

Multi-start Approach for Solving an Asymmetric Heterogeneous Vehicle Routing Problem in a Real Urban Context

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Keywords: Heterogeneous Vehicle Routing Problem, Asymmetric Cost Matrix, Clarke and Wright, Randomized Algorithms, Heuristics.

Abstract: Urban transportation is a strategic domain that has become an important issue for client satisfaction in distribution companies. In academic literature, this problem is categorized as a Vehicle Routing Problem, a popular research stream that has undergone significant theoretical advances but has remained far from practice implementations. Most Vehicle Routing Problems usually assume homogenous fleets, that is, all vehicles are considered of the same type and size. In reality, this is usually not the case as most companies use different types of trucks to distribute their products. Also, researchers consider symmetric distances between customers. However, in intra-urban distribution it is more appropriate to consider asymmetric costs. In this study, we address the Heterogeneous Fixed Fleet Vehicle Routing Problem with some additional constraints: (a) Asymmetric Cost matrix, (b) Service Times and (c) Routes Length restrictions. Our objective function is to reduce the total routing costs. We present an approach using a multi-start algorithm that combines a randomized Clarke & Wright's Savings heuristic and a local search procedure. We execute our algorithm with data from a company that distributes food to more than 50 customers in Barcelona. The results reveal promising improvements when compared to an approximation of the company's route planning.

1 INTRODUCTION

In the last years, logistics and transportation companies are facing growingly demanding situations with fewer available resources. Market instability and the competitive business environment have caused an increasing optimization of logistic processes. Several fields of research have directed their efforts to conceive techniques to fulfil this purpose, like applied mathematics, operations management and computer sciences. The main challenge for these theoretical domains is the consideration of real contexts including real constraints into their approaches.

Vehicle routing is a complex logistics management problem and represents a key phase for the logistic optimization. There are many variations for the routing problem. Particularly, we have considered a special variant where several restrictions are considered at the same time. The set of defined constraints are taken from a real case provided by a food distribution company located in

Barcelona, Spain. The distribution inside cities has special conditions like little time for delivery, congestion, traffic lights, and different types of vehicles related to the size and velocity issues. Also, there are many possible configurations (routes) to visit a customer because the street direction creates a special network of available arcs. The purpose of this study is to develop and apply a randomized multi-start algorithm based on a Clarke & Wright savings heuristic for the Asymmetric Heterogeneous Fleet Vehicle Routing Problem (AHVRP) with service times and routes length restrictions. The main advantage of the proposed approach is to design a simple algorithm that does not need any special fine-tuning.

The paper is organized as follows: *Section 2* describes the theoretical background and previous works. In *Section 3* we develop the details of the proposed algorithm. *Section 4* presents the data instances from the distribution company. *Section 5* shows the results of applying the proposed methodology to a real context case. To conclude, *Section 6* summarizes with some final remarks and

future research lines.

2 THE VRP

The Vehicle Routing Problem (VRP) has been studied for over 50 years (Laporte, 2009). The simplest version is known as the Capacitated Vehicle Routing Problem (CVRP), defined by (Dantzig and Ramser, 1959). In CVRP, a directed graph $G = (V, A)$ is given, where $V = \{0, 1, \dots, n\}$ is the set of $n + 1$ nodes and A is the set of arcs. Node 0 represents the depot, while the remaining nodes $V' = V \setminus \{0\}$ correspond to the n customers. Each customer $i \in V'$ requires a known supply of q_i units, i.e., its demand, from a single depot (assume $q_0 = 0$). This demand is going to be served by exactly one visit of a single vehicle. In this basic form, there is a homogeneous fleet of m identical vehicles with capacity Q to serve these n customers. Each vehicle has also a time limit L for their single trip. A vehicle's trip is a sequence of customers, whose total demand cannot exceed Q that starts from and finishes at the depot with duration no greater than L (used to be a really big value in order to ignore its effect). CVRP aims at finding m trips (vehicles) so that all customers are serviced and the total distance travelled by the fleet is minimized.

2.1 Heterogeneous VRP

VRP's basic version is very theoretical and restrictive. In practice, there exist some other constraints on customers, depot(s), vehicles, etc. that may have a significant impact on solutions. In particular, since companies try to use their resources efficiently and as needed, constraints regarding the type and size of vehicles as well as the number of trips they can make limit the application of basic VRP models considerably. The Heterogeneous Fleet Vehicle Routing Problem (HVRP) overcomes some of these issues.

