Planning and Control Model for a Forest Supply Chain

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Keywords: Decision Making Process, Forest Industry, Mixed-Integer Linear Model, Logistics Planning.

Abstract: This paper presents a mixed-integer linear program (MILP) for a planning problem of multiple activities in the forest industry. The model developed aims at maximizing the total profit of the value chain by optimizing operations in harvesting, transportation, storage, and production. The main motivations for the model is a need to better account for important factors in planning and control, such as quality, freshness, and species of wood products. These factors have a direct influence on costs and supply decisions. In particular, the model developed will improve forest product companies' industrial processes by a better control over the wood fibre freshness. Furthermore, our model is designed for a context where multiple independent companies supply their raw material from the same sources. It can therefore be used as a support tool for collaboration between actors in a forest supply chain.

1 INTRODUCTION

The forest industry is an important economic sector for Canada. In 2011, it provided a value of $ 26.0 billion of Canada's total export with a gross domestic product of $ 23.2 billion. Therefore it ensures about 600,500 direct and indirect employments (FPAC, 2011).

However, the forest industry network is complex, being composed of a set of nodes (i.e., forest, sawmills, paper mills, wholesalers, retailers …) interconnected by flows of materials (i.e., logs, chips, lumber, paper …), information (orders, demand, forecasts …), and financial transactions. The network also includes a large set of constraints, for example those related to product quality (for lumber, paper, and other forest products), raw material availability, and capacity requirements (at the different business unit sites). Among these constraints, product quality has reached standards that require a very precise control over the supply and production processes. For forest products, freshness of raw material such as the logs and wood chips is considered essential to optimise value while satisfying customer needs. Furthermore, as a general rule, the lower the quality of the fibre, the higher the production cost for manufacturing forest products (Beaudoin et al., 2006); (Maness and Norton, 2002).

In order to improve its efficiency, the forest supply chain needs a continuous supply of raw materials to ensure quality and achieve expected standards. On the other hand, the procurement of timber is a real challenge because of the fibre quality variation, especially in the presence of various forest stands and many tree species. The problem becomes even more complex when many independent firms in the same region use wood from the same stands to produce their forest products. If each firm plans its own activities without considering the needs of the others (e.g., small or large trees, fir or spruce ...), the wood fibre will not necessarily be matched to mill demands efficiently. Moreover, the residue of one company (i.e., chips from sawmill) becomes the raw material for another one (i.e., the paper mill).

Therefore when there is no coordination between the stakeholders, it leads to a set of problems such as higher stock levels in the forest or at the different business unit sites, delivery times not respected, unsatisfied demands, poor value of the final product, and so on. From operational and tactical planning perspectives, timber supply is challenging on several levels. it involves several activities such as forest road building and maintenance, selection and scheduling of harvesting areas, transport operations (truck routing and scheduling), and the coordination of interactions between these activities based on

Alayet C., Lehoux N., Lebel L. and Bouchard M..
Planning and Control Model for a Forest Supply Chain.
DOI: 10.5220/0004202200050013
In *Proceedings of the 2nd International Conference on Operations Research and Enterprise Systems* (ICORES-2013), pages 5-13
ISBN: 978-989-8565-40-2

information sharing. Resolving these issues may be achieved through planning and effective monitoring of resource flows and improved business processes in a network of value creation (Suzanne et al., 2004). The present work aims at helping forest companies in better planning and controlling their forest supply activities through a mathematical model that could be used as a decision support tool for value creation. The main motivation of the model is to deal with elements of competition namely cost, quality, and agility. More specifically, the model developed aims at optimizing all activities related to the satisfaction of customer demand, that is the quantity of wood to harvest and to transport to the sawmills, the quantity of lumber and chips to produce, the paper needed to satisfy the demand, and the different products to keep in stock to ensure a certain level of service. To achieve these results, we assume that there is no competition between the different business units. Diverse scenarios are also tested to evaluate the impact of wood freshness variations, price changes, and demand variations on the value network. These scenarios were explored to reflect the reality of the forest industry.

The paper is structured as follows: the next section presents a literature review, followed by the description of the case study. Section 4 describes the modelling of the problem and the assumptions made. The experiment is then explained in detail in Section 5. Finally, Section 6 concludes the paper.

