

On the Accuracy of Representing Heartbeats with Hermite Basis Functions

David G. Márquez¹, Abraham Otero², Paulo Félix¹ and Constantino A. García¹

¹*Centro Singular de Investigación en Tecnoloxías da Información (CITIUS), University of Santiago de Compostela, 15782 Santiago de Compostela, Spain*

²*Department of Information Systems Engineering, University San Pablo CEU, 28668 Madrid, Spain*

Keywords: Heartbeat Representation, Hermite Functions, ECG.

Abstract: Automatic ECG analysis requires choosing a representation for heartbeats. A common approach is using some basis of functions to represent the heartbeat as a linear combination of these functions. The coefficients of the linear combination are used as the features that represent the heartbeat, providing a very compact representation. The most used basis of functions is the one made up of the Hermite functions. Some authors have used as few as 3 Hermite polynomials to represent each heartbeat, while others have used as many as 20. Often little or no justification for the choice of the number of polynomials is given. This paper aims to analyze the impact of using a certain number Hermite polynomials on the accuracy of heartbeat representation. Tests were run fitting the heartbeats of the MIT-BIH arrhythmia database with a number of polynomials ranging from 2 to 20. Three different strategies to determine the heartbeat's position were used. The fitting errors are reported here. Based on these results, some guidelines to choose a suitable number of Hermite polynomials for different applications are given.

1 INTRODUCTION

The electrocardiogram is a simple and inexpensive test for the diagnosis of multiple cardiovascular diseases. Its main disadvantage is probably the large amount of information that it generates; e.g., a 24-hour Holter recording can contain up to 100,000 heartbeats. Thus, visual inspection of the recording can be a tedious and time-consuming task. This is the reason why the biomedical engineering community has attempted to provide tools for the automatic analysis of ECG recordings.

Automatic ECG analysis starts with the detection and characterization of heartbeats. Errors in this task can invalidate the rest of the analysis. In the literature there are three main approaches to represent beats: using the digitized signal (Hu et al., 1993), extracting heartbeat interval features (De Chazal and Reilly, 2006) and using some basis of functions (Lagerholm et al., 2000). Using the digitized signal prevents any loss of information, but this representation is difficult to work with due to its large size, and it is very sensitive to noise. Using heartbeat interval features is the closest representation to the clinicians' modus operandi when they interpret beats. However it is dif-

icult to obtain a robust extraction of these features. The basis of functions have a good performance under noisy conditions and can provide a very compact representation of the beat.

The basis of functions most commonly used is the one made up of the Hermite functions. When using this approach, a choice must be made about the number of Hermite polynomials to be used in the representation of the beats. There are some authors that use as few as 3 polynomials (Braccini and Edenbrandt, 1997), and others use as many as 20 (Park et al., 2008). Usually the authors provide little or no justification for the number of polynomials used.

This paper aims to analyze the impact of using a certain number of Hermite polynomials in the representation of a heartbeat. Section 2 describes the database used in our analysis, the preprocessing applied to the ECG signal, and how the error between the representation obtained from the Hermite basis functions and the original signal was calculated. Section 3 describes the results obtained when fitting the beats with different numbers of Hermite polynomials, and Section 4 discusses these results, providing some guidelines to choose a suitable number of Hermite polynomials.

2 MATERIAL AND METHOD

2.1 ECG Database

The database most commonly used in the papers dealing with automatic beat classification is the MIT-BIH arrhythmia database. Therefore, this will also be the database we shall use in our study. The MIT-BIH arrhythmia database (Moody and Mark, 2001) is made up of 48 ECG recordings of two channels among the modified limb lead II (MLII) and the modified leads V1, V2, V3, V4 and V5. The recordings are digitized at 360 Hz sampling rate. All beats in the database were annotated by two or more cardiologist.

2.2 Preprocessing

To eliminate the baseline drift a wavelet based filter was used. To remove the high frequency noise a low-pass 4 order Butterworth filter with a cutoff frequency of 40 Hz was used. One of the theoretical advantages of representing beats with the Hermite polynomials is the robustness of the representation in the presence of noise. To empirically test this, we shall run our tests both directly on the recordings, and over a filtered version of the recordings.

