

# An Entropy-based Method for Color Image Registration

Shu-Kai S. Fan<sup>1</sup> and Yu-Chiang Chuang<sup>2</sup>

<sup>1</sup>*Department of Industrial Engineering and Management, National Taipei University of Technology, 1, Sec. 3, Chung-Hsiao E. Rd. Taipei 106, Taiwan, Taiwan*

<sup>2</sup>*Department of Industrial Engineering and Management, Yuan Ze University, 135, Yuan-Tung Rd., Chung-Li City, Taoyuan County 320, Taiwan, Taiwan*

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Abstract: In this paper, an entropy-based objective function is developed according to the histogram of the color intensity difference data. The proposed registration method is to orientate the sensed image toward the reference image by minimizing the entropy of the color intensity differences by iteratively updating the parameters of the similarity transformation. For performance evaluation, the proposed method is compared to two noted registration methods in terms of a suite of test images. The experimental study is conducted to verify the effectiveness of the proposed method. Through the experimental results, the proposed method is shown to be very effective in image registration and outperforms the other two methods in terms of the test image sets.

## 1 INTRODUCTION

In the research field of image registration, the cross-correlation method has been widely recognized as a standard intensity-based method. Cross-correlation is used for measuring the similarity between the reference and sensed images over the overlapped region. Henceforth, the matching region aforementioned is referred to as the overlapped region between the reference and sensed images in this study. There have been proposed several variant forms of cross correlation, which additionally took edge information into account for alleviating the influence arising from image monotony. The extended cross-correlation, named as the increment sign correlation, based on the average evaluation of incremental tendency of brightness in adjacent pixels was proposed by Kaneko et al. (2002). This registration method can be used for image scanning, search and registration over a large scene. Another well-known intensity-based method, termed the normalized mutual information (NMI), is constructed based upon information theory, and it is to measure the statistical dependency between two random variables. The NMI method is proposed by Studholme et al. (1999), as defined by:

$$NMI(A, B) = \frac{H(A) + H(B)}{H(A, B)} \quad (1)$$

where  $H(A) = -E_x(\log(p(A)))$  is the entropy of random variable  $A$  and  $p(A)$  is the probability distribution of random variable  $A$ . The MI-based methods, concerning with the histogram of the overlapped region between the reference and sensed images without involving the spatial information of the relative pixels, may lead to local or even “premature” optimum solutions to the transformation model if intensity data in multimodal images are not spatially invariant. In this light, several modifications by adding extra information including gradient, edge and region information have been proposed in the literature. Herein, an entropy-based method is proposed, which only makes use of spatial information on the reference and sensed images without applying additional spatial features. In particular, color information is taken into account in the proposed objective function. The remaining of this paper is organized as follows. Section 2 provides the details of the proposed image registration function and the optimization method. In Section 3, the experimental registration results obtained by using the CC method, the NMI method and the proposed method are compared by means of various

test image sets. Lastly, the conclusion is drawn in Section 4 and some directions for future research are given.

## 2 THE PROPOSED IMAGE REGISTRATION METHOD

The framework of the proposed image registration method is pictorially shown in Fig. 1. To begin, two images under registration are treated as the reference image and the sensed image where applicable. In the proposed image registration method, the parameter set applied to the sensed image would be kept updated according to the optimization method until a pre-specified stopping criterion is met. The proposed objective function (i.e., similarity measure), constructed based on the color intensity differences of corresponding pixels between the reference and sensed images, is used as a yardstick for evaluating the quality of the parameter set. The transformed sensed image will be superimposed onto the reference image according to the obtained parameter set of the transformation function. The proposed objective function and the optimization method will be introduced as follows.

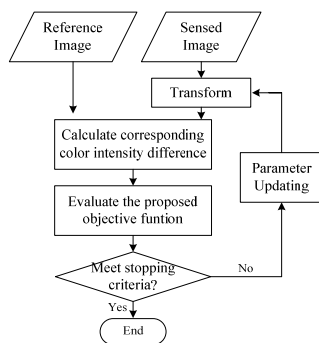


Figure 1: The procedure of the proposed method.