2 LITERATURE REVIEW

Managing the forest supply chain is an important activity due to its impact on value creation and the generation of profits for all business units. It involves different planning decisions that should cover different levels: strategic, tactical, and operational (Martel, 2003). It begins with the harvesting of wood in the forest, followed by species sorting, wood transportation to different mills, log sawing, and processing factories such as pulp, paper, and energy. It ends with the delivery of final forest products to end users (Carlsson et al., 2006). However, planning decisions and their optimization are complex tasks since they have to include many factors such as wood species, wood freshness, lumber price, final product quality, processing time, and so on, as well as multiple independent decision-makers.

Therefore, different planning approaches have been proposed over the time to better use the wood fibre and ensure the synchronization of network activities. Beaudoin et al., (2010) presented supply planning models for multiple forest companies in which supply areas are shared. These planning models were based on coordination and collaboration approaches coupled with distributed and centralized structures. Horne et al., (2006) explained the importance of value creation based on innovation and the development of products and processes from a center of expertise in the forest industry. The objective of their research was to develop a model based on value creation of innovative knowledge to improve decision making and facilitate understanding of complex mechanisms.

It is difficult to think about planning forest operations without considering the control of the different logistic activities. In the literature, there are many definitions of logistic control systems. Among them, we evoke the definition of Meinadier (1998) cited in Trentesaux and Tahon (2010). The authors introduced three activities that defined the driving process: capture, edit, and order.

Several control structures were used to solve complex problems of the forest supply chain. We first distinguish the centralized or integrated structure. This is the classic approach in which all resources are controlled by a single decision center. This center oversees the supply chain, synchronizes and coordinates the various resources, and manages real-time contingencies that occur (Mirdamadi et al., 2009). Among the works that rely on a centralized control structure, we find the work of Walker and Preiss (1988). They developed a model for planning logging (harvesting, quantity of timber harvested per block, etc.) and transportation activities.

A second approach is characterized by a coordinated structure. This structure aims at ensuring coordination between the subsystems and improving resource utilization while promoting a better flow of information (Martel, 2003). This structure usually improves the ability of decision in each sub-control system to effectively solve problems (Mirdamadi et al., 2009). It is within this context that the study of D'Amours et al., (2004) mentioned the importance of coordination in establishing a value forest product network. The authors identified four dimensions for this structure: (1) competitiveness and customer service, (2) integration, (3) coordination, and (4) operational excellence.

Several other driving approaches were treated in the literature depending on the area of study as well as the planning horizon aimed, such as the distributed approach (Gaudreault et al., 2009), the hierarchical approach (Chang et al., 2009), etc. However, the lack of collaboration between network members is a major obstacle for planning and controlling logistic activities efficiently (Lehoux et al., 2008). In the forestry context, few authors treated the topic of collaboration. Beaudoin et al., (2010) used a MILP approach and protocols of negotiation / collaboration to plan the wood supply for multiple forest companies. Lehoux et al., (2011) evaluated various collaborative strategies between a pulp and paper producer and its customer. They presented different MILP models as well as a methodology to solve the various modes of collaboration and measure their impact on partners' benefits.

The different articles show that forest supply chain optimization has an increasing interest, the planning of the different activities and the integration of many factors such as quality representing real challenges. This context will therefore be explored in the following section.

3 CASE STUDY

3.1 Problem Description

The effective management of the forest supply chain certainly requires a better planning and control of logistic activities. Nevertheless, the management of material flows and information is considered complex, because of the interdependence of the stakeholders involved, the quantity and the quality of the information needed, and success factors such as demand satisfaction and product quality. Moreover the freshness of the wood fibre is a particular problem that characterizes the forest product market. It influences the harvesting decisions like labor allocation, harvesting schedules, site selection, etc. Similarly, the wood freshness often causes problems during processing operations such as the choice of the technology and the way to use this technology (sawing processes, parameters setting and cutting setup times for sawmills, etc.).

Thus, the freshness of the raw material has a direct effect on the manufacturing process, the plant performance, and the quality of finished products. In the pulp and paper industry, the chips freshness has a direct impact on the quantity of chemical products (whiteness) to be added during the production. Solving the above enumerated problems represents long, medium, and short term decisions that have a direct influence on production costs.