Theoretically, Hermite polynomials will provide a better characterization of the beat if the point of maximum symmetry is selected as the center of the window of signal to be fitted. This point is usually the peak of the QRS complex, the R wave. Furthermore, setting the beat location in a stable position within the QRS complex will lead to more reproducible results, and therefore to features that will be more easily recognized by an automatic classifier. To try to achieve a more stable beat's position within the QRS complex, and to get as close as possible to the point of maximum symmetry, an algorithm to improve the beats' location provided in the MIT-BIH arrhythmia database was used. The algorithm calculates the mean in a 200 ms window around the annotation provided in the database (the annotation handmade by cardiologists). Usually, the R wave peak is the farthest point from the mean value. This point is selected and a new window of 200 ms around it is extracted from the signal.

The correction to the beat's position can be applied only to one channel or to both channels independently. If it is only applied to one channel, the position of the R wave peak is assumed to be equal for both channels (this is not necessarily true in practice). Otherwise, the location of the R wave peak may be slightly different for each channel.

We have run one test using the beat's positions provided by the MIT-BIH arrhythmia database, the solution most commonly used in the literature. A second test was performed applying the beat location correction algorithm over the first channel and using the same beat location in the second channel. Finally, a third test was run applying the beat location correction algorithm over both channels independently. Each of the three strategies was applied directly over the MIT-BIH arrhythmia database signal recordings, and over the filtered version of the recordings, yielding a total of six different tests.

2.3 Hermite Functions

We will extract each heartbeat's QRS by taking a 200 ms window of sampled ECG centered on the beat's position, being the beat's position calculated by one of the three strategies presented in the previous section. This window is wide enough to encompass the entire QRS complex of a normal beat, but narrow enough not to include the P and T waves. The width of this window is the one normally used in the literature (Lagerholm et al., 2000).

All the Hermite functions converge to zero both in ∞ and in $-\infty$. Thus, we shall add 100 ms zeros on each side of the 200 ms window containing the QRS. Let us denote by $x(t)$ the resulting 400 ms window. $x(t)$ can be represented as:

$$x(t) = \sum_{n=0}^{N-1} c_n(\sigma)\phi_n(t, \sigma) + e(t) \quad (1)$$

where N is the number of Hermite polynomials used in the representation of the beat, $\phi_n(t, \sigma)$ is the n Hermite function, c_n are the coefficients of the linear combination, σ is a parameter that controls the width of the polynomial, and $e(t)$ is the error between $x(t)$ and the Hermite approximation. For details on how to calculate c_n and σ see (Lagerholm et al., 2000).

2.4 Error Measurement

(Lagerholm et al., 2000) used the following measure to quantify the error of the approximation:

$$\epsilon = \frac{\sum_t |e(t)|^2}{\sum_t |x(t)|^2} \quad (2)$$

This measure will be calculated in our test, to be able to compare our results with the ones of Lagerholm et al. We shall also calculate another measure that we believe is more easy to interpret: the normalized root-mean-square error (NRMSE) between the