As two images are correctly orientated, it is reasonable to allege that the pixels around the corresponding positions (or local neighborhood) from the images under registration should exhibit similar intensity patterns in red, green and blue. On this account, the summation of the absolute differences of RGB values in the overlapped regions, intuitively, ought to be minimized. Nonetheless, the corresponding objects or features are, sometimes, not represented in the same color representation due to different modalities on different applications. It may not be appropriate to directly use the summation of

absolute values of the differences of RGB intensity values as the similarity measure.

In reality, there always exists color variation to a certain degree between the sensed and reference images. The color intensity differences between the “transformed” sensed image and the reference image are not possibly all close to zero as expected. Thus, it is not a practically feasible choice to search for the best parameter set of the transformation function by directly minimizing the summation of the intensity differences of RGB values. The overlapped region of the two correctly aligned images should contain the same objects and/or features, but in many practical situations, the intensity values of RGB over corresponding pixels may not be homogeneous. In other words, the corresponding objects or features obtained from different conditions, sensors or viewpoints will possibly have different distributions of the color intensity over the relative pixels. Ideally, the differences of the corresponding pixels in each color are expected as close to a plane surface as possible but not possible to be perfectly zero. On this account, we propose to calculate the entropy of the histogram of the intensity differences of RGB values over the overlapped region between the transformed sensed image and the reference image. The overlapped region of two correctly aligned images shall have minimum entropy in that the color intensity difference distribution is asymptotically convergent from an entropy point of view. Namely, the reduced uncertainty is quantified in lower entropy. Herein, the Shannon entropy (Shannon, 1948) is employed as a measure of registration for a probability distribution  $P$ , and it is defined by

$$-\sum_{p \in P} p \log p. \quad (2)$$

For color images, the similarity measure is proposed and illustrated as follows. The color intensity differences on corresponding pixels in the overlapped region between the reference image and the sensed image are defined as follows:

$$\begin{aligned} d_r(x, y) &= |I_{Rr}(x, y) - I_{Sr}(x, y)|, \\ d_g(x, y) &= |I_{Rg}(x, y) - I_{Sg}(x, y)|, \\ d_b(x, y) &= |I_{Rb}(x, y) - I_{Sb}(x, y)|, \end{aligned} \quad (3)$$

$$(x, y) \in \text{overlapped region},$$

where  $x, y$  indicate the pixel position,  $r, g, b$  represent three different colors, and  $I_r(x, y)$  and  $I_s(x, y)$  are the intensity values of the given position

in the reference image and the sensed image, respectively. The frequency histogram  $h(i, j, k)$  of the absolute difference  $d_r(x, y)$ ,  $d_g(x, y)$ ,  $d_b(x, y)$  in the overlapped region counts according to:

$$\begin{aligned} h(i, j, k) &\leftarrow h(i, j, k) + 1, \\ i &\leftarrow d_r(x, y), j \leftarrow d_g(x, y), k \leftarrow d_b(x, y), \end{aligned} \quad (4)$$

where  $i, j, k$  are the intensity values of three different colors. The histogram is then converted into probability  $p(i, j, k)$  via dividing it by the summation of the designed histogram,  $h(i, j, k)$ . The distribution range of the proposed difference is different from the one for gray level images. The bin number needs to be specified instead of the number of bits for intensity representation in gray level image registration because the frequency histogram is extended to 3 dimensions. The entropy of the intensity difference for color images (denoted by  $EDC$ ) between image  $A$  and image  $B$  is now defined by:

$$EDC(A, B) = -\sum_{k=0}^{kmax} \sum_{j=0}^{jmax} \sum_{i=0}^{imax} p(i, j, k) \log[p(i, j, k)], \quad (5)$$

where  $A$  and  $B$  denote the reference and sensed images,  $imax$ ,  $jmax$ ,  $kmax$ , are the maximum numbers of the intensity values for three different colors, and  $p(i, j, k)$  is the probability of the intensity difference for color images. The optimization method will be used to anchor a possibly lowest value of the objective function  $EDC(A, B)$  for obtaining the best registration result. If two images are correctly aligned without any different intensity contrast, the objective function  $EDC(A, B)$  is theoretically zero.