Different variants of HVRP have been proposed in literature. (Baldacci et al., 2008), for instance, present a comprehensive description of some of them. One of the most relevant works on this area is (Li et al., 2007). The realist aspect of this research line has produced several recent studies, like that in (Subramanian et al., 2012). In general HVRP context, there is a heterogeneous vehicle fleet M composed by m different vehicle types, i.e., $M = \{1, \dots, m\}$. For each vehicle type, there are m_k vehicles, a number that might be very large or, essentially, unlimited. The m_k vehicles of type $k \in M$

have capacity Q_k , fixed cost F_k , and variable cost per arc (i, j) travelled c_{ij}^k ($i \neq j$). The number of trips performed by type k vehicles must not be greater than m_k . The cost of a route results from adding the costs of arcs included in the route and the vehicle's fixed cost F_k .

In this paper, we consider the HVRP with the following additional considerations regarding the available fleet and its costs:

- The number of vehicles of each type, m_k , is limited (fixed fleet) and their use must be determined. This is known in literature as Fixed Fleet HVRP, and;
- For each vehicle type: (1) its fixed costs are ignored (i.e. $F_k = 0, \forall k \in M$); (2) its routing costs are *vehicle-independent* ($c_{ij}^{k_1} = c_{ij}^{k_2} = c_{ij}, \forall k_1, k_2 \in M, k_1 \neq k_2$).

2.2 Asymmetric VRP

Also notice that the cost, c_{ij} , of each travelled arc (i, j) could not be the same for inverse direction (j, i) , i.e. $\exists i, j; i \neq j; c_{ij} \neq c_{ji}$. This is the basic definition of Asymmetric CVRP (ACVRP), where a directed-graph is created and the cost of each arc is independent. (Laporte et al., 1986) develop an exact algorithm for the asymmetrical CVRP. The authors use a Branch-and-Bound tree in which sub-problems are modified assignment problems subjected to some restrictions. Computational results for problems involving up to 260 cities are reported. (Vigo, 1996) proposed a heuristic algorithm using additive bounding procedures for the ACVRP. Randomly test problems involving 300 customers are used to show the promising performance of his approach. (Toth and Vigo, 1999) addressed a different problem, the symmetric and asymmetric VRP with Backhauls. The authors proposed a Cluster-first-Route-second heuristic. Randomly generated instances are used to produce computational results. (Rodríguez and Ruiz, 2012) have made experiments to study the effect of asymmetric matrix on CVRP instances. On this, the authors have considered classical heuristics and current state-of-the-art metaheuristics. They highlighted that "a higher asymmetry degree in the instances affects in a statistically significant way the CPU time needed by the algorithms and deteriorates the quality of the solutions obtained".

However, the combination of these two restrictions, Heterogeneous Fleet and Asymmetric Cost matrix, is not frequent in the literature. In summary, the original problem we consider in this paper is the Asymmetric Heterogeneous Fixed Fleet

VRP (AHVRP). We also assume that (a) any vehicle type can visit any individual customer (the smallest vehicle capacity is bigger than the biggest demand); (b) there are independent service times for each node (the delivery time spent in each client for unloading of merchandise) that follows a specific statistical distribution; and (c) the length of routes is controlled by a maximum value. The objective function is focused on minimizing the total routing costs, considering travelling plus service times and a duration restriction of routes.

3 OUR APPROACH

Our approach is based on the algorithm called Simulation in Routing via the Generalized Clarke and Wright Savings heuristic (SR-GCWS) proposed by (Juan et al., 2010). This randomized procedure was originally made for solving the CVRP. Figure 1 presents an overview of our approach, where a multi-start process is started during a specific period of time, and, at each iteration, a solution is constructed using a randomization version of the classical parallelized Clarke and Wright Savings (CWS) heuristic (Clarke and Wright, 1964). CWS is probably one of the most cited heuristic to solve the CVRP. This procedure uses the concept of savings. On general, at each step of the solution construction process, the edge with the most savings is selected if and only if the two corresponding routes can feasibly be merged using the selected edge. The CWS algorithm usually provides relatively good solutions in less than a second, especially for small and medium-size problems. In the literature, there are several variants and improvements of the CWS. The original version of CWS is based on the estimation of possible savings originated from merging routes, i.e., for unidirectional or symmetric edges $sav(i, j) = c(0, i) + c(0, j) - c(i, j)$. These savings are estimated between all nodes, and then decreasingly sorted. Then the bigger saving is always taken, and used to merge the two associated routes. On the randomized version of this algorithm, we use a pseudo-geometric distribution to induce a biased randomization selection of savings. Moreover, this selection probability is coherent with the savings value associated with each edge, i.e., edges with higher savings will be more likely to be selected from the list than those with lower savings. Therefore, each combination of edges has a chance of being selected and merged with previously built routes. This allows obtaining different outputs at each iteration of the multi-start procedure.