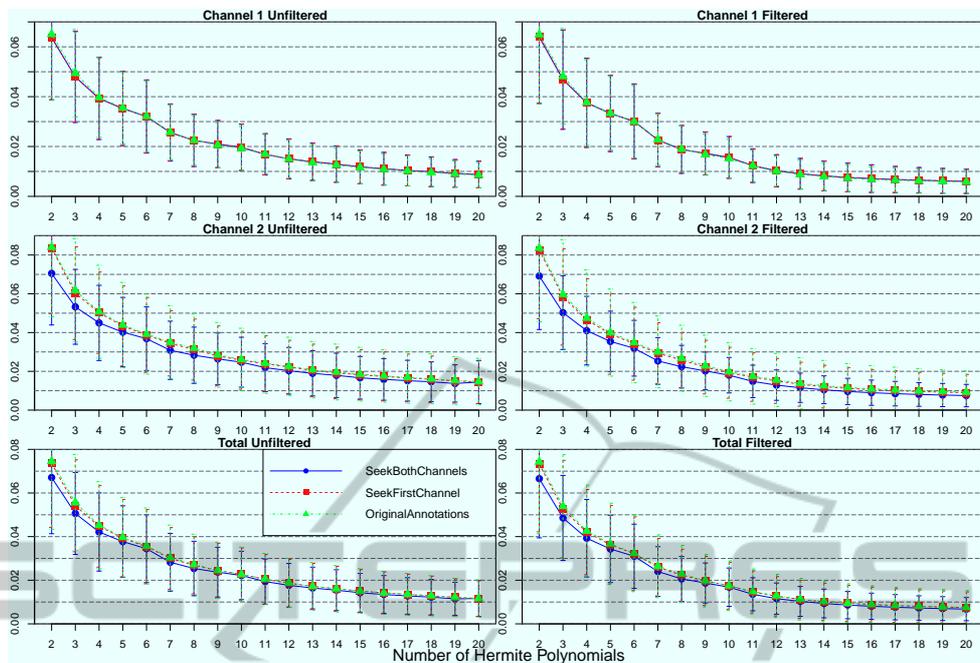


Figure 1: NRMSE results for the unfiltered and filtered signal for the three different strategies to determine the beat's position.

Hermite reconstruction and the sampled signal:

$$NRMSE = \frac{RMSE}{x_{max} - x_{min}} = \frac{\sqrt{\frac{\sum_t |e(t)|^2}{N}}}{x_{max} - x_{min}} \quad (3)$$

where N is the size of the window in samples. The NRMSE can be interpreted as the average error expressed as a percentage of the range of values in the signal fragment ($x_{max} - x_{min}$).

3 RESULTS

The results of the average NRMSE (see Equation 3) through all recordings are shown in Figure 1. The errors of each channel and the average error of the two channels are shown. The results corresponding with the beat's positions provided in the database, the beat's position correction applied to the first ECG channel, and the beat's position correction applied to both channels are marked with triangles, squares and circles, respectively. The bar shows the standard deviation of each error. The graphs on the left are the results for the unfiltered signal and the graphs on the right are the results for the filtered signal.

Figure 2 shows Lagerholm's error measure (see Equation 2) when using the beat positions provided in the database, and when the correction is applied to both channels. Results are shown both for the filtered and unfiltered signal.

4 CONCLUSIONS AND DISCUSSION

The results in the previous section show that even with a small number of Hermite functions, beats can be represented acceptably. This is not surprising at all; there are authors in the literature that use as few as 3 functions to represent the beats (Braccini and Edenbrandt, 1997). 7 polynomials may be a sweet spot; between 6 and 7 we can still appreciate a significant improvement in Figure 1 and Figure 2; but after 7 the improvements are smaller. At least when the final goal is to obtain a beat classification, it is questionable whether it is worth using a number as high as 20 polynomials (Park et al., 2008), since the benefits obtained from a slightly more accurate representation of the beats may be overtaken by the disadvantages of training classifiers in a higher dimension space: going from 12 functions to 20 produces a decrease of approximately 0.005 in the total NRMSE both over the filtered and the unfiltered signal (see Figure 1).

The beat's position correction algorithm, especially when applied to both channels, provides noticeable improvements of the results. These improvements are more marked in the second channel, especially when using low numbers of Hermite functions. The reason why the correction provides better results on the second channel is probably because the MIT-BIH arrhythmia database has been annotated over the first channel (Moody and Mark, 2001). The reason

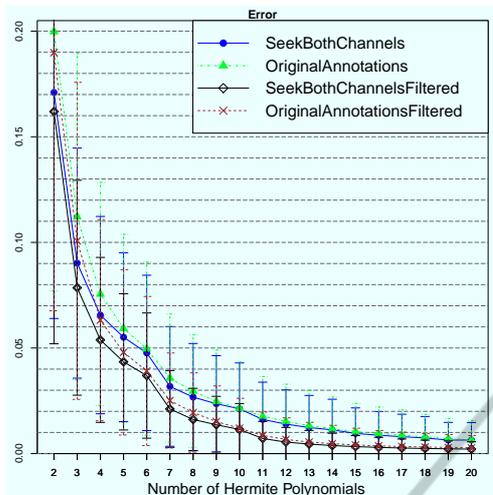


Figure 2: (Lagerholm et al., 2000) error measure.

why more improvement is obtained for a low number of polynomials is because when using a high number of polynomials it is possible to represent the beat accurately even if the point chosen as the center of the fitting window is not the point of maximum symmetry (see Figure 1).