In this paper, it is also assumed that the scene is far from the camera, so the perspective deformation between images can be neglected. Under such circumstances, the similarity transformation function is employed to transform the sensed image while aligned to the reference image. The function is defined as follows:

$$\begin{aligned} X &= S[x \cos \theta + y \sin \theta] + h, \\ Y &= S[-x \sin \theta + y \cos \theta] + k, \end{aligned} \quad (6)$$

where  $(x, y)$  and  $(X, Y)$  are the original location and the transformed location;  $S, \theta, (h, k)$  are the scaling, rotational, and translational parameters for the sensed image, respectively. The optimization task attempts to locate the optimal parameter set

$\mathbf{p} = (S, \theta, h, k)^T$  for the transformation function that minimizes the objective function  $EDC(A, B')$ , i.e.,

$$\begin{aligned} B' &\leftarrow \text{Transform}(\hat{\mathbf{p}}) \cdot B, \\ \hat{\mathbf{p}} &\leftarrow \arg \min EDC(A, B'). \end{aligned} \quad (7)$$

To optimize the proposed objective function, the Powell's method [9] will be used to solve the objective function  $EDC(A, B')$ . To initialize the search step, the scaling parameter is set to 1.0, and the remaining ones are all set to zero. In the next section, the registration performance of the proposed method will be assessed in terms of several test image sets.

### 3 EXPERIMENTAL STUDY

In this section, an experimental study that evaluates the proposed image registration method with the other existing methods, the normalized mutual information (NMI) method and the cross correlation (CC) method, will be conducted in terms of different test image sets. To perform fair comparisons among different registration methods, the Powell's method with the same initial parameter setting and the similarity transformation are applied to all these three methods as the optimization tool. The test image sets are shown in Fig. 2; from left to right are the reference image and the sensed image, respectively. The size of test images is of the size  $256 \times 256$  pixels. For the NMI method, the histogram of the intensity differences of the corresponding pixels between the reference image and sensed image is created with 256 bins due to the 8-bit representation. To achieve better execution efficiency, the numbers of bins are set to  $32 \times 32 \times 32$  for the histogram of the color intensity differences of the proposed image registration method.

Four test image sets shown in Fig. 2 are used for evaluating three image registration methods. To optimize these three objective functions by using the Powell's method, the search range of each parameter for one-dimensional search is restricted within  $[S_c - 0.5, S_c + 0.5]$  for the scaling parameter,  $[\theta_c - 45, \theta_c + 45]$  for the rotational parameter,  $[h_c - (width / 4), h_c + (width / 4)]$ ,  $[k_c - (height / 4), k_c + (height / 4)]$  where  $width$  and  $height$  are obtained from the test image size, for the translational parameter, and a multiple of  $\pm 1.5$  to the combined direction for pattern search. Note that

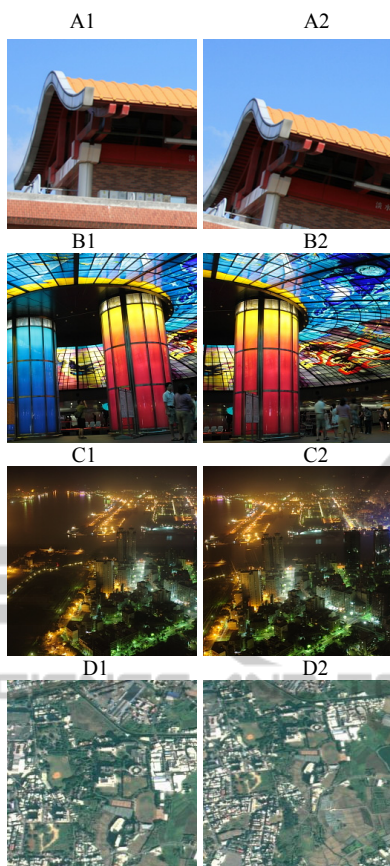


Figure 2: Four test image sets used in the experiment.