However, the savings construction is modified for being applied to the AHVRP, because the inversed edges are also considered in the set of options (multiplying the original quantity on the symmetric version by two), i.e., for two different nodes i and j : $sav(i, j) = c(i, 0) + c(0, j) - c(i, j)$ and also $sav(j, i) = c(0, i) + c(j, 0) - c(j, i)$. Therefore, all savings will be competing to be taken in the biased randomized process, and those with higher savings will define the orientation of routes.

Likely the routes construction process will consider the direction of savings edges. Once a route takes a direction then all considered candidate routes to be merged with the first one must follow the same direction.

Just before the construction process, the total route duration (travelling plus service times) and the candidate vehicle taking care of the new route are validated. The bigger vehicle between the two processing routes will be responsible of the new route. This vehicle assignment promotes the merging of routes as possible (Cáceres-Cruz et al., 2012). If a route does not have an assigned vehicle, then the first vehicle on the available vehicle list (decreasingly sorted by capacity) is selected. For this, several fictitious vehicles will be required mainly at the beginning of the CWS process. The fictitious vehicle should be defined using the minimum possible capacity on the instance. At the end, the fictitious vehicles must be discarded, if not the solution is unfeasible. This vehicle assignment rule does not add any computational time on to the algorithm execution keeping the overall complexity of the algorithm controlled. However there is a remark: any individual demand can be carried out by any truck (even the smallest and fictitious).

After construction, the solution is improved with a local search method based on a memory cache (Juan et al., 2011). This technique keeps in memory the best known routes so far with the different combination of customers. This procedure compares and saves the best order for visiting the nodes on all solutions generated so far. The previously assigned vehicle to each route remains unchanged during this process. At the end, the best solution is recorded.

4 COMPANY INSTANCES

With the analysis based on (Pessoa et al., 2008); (Baldacci et al., 2008), we have identified standard benchmarks such as the ACVRP and HVRP. We could not find a general accepted dataset for the combination of these two problems. The most

