Evolutionary Systems Agents' Mathematical Models

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Abstract: General mathematical theory of evolutionary system developed earlier is implemented to various problems of artificial intelligence and intelligent agent mathematical modeling. Examples of application of this general theory to the evolutionary systems such as economics, education, and health care are also considered.

1 INTRODUCTION

The general mathematical theory and mathematical models (MM) of the evolutionary system (ES) and its various applications were launched by the papers (Glushkov, 1977), (Glushkov and Ivanov, 1977) and can be seen in the monographs (Hitronenko and Yatsenko, 1999),(Hitronenko and Yatsenko, 2003), (Ivanov and Ivanova, 2006),(Korzhova et al., 2011). The notation of intelligent agents (IA) can be seen in (Wikipedia, 2011).

The present paper is devoted to another interpretation of the notions of work place and active center adapted to the notion of IA from the point of view of artificial intelligence.

This interpretation allows researchers and constructors to apply the results in (Hitronenko and Yatsenko, 1999),(Hitronenko and Yatsenko, 2003),(Ivanov and Ivanova, 2006) for the creation of artificial intelligent systems and the respective robotics.

2 THE BASE MM OF ES

The basic minimal or simplest MM has the form

$$m(t) = \int_{a(t)}^{t} \alpha(t,s)y(s)m(s)ds,$$

$$0 \le y \le 1, 0 \le a(t) \le t, \alpha \ge 0,$$

$$c(t) = \int_{a(t)}^{t} \beta(t,s)[1-y(s)]m(s)ds, \beta \ge 0,$$

$$R(t) = \int_{a(t)}^{t} m(s)ds, M(t) = \int_{0}^{t} m(s)ds,$$

$$G(t) = M(t) - R(t),$$

$$f(t) = m(t) + c(t), t \ge t^* \ge 0. \quad (1)$$

where $m(t)$ is the rate of creation of the first kind new generalized product (resource) quantity at the time instant t, which provides the fulfillment of the internal functions of ES, that is, restoration of itself and creation of the second kind product; $y(t)m(t)$ is a share of $m(t)$ for fulfillment of internal functions in the subsystem A of restoration and perfection of the system as a whole; $\alpha(t,s)$ is the efficiency index for functioning of the subsystem A along the channel $\alpha(t,s)y(s)m(s) - m(t)$, i.e., the number of units of $m(t)$ created in the unit of time starting from the instant t per one unit of $y(s)m(s)$; $a(t)$ is a special temporal bound: the new product created before $a(t)$ is never used at the instant t, but created after $a(t)$ is used entirely; $c(t)$ is the rate of creation of the second kind new generalized product quantity at the instant t, which provides the realization of the external functions of ES;$[1-y(s)]m(s)$ and $\beta(t,s)$ are similar to ym and α respectively but for the subsystem B of creation of the second kind product; $R(t)$ is the total quantity of the first kind product functioning at the instant t; $M(t)$ is the total quantity of the first kind product to be created during the time $[0,t]$; $G(t)$ is the total quantity of the obsolete product at the instant t; $f(t)$ is the rate of the resource inflow from the outside ($m(t)$ and $c(t)$ are measured in the units of $f(t)$); t^* is the starting point for modeling; $[0,t^*]$ is the prehistory of ES, for which all the functions are given (their values will be noted by the same symbols but with the sign "$*$",

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e.g., $m(t) = m^*(t)$, $t \in [0,t^*]$).

It is obvious that all the relations (1) are faithful representations by definition. In a general case, the indices α and β depend on m, c, a, y, R, M, G, and f.

It can be seen that (1) consists of 7 equalities and 7 inequalities connecting 14 values, namely: m, c, α, y, β, $1-y$, a, R, M, G, t, t^*, f, and 0, all of which are nonnegative. Usually, α, β, y, f, and/or R are given, and the others are to be found. Even in the simplified formulation, MM (1) is the system of nonlinear functional relations, in which along with the nonlinear integral equation of the unusual form (the lower bound a(t) can be unknown function) we have the system of functional inequalities.

The respective new IA at the time t in the subsystem A, B of ES are $y(t)m(t)$, $[1-y(t)]m(t)$, functioning in accordance to MM (1). It should be noted that this magnitudes most properly include combination of IA and usual work places (WP) labor functions, which are fulfilled by human beings. The base simplest self-organized ES has the following MM:

$$\begin{aligned}
&\alpha'(t) = \int_{a(t)}^{t} \alpha(t,s)x(s)m(s)ds,\\
&m(t) = \int_{a(t)}^{t} \alpha(t,s)y(s)m(s)ds,\\
&c(t) = \int_{a(t)}^{t} \alpha(t,s)z(s)m(s)ds,\\
&\alpha(s,t) = \alpha(s)e^{d(s-t)}, d \geq 0,\\
&0 \leq x,y,z \leq 1,\ x+y+z = 1,\\
&f(t) = \alpha'(t) + m(t) + c(t),\\
&t \geq t^* > 0, \qquad (2)
\end{aligned}$$

where xm is a share of m for creation in the subsystem C new technology of ES.