Filtering provides significant improvements in the results (see Figure 1 and Figure 2). We have performed independent tests using only high frequency filtering and only baseline drift removal. The removal of baseline drift alone produced virtually identical results to working directly with the unfiltered signal; almost all the improvements that can be seen in Figures 1 and 2 when using the filtered signal arise from the high frequency filtering. This suggests that Hermite approximation is more affected by high frequency noise than by baseline drift. For example, a 2% of NRMSE can be achieved without filtering with 11 polynomials but with filtering only 8 are required; and we cannot reach a 1% of NRMSE without filtering, not even with 20 polynomials, while with filtering is possible to reach this error with 13 (see Figure 1).

Among the papers we have reviewed only (Lagerholm et al., 2000) reports error results for the Hermite approximation. Lagerholm et al. calculated the error with Equation 2. They only report the values for 3, 4, 5 and 6 Hermite polynomials; the errors are 9.7%, 6.8%, 5.5% and 4.5%, respectively. These results are slightly lower than the results we obtained with our beat correction algorithm applied over both channels with the unfiltered signal. However, when using the filtered signal the errors we obtain are lower than the results of Lagerholm et al., both when using the original beat annotations from the database, and when using the beat position correction over both channels. It should be noted that Lagerholm et al. applied no

high-frequency filtering.

In this paper we have determined the accuracy of the representation with a measure of the error between the reconstruction obtained from the Hermite polynomials and the original signal. However, if the final goal of representing beats with Hermite polynomials is to classify them in different morphological families (instead of, for example, compression of the ECG (Jane et al., 1993)), the features that minimize this error need not to be those that provide the best separation between the different classes of beats. It would be interesting to study how the features obtained when representing the beats with a different number of Hermite polynomials enable the different beat families to be separated by an automatic classifier. This will be one of our lines of future work.

ACKNOWLEDGEMENTS

This work was supported by the Spanish Ministry of Science and Innovation (MICINN) under grant TIN2009-14372-C03-03.

REFERENCES

- Braccini, G. and Edenbrandt, L. (1997). Self-organizing maps and Hermite functions for classification of ECG complexes. *in Cardiology* 1997, 24:425–428.
- De Chazal, P. and Reilly, R. B. (2006). A patient-adapting heartbeat classifier using ECG morphology and heartbeat interval features. *IEEE Transactions on Biomedical Engineering*, 53(12 Pt 1):2535–43.
- Hu, Y., Tompkins, W., and Urrusti, J. (1993). Applications of artificial neural networks for ECG signal detection and classification. *Journal of Electrocardiology*, 26:66–73.
- Jane, R., Olmos, S., and Laguna, P. (1993). Adaptive Hermite models for ECG data compression: performance and evaluation with automatic wave detection. *Computers in Cardiology*.
- Lagerholm, M., Peterson, C., Braccini, G., Edenbrandt, L., and Sörnmo, L. (2000). Clustering ECG complexes using hermite functions and self-organizing maps. *IEEE Transactions on Biomedical Engineering*, 47(7):838–48.
- Moody, G. and Mark, R. (2001). The impact of the mit-bih arrhythmia database. *Engineering in Medicine and Biology Magazine, IEEE*, 20(3):45–50.
- Park, K., Cho, B., Lee, D., Song, S., Lee, J., Chee, Y., Kim, I., and Kim, S. (2008). Hierarchical support vector machine based heartbeat classification using higher order statistics and hermite basis function. In *2008 Computers in Cardiology*, pages 229–232. IEEE.