The Effect of Magnetomechanical Resonance on Stokes Vector in Magneto Optical Crystals

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Abstracts: The peculiarities of changes in the polarization of light passing through a crystal have been investigated for the case of magnetomechanical resonance within the model of longitudinal oscillations of a magnetooptical crystal shaped as a thin rod. It is shown that the components of the Stokes vector experiencing frequency amplitude and phase changes.

1 INTRODUCTION

It was shown (Linchevskyi and Petrishchev, 2011) that mechanical stresses arise in a magneto-optical crystal (MOC) under the conditions of magnetomechanical resonance (MR). These stresses cause additional changes in the magnetization and rotation of the polarization plane of light due to the Faraday effect.

If the magnetic field direction differs from the propagation direction of light, a quadratic birefringence (Cotton–Mouton) effect of comparable magnitude arises in cubic ferrimagnets simultaneously with the Faraday effect (Smolenskii et al., 1975). The distribution of mechanical stress over the MOC volume at MR makes the crystal magnetization inhomogeneous. This circumstance hinders the use of a Mueller matrix for an active medium exhibiting linear and quadratic magneto-optical effects when the MOC is homogeneously magnetized (Tron'ko, 1970).

In this paper, we report the results of studying the amplitude and phase-frequency relations for the Stokes vector at the output of an inhomogeneously magnetized MOC using Mueller matrices.

2 MATHEMATICAL MODEL

Figure 1 shows an MOC shaped as a thin rod of length $2l$ oriented along the axis, which coincides with the propagation direction of polarized light with a wave vector ***k***. The light-magnetization axis of MOC coincides with the axis OZ. The electric field component E_y corresponds to the highest velocity of light propagation in the MOC. The magnetic field vector contains a constant component H_0 directed along the axis plays the role of bias field. Its value is chosen so as to provide maximum sensitivity of ferrimagnet magnetization to strains. According to (Bozorth, 1951), the value H_0 should provide magnetic induction at a level of $\approx 0.6B_s$ in the ferrimagnet (B_s is the saturation induction).

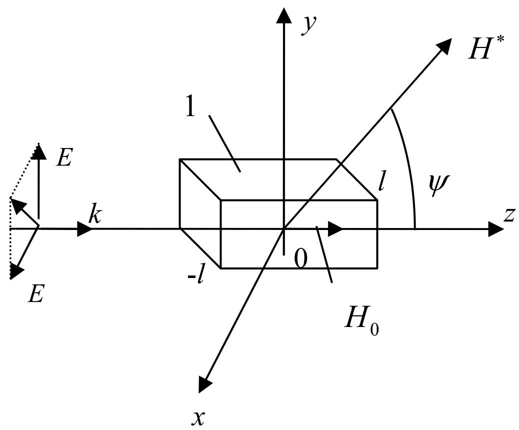

Figure 1: Schematic of the mathematical model: (1) a magneto-optical crystal.

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A harmonically oscillating field $H^* = he^{i\omega t}$ of specified frequency is directed in the drawing plane and makes an angle ψ with the vector ***k***. The amplitudes of the fields and satisfy the requirement $H^* << H_0$. MOC is maintained in a free state (i.e., is not clamped by the design elements) to provide high Q mechanical vibrations.

Under these conditions, periodic (with frequency ω) mechanical stresses arise along the axis due to the magnetostriction in MOC; these stresses induce additional changes in the magnetization J. Note that, within the thin-rod model, the change in the position of the field component with respect to the longer rod axis may change the amplitude of longitudinal mechanical stresses. We assume that the rod performs longitudinal vibrations and that there are no strains in the transverse direction with respect to the rod axis.

At $\psi \neq 0$, the light transmission through an inhomogeneously magnetized MOC is accompanied by both Faraday and Cotton–Mouton effects. The components of the magnetization vector can be written as:

$$\begin{cases} J_y = \chi H^* \sin\psi \\ J_z = \left[\chi + \tilde{m}\Lambda(\dfrac{\cos\gamma z}{\cos\gamma l} - 1) \right] H^* \cos\psi + \chi H_0 \end{cases}, \qquad (1)$$

where: $\Lambda = 0,77\lambda_s J_s / K_1$, J_s - is the saturation magnetization; λ_s - is the saturation magnetostriction; and K_1 is the MOC anisotropy constant, $\tilde{m}$ - piezomagnetic constant.

To construct the Mueller matrix, we divide the MOC into several layers, each of thickness Δz. This value is chosen to be small enough to neglect magnetization inhomogeneity within the layer. Then, using the Mueller matrix for a homogeneous medium, which exhibits Faraday and Cotton–Mouton effects simultaneously (Tron'ko, 1970), and taking into account relations (1), one can express the Mueller matrix of the sample, $[M]$ in terms of the product of matrices of its homogeneous layers:

$$[M] = \prod_{j=n}^{j=-n} \left[M_j(\Delta z) \right], \qquad (2)$$

where: $n = l/\Delta z$ is the number of layers (in the limit, $n \to \infty$),

$$\left[M_j \right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_1 & -s_1 c_2 & s_1 s_2 \\ 0 & s_1 & c_1 c_2 & -c_1 s_2 \\ 0 & 0 & s_2 & c_1 \end{bmatrix},$$

$c_1 = \cos 2a\Delta z; c_2 = \cos b\Delta z; s_1 = \sin 2a\Delta z;$,
$s_2 = \sin b\Delta z$

$$2a = \alpha \left\{ \left[\chi + m\Lambda(\frac{\cos\gamma j\Delta z}{\cos\gamma l} - 1) \right] H^* \cos\psi + \chi H_0 \right\}$$

$$2b = \beta\left(\chi H \sin\psi\right)^2, \dot{\gamma} = \gamma\left(1 - i/2Q\right),$$