Thus, the respective new IA and WP at the time t in the subsystem A, B, and C of ES are $y(t)m(t)$, $z(t)m(t)$, *and* $x(t)m(t)$, functioning in accordance to MM (2).

3 MORE COMPLICATED MM OF ES

The n-product MM, $n > 2$, can be formally written in the same form (1), where m, a, and c are the vector functions, and α, y, β, and z are the respective matrices (where the inequalities for the vectors and matrices are the same inequality for their appropriate components). The continuous MM can be described in the same form considering t and s as many-dimensional variables and examining the appropriate integrals as multivariate ones. The stochastic MM can be obtained by considering α, β , and f as functions of a random factor ω. The discrete MM can be represented in the same form if the integrals in (1) are understood in the sense of Stieltjes. The MM of ES (2) can be generalized by the similar way. Thus, according to those MM we have IA of the previous types for more complicated systems.

4 PROBLEMS IF ... THEN ...

In the case of MM (1) the problem 'if ... then ...' means that, for example, when α, β, y, f and/or R are given (and all the functions on the prehistory of ES functioning), the other functions are to be found using MM (1). In the case of MM (2), the problem means that, for example, when x, y, f, and/or R are given, and the other functions are to be found using MM (2). For intelligence systems, those kind of problems are rather important because they allow us to make theoretical experiments before practical realization.

5 AN OPTIMIZATION PROBLEM

One of the important typical optimization problems for ES is maximization of the functional

$$I(y) = \int_{t^*}^{t} c(t)dt = \int_{t^*}^{t} \left(\int_{a(t)}^{t} \beta(t,s)[1-y(s)]m(s)ds\right)dt, \qquad (3)$$

over y with regard to MM (1).

The first essential result on the properties of solutions of the problem (3) has been obtained in (Glushkov and Ivanov, 1977)(the first law): *The record of an external function for any ES can be obtained only under the conditions of its sufficiently comfortable guarantee, that is, under the significant fraction of resources sent to the internal needs of ES.*

As to the same problem (3) and MM of ES of (2)-type, it was proven under certain conditions that the second law (Korzhova et al., 2011) is valid:

The record of an external function for any ES can be obtained only under the following priority of resource distribution: the highest priority has subsystem C, then subsystem A, and then subsystem B.

6 ON APPLICATIONS

6.1 Economics IA Models

In the case of economics, the agents besides the skill of using MM (1) or (2), have to know how to use the base of data on labor functions and product and services in order to properly change, e.g., the function $c(t)$ with regard to the up-to-date state of affairs.

6.2 Education IA Models

In this case, the external product $c(t)$ is a rate of graduate specialists number at the instant t, so that the respective agents have to know how to use the base of data on various tests and up-to-date needs of new professionals.

6.3 Health Care IA Models

In this case, the external product $c(t)$ is a rate of patients number subjected to prophylaxis, diagnosis, or therapy at the instant t, so that the respective agents have to know how to use the base of data on various tests of the norm and pathology for human beings as well as on various up-to-day remedies of prophylaxes, diagnosis and curing. Let us dwell on this application with much more detail.

6.3.1 MM of Organism Subsystems

For many applications the following MM of organism subsystem (OS) should be considered:

$$\begin{aligned}
&m(t) = \int_{a(t)}^{t} \alpha(t,s)u(s)v(s)v^{-1}(t)y(s)m(s)ds,\\
&c(t) = \int_{a(t)}^{t} \beta(t,s)[1-y(s)]m(s)ds, \beta \geq 0,\\
&R(t) = \int_{a(t)}^{t} m(s)ds,\\
&G(t) = \int_{a(t)}^{t} \beta(t,s)m(s)ds,\\
&M(t) = R(t)+G(t),\\
&f(t) = m(t)+c(t) = k(f^*,t)c(t), t \geq t^* \geq 0,
\end{aligned} \quad (4)$$

where α, u, v, y, and β are matrices, 1 is the unit matrix (inequalities for matrices mean the respective inequalities for their components) α, a, m, c, R, G, M, and k are vectors, and the relations for f (f^* is the qualitative structure of f) should be replaced by

$$f(t) = \sum_{i=1}^{r} m_i(t) + \sum_{j=1}^{s} s_j(t) + \sum_{j=1}^{s} k_j(t)c_j(t),$$
$$r+s = n. \quad (5)$$

In addition, the equations of homeostasis should be included; that is,

$$R_i'(t) = m_i(t) - m_i(t)(a_i(t))a_i'(t),$$
$$i = 1,...,r. \quad (6)$$

It should be emphasized that considering the given subsystem of organism with the same kind of MM in detail, we should include in its MM an aggregate MM of the rest of the whole organism.

