voronoi cells as many seeds have been detected. To this aim, distance transformation with identity label propagation is accomplished from the seeds on the zero voxels of BH , by extending to 3D the algorithm suggested in (Fischler and Barrett, 1980) for the 2D case. In this way, the zero voxels of BH are assigned the identity label of the seed to which they are closer. All voxels in the same Voronoi cell are associated the color of the corresponding seed.

The quantized version of I is built during an inspection of I . Each pixel p of I , whose color components have values x , y , and z respectively, receives the color associated to the Voronoi cell including the voxel in position (x, y, z) .

4 EXPERIMENTAL RESULTS

We have tested the quantization method on about 100 color images with different size and color distribution, taken from available repositories (e. g., <http://www.hlevkin.com/>, <http://sipi.usc.edu/database/>, <http://r0k.us/graphics/kodak>, <http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/>). A small set of six test images is shown in Figure 2.

The performance of our method can be appreciated quantitatively by referring to Table 1, where the results for the six test images are summarized. For each test image, $PSNR$, $SSIM$, CL and CR are computed for the quantized images obtained in correspondence with different values of n_f , namely $n_f = 512$, $n_f = 256$, $n_f = 128$, and $n_f = 64$. The value $r_f \times c_f$ is also indicated, as well as the number of final colors N_d .

Table 1: Results for the six test images.

image		$n_f=512$	$n_f=256$	$n_f=128$	$n_f=64$
1)	$r_f \times c_f$	22×22	15×15	11×11	8×8
	N_d	418	210	117	63
	SSIM	0.97	0.96	0.95	0.93
	PSNR	37.53	35.33	33.88	32.05
	CL	4.07	5.44	6.47	8.08
	CR	0.57	0.50	0.45	0.39
2)	$r_f \times c_f$	22×22	15×15	11×11	8×8
	N_d	406	204	116	63
	SSIM	0.95	0.93	0.90	0.87
	PSNR	36.84	34.15	31.98	29.44
	CL	4.76	6.41	8.27	11.10
	CR	0.54	0.47	0.42	0.37
3)	$r_f \times c_f$	18×27	13×19	9×13	6×9
	N_d	438	235	114	53
	SSIM	0.96	0.95	0.93	0.89
	PSNR	36.89	34.96	33.35	30.45
	CL	4.18	5.44	7.00	10.15
	CR	0.57	0.51	0.44	0.37
4)	$r_f \times c_f$	22×23	15×16	11×11	7×8
	N_d	478	235	120	56
	SSIM	0.96	0.94	0.92	0.88
	PSNR	37.78	36.11	13.83	31.34
	CL	4.41	5.49	7.18	9.74
	CR	0.53	0.47	0.41	0.35
5)	$r_f \times c_f$	20×25	14×17	9×12	7×9
	N_d	458	230	105	63
	SSIM	0.96	0.94	0.92	0.90
	PSNR	36.53	34.50	31.87	30.46
	CL	5.20	6.60	8.97	10.61
	CR	0.53	0.47	0.40	0.36
6)	$r_f \times c_f$	18×27	13×19	9×13	6×9
	N_d	260	154	91	47
	SSIM	0.98	0.97	0.96	0.94
	PSNR	37.07	35.11	31.86	30.14
	CL	2.98	3.78	5.40	7.70
	CR	0.55	0.50	0.45	0.38



Figure 2: The six test images. For each image, the size and the number of colors are given.



Figure 3: From top to bottom, the input image and quantization results obtained by using different lower resolution images.

To show qualitatively the performance of our method, refer to Figure 3, where the results obtained by using different lower resolution images to detect the palette are shown for the test image 3).

Figure 4 shows the three lower resolution images with $n_f = 256$, $n_f = 128$, and $n_f = 64$ respectively used to obtain the three results given in Figure 3.



Figure 4: From left to right, the lower resolution images used to obtain the results shown in Figure 3. Pixel size has been increased for visualization purposes.

The quantization method is easy to implement and computationally advantageous. The results are generally satisfactory. Of course, we are aware of the limits of our approach. One drawback is that if n_f is set to a small value (less than 64), the quality of the resulting quantized image decreases significantly. This is due to the fact that the value of n_f conditions the size of the lower resolution image. If such a size is very small, the lower resolution image is not adequate to represent the input image in a satisfactory way.

Another drawback that can affect the method is that colors characterized by large occurrence in the input image, but forming regions with small size (say, smaller than the size of the cells of the decimation grid used by scaling down), are likely to be not considered as colors of the palette.

We are working on possible solutions to alleviate

the above problems.

As for the first drawback, if the desired n_f is very small, the following process can be done. Instead of computing a lower resolution image with size n_f , we build a lower resolution image with a larger size, regarded as adequate for a satisfactory representation of the input image and use it to fix the palette. If the number of colors of the palette is larger than n_f , then colors that are sufficiently close are clustered before computing the Voronoi Diagram.

As for the second drawback, once the lower resolution image has been generated, the palette is enriched by adding colors that, though characterized in the original image by large occurrence, do not exist in the lower resolution image.

5 CONCLUDING REMARKS

A new color quantization algorithm has been presented, which is based on the detection of the representative colors in a lower resolution version of the input color image. The maximum number of desired colors is used to fix the reduction factor and build the lower resolution image. Colors found in the lower resolution image are taken as seeds for the computation of the Voronoi Diagram. Colors of the input image in the same Voronoi cell are associated the color of the corresponding seed.

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