In this context, we study the case of a forest supply chain located in Côte-Nord, a Quebec region in Canada. The network includes several harvesting areas, covering 103,146 km2 while 84,382 km2 are accessible and productive forest lands (MRNFP, 2004). Several sawmills with variable processing capacities procure their raw material/logs/timber from these harvesting areas. The wood is used to manufacture lumber for construction market as well as chips delivered to a pulp and paper mill located in the same region. In recent years, the network has faced many difficulties such as an overcapacity of sawing, a decrease in wood fibre quality, and higher operational costs. Combined with a lower demand for forest products on traditional markets, these factors lead to a loss of 6,300 jobs in the last five years. Different studies suggested that procurement cost could be reduced and final product quality increased if raw material quality could be better matched to mill demands. To make this possible, a global strategy involving a better planning and control over the network activities is required.

To address this challenge, we have first developed an integrated or centralized planning model adapted to this context. The model includes five forest areas and the four sawmills of the region. A fictional bioenergy plant has been added because the region is considering using wood residues for energy. The forest is made up of two different species, fir and spruce, and four intermediate products are delivered to the sawmills (small and large spruce logs + small and large fir logs). Sawmills can also be supplied from external sources to cover the lack in case of high demand. Sawmills consume raw material to produce chips for paper making as well as lumber for the construction market. The model also takes into account a paper mill supplies by the four sawmills. The paper mill gets all its chips from the sawmills. It produces newsprint and magazine paper.

The main objective of this study is to determine an effective supply plan for this network that could maximize the profit of all the stakeholders. An illustration of the network is given in Fig 1.

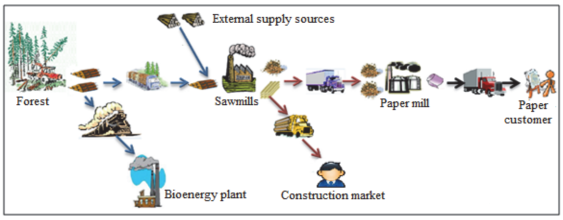

Figure 1: Logistics network of the case study.

3.2 Assumptions

The model developed is based on a one-year planning period divided into fifty-two weeks. Our experiment is performed on a rolling horizon of four weeks (Fig. 2). For each scenario, we solve the model for the first four weeks, for example weeks 1 to 4. Then, we consider the results of the first week to solve the next four weeks, weeks 2 to 5, and so onBy using a rolling horizon, we can develop a 4 week schedule for forest operations while considering updates, revisions, and adjustments when necessary. This assumption has been made to reflect companies' reality.

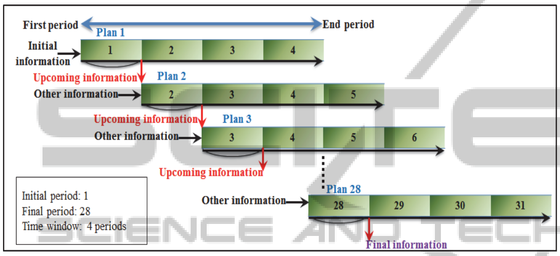

Figure 2: An example of the rolling horizon approach used.

We also assume that the level of freshness of the wood fibre is divided into three categories: green (young or fresh), yellow (medium or less fresh), and red (old or not fresh).

In particular, we use "θ" to reflect the percentage of aging during the period "t", and these parameters are set according to product types and seasons. Furthermore, φ is the percentage of aging per time unit. So if the percentage of aging per week, θ, is seven days, the percentage of aging per day is:

$$\varphi = \sqrt[7]{\theta} \tag{1}$$

4 MATHEMATICAL MODELLING

In this section, the mathematical model for Côte-Nord network is presented. The model, based on a centralized driving approach, reflects the network shown in Figure 1.