6.3.2 Conception of the Norm and Pathology

All the values in MM under consideration, (see, for example, (1-3) have rather profound sense from the point of view of the structure and functions of the subsystem under consideration. So, if the vector of these values is denoted by $R = (r_1,...,r_q)$, then it is natural to suppose that one of the conceptions of the norm consists in the validity of the relations

$$r_s^- \leq r_s \leq r_s^+, \; s = 1,...,q, \quad (7)$$

where r_s^- and r_s^+ are admissible bounds for variations of the introduced values from point of view of the norm. On the other hand, if at least one of the relations in (7) is violated, then it is natural to speak about conception of the pathology or the pathological state of the subsystem.

6.3.3 Norm Restoration Problems

The most simple and expanded restoration is that the determined values of the group A") are injected into the organism directly or, on the contrary, are removed from the organism. The respective medicines should be injected into the organism to achieve validity of the necessary bounds in (7) relative to this group. However, under the condition that the indices of violations of the group A") are accompanied by those of the group A') and class B), such a simple method is usually the temporal measure and does not deduce restoration.

For increasing (decreasing) values of $m(t)$-type, provided that the values of the other magnitudes are not to be broken, it is usually necessary to increase (decrease) the value components of the distributive matrices.

Thus, the crucial condition for restoration is the increase of a share of all resources of the subsystems to its internal requirements.

The most serious and profound violations are connected to the structural shift of the subsystem, i.e., with the violation of (7) for the indices of the class B. Here only the interference into the structure of f^* and the genome apparatus can likely restore the norm.

Although in principle, in this case, there can be developed a theory similar to the above theory. It means the following principle of the norm of organism restoration:

The crucial condition for restoration (under weakened functions of the classes A and B) is the increase of the share of all the resources to the needs of a comfortable state of OS structure first, then its internal sphere, and after that its external sphere.

6.4 On MM of a Doctor's and/or Health Care Intelligent Agent's Business

The simplest, base MM of a doctor's and/or health care IA's (D and/or IA) business has the form

$$
\begin{aligned}
&m(t) = \int_0^t \alpha(t,u)\lambda(t,u)y(u)m(u)du, \\
&c(t) = \int_0^t \beta(t,u)\mu(t,u)[1-y(u)]m(u)du, \\
&R(t) = \int_0^t \lambda(t,u)y(u)m(u)du + \\
&\int_0^t \mu(t,u)[1-y(u)]m(u)du, \\
&G(t) = \int_0^{a(t)} \beta(t,s)m(s)ds, \\
&f(t) = M(t) - R(t), \\
&M(t) = \int_0^t m(u)du, f(t) = m(t) + c(t) = \\
&kc(t),\ t \geq t^* \geq 0,\ t > 0, \qquad (8)
\end{aligned}
$$

where m is the new resource (per unit of time) of a D and/or IA WP, providing its internal functions; c is some new resource providing external functions, including the function of curing; λ and μ are the intensities of functioning along the channels ym–m, $[1-y]m$–c respectively; $R(t)$ is the total amount of functioning internal resource; y, α, β, G, k, f, and t^* have the same definitions as many times above.

The function of curing decides a D and/or IA business. The number of patients' c_i who have been cured for i^{th} disease in the unit of the time is actually decided by the same function. Therefore, this function should be well estimated. It should be estimated by the total number of the patients who have been successfully cured in a certain period of time. The methods of curing or restoring to the norm depend on many factors. We would like to dwell only on some approaches to curing, following from the models.

To increase (decrease) the values of $m(t)$-type, if the values of the other magnitudes are not broken, it is usually necessary to increase (decrease) the values $y(s)$, $s < t$. Thus, the crucial condition for restoration (under weakened functions of the class A) is the increase of a share of all the resources of a D and/or IA to his/her/IA internal requirements: development of a new, much more effective technology.

One may hope that any modes of action, among them nontraditional ones, including the various physical loading on a certain part of an organism and contributing to increasing of resource distribution in advantage to the internal sphere of the sick subsystem, will usually bring an essential positive effect.

In conclusion of this subsection, we would like to note that (Ivanov and Ivanova, 2006), (Korzhova et al., 2011) contain MM and their applications to many concrete diseases such as AIDS, cancers, diabetes, etc.

7 CONCLUSIONS

In conclusion, we would like to emphasize that the theory and application of MM of ES and all the techniques under consideration may be expanded far beyond the examples already described.

However, the creation of the respective intelligent systems and robotics requires still much combined effort of various specialists, which is much more ahead of us.

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