$\mathbf{p}_c = (S_c, \theta_c, h_c, k_c)^T$  is the “incumbent” parameter setting in each iteration. The stopping criterion used for halting the Powell’s method is satisfied if the improvement of the current objective value over the previous one is less than  $10^{-4}$ . Once the obtained parameter set transforms the sensed image outside of the reference image, meaning no overlapped region located, the optimization step in the Powell’s method will be immediately stopped. The first test image set is obtained from the regular digital camera, and there exist small translational, rotational and scaling differences between test images. Therefore, it is anticipated that all these three methods would align these two images successfully. Now consider the registration result of the first image set shown in Fig. 3, which are taken from Dan-Shui MRT station, New Taipei City. The CC method, the NMI and the proposed methods produce satisfactory registration results. In the second test image set, there are many edge points and some regular patterns in images, and also there exists a large horizontal translation between images. The complexity in structure and the intensity contrast pose a great challenge to image

registration. The resulting images obtained from the CC method show that the right pillar in the sensed image is mistakenly aligned to the left one in the reference image due to the similar pattern between two pillars. The NMI method yields the non-overlapped result, meaning that the sensed image is transformed outside of the reference image. The proposed EDC method correctly aligns this test image set through the different color information of the pillars. The third test image set is the night scene image set. Apparently, the CC method and the proposed methods both generate accurate registration results, and these two combined registration results are very close to each other. Unfortunately, the NMI method presents an incorrect result which aligns the sensed image to the upper right side of the reference image. For the last test image set, these two images are very difficult to locate the correct parameter set because of the limit of the transformation model. The similarity transformation model can not compensate for the difference of deformation from different viewpoints. Therefore, the CC method and the proposed EDC method can, at best, gain the approximate parameter set. The combined resulting image of the NMI method indicates that the sensed image is transformed outside of the reference image.

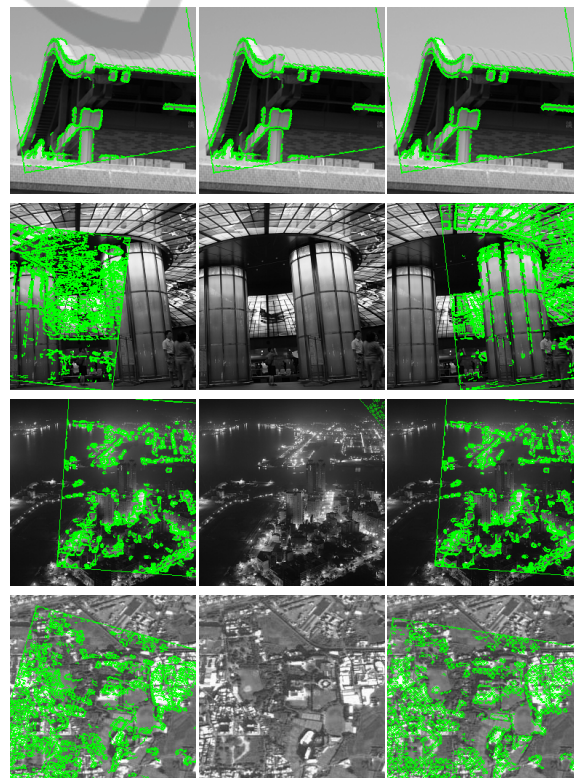


Figure 3: Registration results of test image sets returned by using the CC method, the NMI method.

On the other hand, the proposed EDC method obtains more correctly registered region on the bottom right side of the reference image than the CC method. The proposed method is coded by Borland C++ Builder 6.0. It takes, in general, less than 2 minutes for the whole image registration process with one test image pair in the experiment under Intel Core 2 Duo-2.80GHz platform with 4 GB DDR SDRAM on the Win7 system.

#### 4 CONCLUSIONS

In this article, a new image registration method with color information is presented. The new method is developed based on the entropy of the color intensity differences of corresponding pixels on the overlapped region. The proposal attempts to reduce the effects arising from different image-taking conditions or sensors while registration. Toward this end, the color information is taken into account in the proposed objective function. To help search the possibly best parameters in the similarity transformation function, a well-known function minimization algorithm, called the Powell's method, is borrowed from the field of numerical optimization to solve the proposed objective function. A suite of 4 test image sets, with varying degrees of difficulty and complexity, serve as the test bed for performance evaluation. The experimental results show that the proposed objective function presents more robust convergence properties than the CC method and the NMI method. Building upon this research, there are still a number of interesting topics worth further study in this area. For instance, non-rigid image registration can be a very fruitful application area, and other relevant image processing applications should take place to further validate the effectiveness of the proposed method.

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