Through this model, we try to maximize profits for forest companies by determining harvested volumes, the quantities to keep in stock and to transport at each node, as well as the quantities produced by the processing units. The decision variables, parameters, and the complete mathematical model are described in Appendix A. The objective function is summarized by equation (2).

$$z = R - C^h - C^e - C^t - C^s - C^m \tag{2}$$

Where R is the revenue of the value network, C^h, the cost for harvesting operations, C^e, the cost for buying wood from an external supply, C^t, the total transportation cost (transport between network nodes: forest, plants, and customers), C^s, the inventory cost for the whole network, and C^m, the cost for processing the wood at the different business units.

The network revenues are generated from the sale of lumber, paper and the delivery of wood residues to the bioenergy plant. Costs are divided into several categories. Specifically, the harvesting cost includes the cost for forest road construction and maintenance, as well as the administrative cost. The cost of external supply includes all costs induced by moving logs from an outside supplier to the sawmill (purchasing cost, ordering cost …). The transportation cost includes product delivery costs as well as loading and unloading costs. There is also an inventory cost that includes, among other things, material handling and equipment costs. The processing cost then covers the costs and expenses for producing lumber, wood chips, and paper.

As presented in detail in Appendix A, constraints have been defined to represent the Côte-Nord context. Two constraints were used to ensure customer demand satisfaction. Two other constraints have been added to ensure a product flow balance between the sawmills and the paper mill. In order to reflect aging at the different storage areas (i.e., forest, sawmills and paper mills sites), different constraints were used. Constrains for processing capacity of each business unit, capacity of the different storage areas, and transportation capacity were also considered. Finally, a constraint has also been used to specify the maximum capacity of supply from external sources.

5 EXPERIMENTATION AND DISCUSSION

5.1 Scenarios and Results

The mathematical model was solved using the CPLEX solver under ILOG OPL environment. Several scenarios were tested to solve the problem. First, different levels of freshness were considered in

order to evaluate its effect on network profits. Demand variability was also taken into account to reflect the reality of the forest industry. A disturbance of market prices for lumber was then analyzed because the price for forest products is usually far from linear so it becomes necessary to understand the impact of this change on the system.
If we look at the first scenario, five levels of wood freshness have been explored. The percentage of aging assumed for each level is presented in Table 1.

Table 1: Levels of wood freshness considered in the experimentation.

Levels	1	2	3	4	5
θ (%)	0	10	30	50	60

Results summarized in Table 2 show that a decrease in fibre quality (freshness) has a direct impact on the total network profit.

Table 2: Results for variations of wood freshness.

Costs (M $) \ Instances	1	2	3	4	5
Harvest Cost	861.3	861.5	852	801.1	751.9
Cost of External Supply	268.7	265.8	272.4	317	392.4
Storage Cost	31.5	32	33.1	36.1	40.9
Total Cost	1953.6	1950.3	1945.2	1931.4	1972.6
Revenue	2055.6	2052.2	2050	2039.3	2051.7
Network profit	102	101.9	104.8	107.9	79.1

We also note an improvement in profit when some of the forest products become older from one period to another. This improvement is justified by the demand for products of lower quality that necessarily cost less to the customer and that cannot be satisfied when all the wood fibre is considered fresh. However, it is clear that when the percentage of aging is very high (i.e., 60%), the network profit decreases abruptly. Indeed, the rapid aging forces sawmills to buy wood from external supply sources which significantly increases the cost of external supplies. We can also point out the storage cost that becomes more and more significant. This increase is justified by the accumulation of low quality wood fibre that remains in stock from one period to another. This inventory cost will therefore decrease the total profit. Similarly, the cost for harvesting and logging significantly decreases when the percentage of aging increases. In fact, when there is a rapid aging, the network optimization requires a decrease of harvesting because the harvested logs quickly get old and remain in storage due to low demand for low quality.

The second scenario considered different variations of the lumber demand, that is, few perturbations, seasonality, and cyclical seasonality (Fig 3).

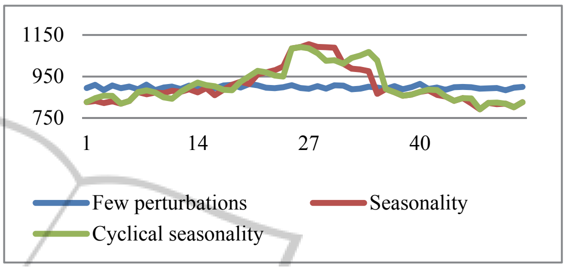

Figure 3: Variations of the lumber demand (m^3/Week).