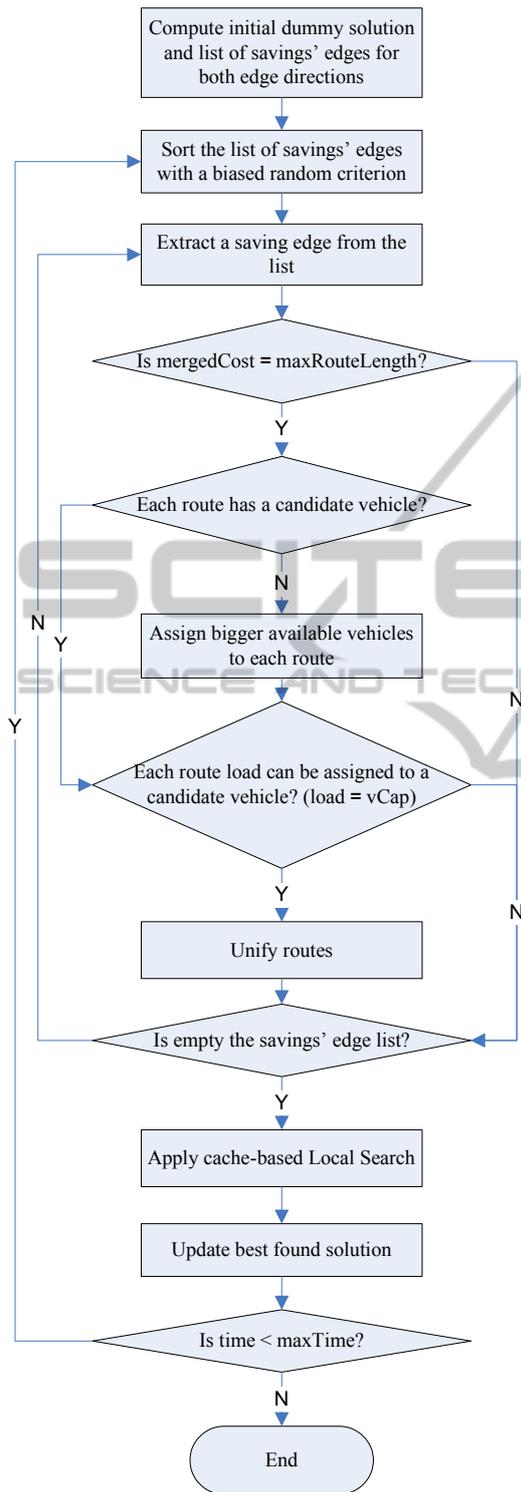


Figure 1: Overview of our approach.

appropriate dataset presented in (Marmion et al., 2010) is related to our specific problem. The experiments are based on a set of real instances

related to ACVRP (Fischetti et al., 1994). The authors simulate the heterogeneous fleet over a range of values for testing some operators on four different algorithms. However, the proposed benchmarks have only considered the effect of variable cost on vehicles selection by ignoring the different capacities. As a result, there is no specific dataset for the above studied problem.

As the case of study, we used the information of a food distribution company located in Barcelona, Spain. The company has provided us with the delivery address of their customers in six independent days along with their demands for those days. The transportation limits are defined inside of the city borders (urban distribution).

The main interest of the company is to apply the proposed approach to bigger datasets using a web information tool. For this reason, the company just compile the information during a short period (as a sample) in order to produce a preliminary result. In addition, the compiling process represented an important investment of resources considering the size of the company. Therefore on a daily basis, this company receives requests from around 50 customers. This information serves as input to manually design the company's routing planning.

According to the size of the company it is not possible to employ a person specialized in mathematical software in order to apply exact methods. Therefore they prefer to have an approximated solution algorithm embed in a web tool which could be used to give automatic solution in little time.

There is a specific constraint: each vehicle must visit all customers of a route in a maximum period of 180 minutes. This route length restriction must to include the travelling time and the service time. So far, the company uses two types of vehicles, which are described in the Table 1. The columns of this table show the capacity (Q_k) and quantity (m_k) of available vehicles for each type (k). Actually the company used four vehicles, but they needed to determine if it is possible to reduce the total routing costs and also execute the same deliveries with fewer routes.

Table 1: Composition of the current company fleet.

Vehicle Type k	Q_k	m_k
1	20	2
2	30	2

We have used a map-location service, like Google Maps to generate the asymmetric cost matrix

between every pair of nodes (50 x 50 maximum cells). Even when this kind of routing considers all possible streets of the city, the cost matrix will only represent the best travelling time between each two nodes.

The main features of given six data instances are summarized in the Table 2. On the first column, we present the identification of each instance that represents a day. The second column shows the number of customers with demands. Third column is the total demand. And the last column represents the total service time of all the nodes on the instance.

As commented before, the company provides us with the historic data of some of their service times and routes. We have randomly generated the respective values for the instances, using simulation theory (Monte Carlo Simulation) and the provided data. Then, we have defined that the service time for each client follows a triangular distribution with $min = 1$, $max = 12$ and $mode = 3$ minutes. This distribution is often used to represent time in general simulation models. However, the routes used differ among all days. Notice that the company did not save exact information of all their routes, even within a whole day. Likely they do not apply any specific routing method. A person in charge, who tries to assign routes to all drivers, designs the routing planning.

Table 2: General features of real instances.

Instance (day)	Number of Customers	Total Requested Demand	Total Service Time (min)
A	40	53	163
B	50	75	213
C	40	60	163
D	39	54	159
E	40	57	162
F	18	28	75

5 NUMERICAL RESULTS

Our algorithm was implemented as a Java application and used to run the six instances described above on an Intel Xeon E5603 at 1.60 Ghz and 8 GB of RAM. For each instance, a single run with a total maximum time of 500 seconds was employed. The limitation in computing time is due to the fact that we wanted to obtain results in a ‘reasonable’ amount of time. We employ the Random Number Generator (RNG) library for

Stochastic Simulation developed by researchers of the Montreal University (<http://www.iro.umontreal.ca/~simardr/ssj/>).

Table 3 shows the results obtained in experiments. The first column shows the instance id; the second, the number of routes defined in the solution; the third column, the total travelling times of routes; the fourth column, the total routing costs considering the travelling times plus the service times of the instance; and the last column, the computational time needed to find the best solution.