$\gamma = \omega\sqrt{\rho/Y}$, Y, ρ and χ are, respectively, the Young's modulus, density, and magnetic susceptibility of the magnetized rod; α - the proportionality factor between the angle of rotation of polarization plane, normalized to the length unit of MOC and its magnetization; β - relative phase shift between the components of the field E_x and E_y .and is the specific phase shift between the field components and at a specified field. Using matrix (2), we determine the variable components of the Stokes vector at the MOC output for $\psi = 45^0$ using the example of yttrium garnet ferrite ($Y_3Fe_5O_{12}$):
$2l = 15\text{mm}, Y = 138\text{GPa}, \rho = 5{,}17 \cdot 10^3\ \text{kg/m}^3$,
$m = 1060\text{T}, K_1 = 6{,}2 \cdot 10^2\ \text{J/m}^3$,
$\lambda_s = -1.4 \cdot 10^{-6}, Q = 200, J_s = 11{,}4\,\text{êÀ/m}$,
$\alpha = 1{,}3\,\text{deg/À}, \beta = 3{,}9 \cdot 10^{-4}\ \text{deg×m/À}^2$

Parameters of the magnetic field were
$H_0 = 635\,\text{À/m}, h = 20\,\text{À/m}$.

For definiteness, we assume that the initial light is plane-polarized with an azimuth of the electric vector oscillation plane equal to 45° (the Stokes vector at the input is $(V_1) = (1,0,1,0)$). At the MOC output, the Stokes vector has the form

$$(V_2) = [M](V_1) \qquad (3)$$

Following the designations of the Stokes vector components (V) = (I, M, C, S) according to (Shercliff, 1962), we should note that the elements M, C, and S of the vector (V_2) contain both constant and variable components:

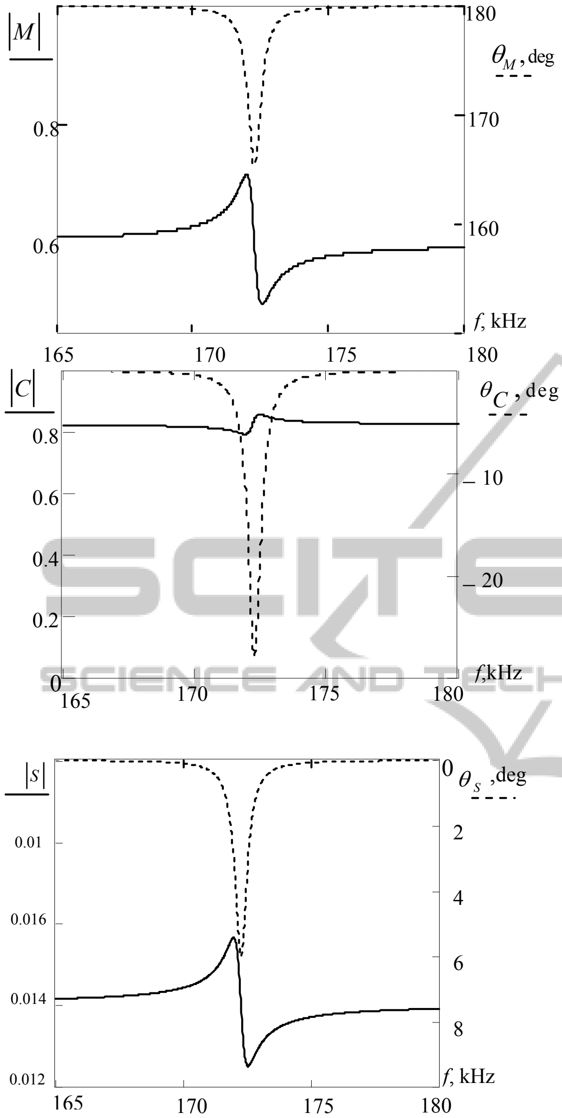

Figure 2: Linear-frequency dependences the magnitude (—) and the initial phase shifts (---) of the M, C, S Stokes vector elements.

$$\left(V_2\right) = \begin{pmatrix} 1 \\ M_0 \\ C_0 \\ S_0 \end{pmatrix} + \begin{pmatrix} 0 \\ me^{i\theta_M} \\ ce^{i\theta_C} \\ se^{i\theta_S} \end{pmatrix} \quad (4)$$

The constant components M_0, C_0, S_0 are determined by the magnetic field H_0, whereas the variable components s m, c and their initial phase $\theta_M, \theta_C, \theta_S$ depend on a number of factors: H^*, H_0 and ω (at MR).

Figure 2 shows the results of calculating the magnitude dependence $|M|, |C|, |S|$, and the initial phase $\theta_M^0, \theta_C^0, \theta_S^0$ of the variable components m, c, s of the linear frequency f. Note that the component for polarized light can also be found from the relation:

$$\left(M_0 + me^{i\theta_M}\right)^2 + (C_0 + ce^{i\theta_C})^2 + +(S_0 + se^{i\theta_S})^2 = 1 \quad (5)$$

3 CONCLUSIONS

Magneto mechanical oscillations induced in MOC due to the change in the variable component of magnetization change the polarization of light.

When the direction of the dc (bias) magnetic field and the light propagation direction does not coincide with the propagation direction of light, the inhomogeneity of MOC magnetization due to the MR causes additional amplitude and phasefrequency changes in the variable components of ellipticity and the components X and Y of polarized light.

The results obtained can be used to determine the MOC material constants and design magnetic field sensors on their basis.

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