Table 3: Results for variations of the lumber demand.

Costs (M $) \ Instances	Few perturbations	Seasonality	Cyclical seasonality
Harvest Cost	873.0	867.9	875.2
Cost of External Supply	250.3	252.1	254.3
Storage Cost	31.8	30.6	30.4
Total Cost	1945.6	1934.7	1947.1
Revenue	2080.5	2035.4	2050.5
Network profit	134.9	100.7	103.4

The benefits of having a stable timber demand are highlighted by Table 3. In fact, stability is difficult to achieve since demand for lumber, at least in Canada, is characterized by a seasonal structure related to the construction market. To ensure a more constant demand, forest product companies are therefore trying to develop new external markets by producing more value-added products.

Results in table 3 show that the harvesting cost is lower when the demand is seasonal. This is due to the fact that it is cheaper to harvest during the summer (favorable climatic condition). The cost related to external supplies is also not negligible, since it represents almost a quarter of the total cost. This value is justified by a harvesting capacity that is limited and even null during the thawing season.

The sale price is an important and a classic factor to consider in the analysis of a value creation network. For our case study, results show that the network profit may be doubled if the lumber price increases by 7%. These results are justified by the importance of lumber demand that represents almost 80% of the total network demand. Thus, it becomes essential to plan the supply chain efficiently in order to deliver the right product to the right customer with the right quality. By managing quality standards (freshness) it becomes also possible to offer a more advantageous and stable price.

6 CONCLUSIONS

This article proposes an integrated model to plan supply chain operations for the forest industry while considering key constraints related to the freshness of the wood fibre. In particular, we analyze a case study, which includes four sawmills and one paper mill located in eastern Canada. To ensure a better use of the wood fibre and a greater synchronization of the network activities, the model provides harvesting, transportation, production, and storage plans for the forest companies of this region. The model aims at improving the management of the wood fibre quality while reducing the operating costs such as storage, transportation, and processing.

The proposed model has been tested using three different scenarios: variations of the wood freshness, different patterns of lumber demand, and variations of the lumber price. Results show that the wood fibre freshness is a key criterion to consider for increasing the benefit of the value network. On the other hand, scenario analysis based on lumber price and demand confirm the necessity for Canadian forest product companies to expand their market to avoid the effects of the relative instability of the Canadian lumber market.

The next step will be to develop multiple models based on a coordinated driving strategy for addressing the fact that each company is "independent" or autonomous. It will also be necessary to develop some mechanisms to ensure a fair benefit sharing generated of a better synchronization of network members' operations.

REFERENCES

Beaudoin, D., Frayret, J. M., LeBel, L., 2010. Negotiation-based distributed wood procurement planning within a multi-firm environment. *Forest Policy and Economics*, vol. 12, no. 2, pp. 79–93.

Beaudoin, D., LeBel, L., Frayret, J. M., 2006. Tactical Supply Chain Planning and Robustness Analysis in the Forest Products Industry. *Canadian Journal of Forest Research*, vol. 37, no. 1, pp. 128-140(13).

Carlsson, D., D'Amours, S., Martel, A., Rönnqvist, M., 2006. *Supply Chain Management in the Pulp and Paper Industry*. CENTOR, Working Paper, DT-AM-3.

Chang, Y. H., Weyb, W. M., Tseng. H. Y., 2009. Using ANP priorities with goal programming for revitalization strategies in historic transport: A case study of the Alishan Forest Railway. *Expert Systems with Applications*, vol. 36, no. 4, pp. 8682–8690.

D'Amours, S., Frayret, J. M., Rousseau, A., 2004. *De la forêt au client – Pourquoi viser une gestion intégrée du réseau de création de valeur ?* Université Laval, FORAC, CENTOR, Document de travail.

Forest Products Association of Canada (FPAC)., 2011. *Industry by the number. Key economics facts*.

Gaudreault, J., Frayret, J. M., Pesant, G., 2009. Distributed search for supply chain coordination. *Computers in Industry*, vol. 60, no. 6, pp. 441–451.