The travelling costs on instances B and E represent the higher values obtained. Both of them travelling costs are bigger than the previously commented restriction of 180 minutes. However, this restriction is applied to the route duration and also it considers the service time on each node. On these two instances, the average total routing cost of routes has to be considered. For this, the total routing cost is divided by the number of routes on the solution producing 134 and 174 minutes respectively.

Notice that even when the running time is set to a maximum limit of 500 seconds, the average time for finding the best solutions is less than 131 seconds.

Table 3: Results of Best Solutions after 500 seconds running.

Instance (day)	Routes	Total Travelling Cost (min)	Total Routing Cost (min)	Time (sec)
A	2	173	336	1.14
B	3	189	402	114.76
C	2	170	333	137.52
D	2	172	331	275.90
E	2	186	348	253.42
F	2	116	191	0.25
Average	2.17	167.67	323.50	130.50

In order to validate the solution quality of our approach, we have compared our results against an approximated value of the current total routing costs. As we said before, the company does not have the exact values of routing costs. However, they tend to use all four vehicles as an attempt to reduce delivery times, in an intuitive way. Therefore we have forced our algorithm to use four vehicles in order to produce a near value of current company solutions. The output represents the best solution found in 500 seconds. We delivered the forced four-route solution to the company in order to validate it with the real planning, and we obtained a positive confirmation. Table 4 presents the travelling times for each

scenario and the gap between these two solutions.

The difference between the approximated company solutions and our approach results is around 13%. In the next two images, we have illustrated both routing solutions of the approximated planning (Figure 2), and the new proposed solution (Figure 3) for the instance B, where the number of routes was reduced to 3. Notice that the average number of routes of our approach is around 2 which represents a considerable reduction of the amount of routes.

Table 4: Comparison with extreme case using whole fleet (four vehicles).

Instance (day)	Best Costs using 4 routes (min) (2)	Best Costs (min) (1)	GAP (2-1)
A	192	173	-9.90%
B	205	189	-7.80%
C	206	170	-17.48%
D	190	172	-9.47%
E	211	186	-11.85%
F	153	116	-24.18%
Average	192.83	167.67	-13.45%

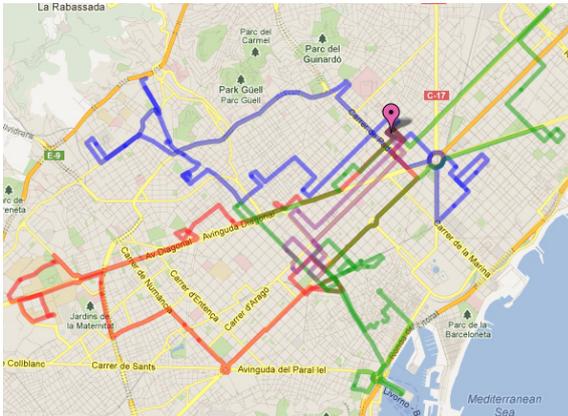


Figure 2: Approximated routing planning of the company for instance B, using Google Maps.

6 CONCLUSIONS

In this paper, we have presented a multi-start approach for solving the Asymmetric Heterogeneous Vehicle Routing Problem (AHVRP) with service time consideration and routes length restrictions. The proposed approach integrates a randomized heuristic approach with a local search. Our results

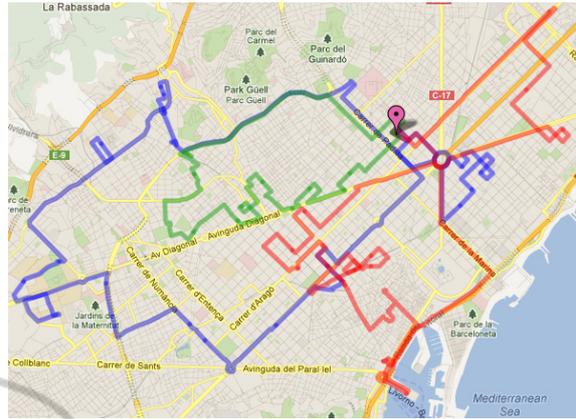


Figure 3: Designed routes in the proposed solution for instance B, using Google Maps.

are based on data obtained from a distribution company and we compare our solutions with an approximation value of the actual ones implemented by the company. These results revealed promising improvements.

Through this experience it was possible to support a food distribution company to: (a) realize the current situation with quantitative methods; and (b) improve their routing planning with a simple approach. We used Monte Carlo Simulation to complete the missing data from the company, and obtain the information required for testing.

A popular way to evolve a study related to savings algorithms is to propose new savings definitions. The proposed definition of savings for asymmetric VRPs could change in order to promote other types of route constructions. Likely the inclusion of other real constraints for urban distribution is also being considered in the next steps of our research, such as manage open routes and balanced loads on routes. In fact this last restriction is important because there are some routes with fewer planned visits whereas others with more.

ACKNOWLEDGEMENTS

This work has been partially supported by the Spanish Ministry of Science and Innovation (TRA2010-21644-C03). It has been developed in the context of the IN3-ICSO program and the CYTED-HAROSA network (<http://dpcs.uoc.edu>).

REFERENCES

- Baldacci, R., Battarra, M., Vigo, D. 2008. Routing a Heterogeneous Fleet of Vehicles. In *The Vehicle Routing Problem: Latest Advances and New Challenges*. B. Golden, S. Raghavan and E. E. Wasil, Springer: 3-27.
- Cáceres-Cruz, J., Lourenço, H., Juan, A., Grasas, A., Roca, M., Colome, R. 2012. Aplicación de un Algoritmo Híbrido para la Resolución de un Problema de Enrutamiento de Vehículos Heterogéneos en una Empresa de Distribución. In *Proceedings of Congreso Español sobre Metaheurísticas, Algoritmos Evolutivos y Bioinspirados (MAEB)*, 767-773. February 8-10. Albacete, Spain.
- Clarke, G., Wright, J. 1964. Scheduling of vehicles from central depot to number of delivery points. *Operations Research*, 12(4):568–581.
- Dantzig, G. B., Ramser, J. H. 1959. The Truck Dispatching Problem. *Management Science* 6:80–91.
- Fischetti, M., Toth, P., Vigo, D. 1994. A branch-and-bound algorithm for the capacitated vehicle-routing problem on directed-graphs. *Operations Research* 42(5):846–859.
- Juan, A. A., Faulín, J., Ruiz, R., Barrios, B., Caballe, S. 2010. The SR-GCWS Hybrid Algorithm for Solving the Capacitated Vehicle Routing Problem. *Applied Soft Computing* 10: 215–224.
- Juan, A. A., Faulin, J., Jorba, J., Riera, D., Masip, D., Barrios, B. 2011. On the Use of Monte Carlo Simulation, Cache and Splitting Techniques to Improve the Clarke and Wright Savings Heuristics. *Journal of the Operational Research Society* 62: 1085-1097.
- Laporte, G., Mercure, H., Nobert, Y. 1986. An exact algorithm for the asymmetrical capacitated vehicle-routing problem. *Networks* 16(1): 33–46.
- Laporte, G. 2009. Fifty Years of Vehicle Routing. *Transportation Science* 43(4): 408-416.
- Li, F., Golden, B., Wasil, E. 2007. A record-to-record travel algorithm for solving the heterogeneous fleet vehicle routing problem. *Computers & Operations Research* 34: 2734-42.
- Marmion, M. E., Humeau, J., Jourdan, L., Dhaenens, C. 2010. Comparison of neighborhoods for the HFF-AVRP, *Computer Systems and Applications (AICCSA), 2010 IEEE/ACS International Conference on*, pp.1-7, 16-19 May 2010.
- Pessoa, A., Poggi de Aragão, M., Uchoa, E. 2008. Robust Branch-Cut-and-Price Algorithms for Vehicle Routing Problems. In *The Vehicle Routing Problem: Latest Advances and New Challenges*. B. Golden, S. Raghavan and E. E. Wasil, Springer: 297-325.
- Rodríguez, A., Ruiz, R. 2012. A study on the effect of the asymmetry on real capacitated vehicle routing problems. *Computers & Operations Research* 39(9): 2142–2151.
- Subramanian, A., Penna, P. H. V., Uchoa, E., Ochi, L. S. 2012. A hybrid algorithm for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research* 221: 285-295.
- Toth, P., Vigo, D., 1999. A heuristic algorithm for the symmetric and asymmetric vehicle routing problems with backhauls. *European Journal of Operational Research* 113: 528-543.
- Vigo, D. 1996. A heuristic algorithm for the Asymmetric Capacitated Vehicle Routing Problem. *European Journal of Operational Research* 89: 108-126.