Horne, C. V., Frayret, J. M., Poulin, D., 2006. *Creating value with innovation: From centre of expertise to the forest products industry*. Forest Policy and Economics,vol. 8, no. 7, pp. 751–761.

Lehoux, N., D'Amours, S., Langevin, A., 2008. Collaboration et coordination dans les réseaux, une revue des travaux clés. *CIRRELT*-2008-51.

Lehoux, N., D'Amours, S., Frein, Y., Langevin, A., Penz, B., 2011. Collaboration for a two-echelon supply chain in the pulp and paper industry: the use of incentives to increase profit. *Journal of the Operational Research Society*, vol. 62, pp. 581-592.

Maness, T. C., Norton, S. E., 2002. *Multiple Period Combined Optimization Approach to Forest Production Planning*. Scandinavian Journal of Forest Research, vol. 17, no. 5, pp. 460-471.

Martel, A., 2003. *Le pilotage des flux : concepts de base et approches contemporaines.* DF-3.1.1. La conception de réseaux logistiques. CENTOR, Université Laval.

Ministère des ressources naturelles, de la faune et des parcs, direction régionale de la Côte-Nord (MRNFP)., 2004. *Région de la Côte-Nord. Document d'information sur la gestion de la forêt publique*.

Mirdamadi, S., Dupont, L., Fontanili, F., 2009. *Modélisation du processus de pilotage d'un atelier en temps réel à l'aide de la simulation en ligne couplée à l'exécution*. Thèse de doctorat. Institut National Polytechnique de Toulouse.

Suzanne, T., Roy, D. S., Ari-Pekka, H., 2004. From supply chain to demand chain: the role of lead time reduction in improving demand chain performance. *Journal of Operations Management*, vol. 21, no. 6, pp. 613–627.

Trentesaux, D., Tahon, T., 2010. *Pilotage hétérarchique des systèmes de production. Habilitation à diriger des recherches.* L'Université de Valenciennes et du Hainaut-Cambresis. LAMIIH. UMR CNRS 8530.

Walker, H. D., Preiss, S. W., 1988. Operational planning using mixed integer programming. *The Forestry Chronicle*, vol. 64, no. 6, pp. 485-488.

APPENDIX

The data required to formulate the problem are:

r: Set of products
w: set of supply sources
e: Set of external supply sources
u: Set of sawmills
u': Other Plants (bioenergy plant ...)
u'': Set of paper mills
c: End clients of paper mills
b: Timber clients
a: Level of freshness (age group: 1: green (young), 2: yellow (medium), 3: Red (old))
t: Number of periods

The decision variables, parameters and coefficients used for mathematical modeling are:

Parameters

C^{H}_{rwt}: Unit harvesting cost of product *r* belongs to the supply source *w* during period *t*
C_{reut}: Unit supply cost of product *r* from external supply source *e* to the plant *u'* during period *t*
C^{SW}_{rwt}: Unit storage cost of product *r* in the supply source *w* during period *t*
C^{T}_{rwut}: Unit transporting cost of product *r* from the source *w* to the sawmill *u* during period *t*
$C^{T}_{rwu't}$: Unit transporting cost of product *r* from the source *w* to the plant *u'* during period *t*
$C^{T}_{ruu''t}$: Unit transporting cost of product *r* from the sawmill *u* to the paper mill *u''* during period *t*
C^{T}_{rubt}: Unit transporting cost of product *r* from the sawmill *u* to the timber client *b* during period *t*
C^{SI}_{rut}: Unit storage cost of raw materials *r* in the sawmill *u* during period *t*
$C^{SI}_{ru''t}$: Unit storage cost of raw materials *r* in the paper mill *u''* during period *t*
C^{SF}_{rut}: Unit storage cost of finished product *r* in the sawmill *u* during period *t*
$C^{SF}_{ru''t}$: Unit storage cost of finished product *r* in the paper mill *u''* during period *t*
C^{PU}_{raut}: Unit production cost of product *r*, aged *a*, in the sawmill u during period *t*
$C^{PU}_{rau''t}$: Unit production cost of product *r*, aged *a*, in the paper mill *u* during period *t*
$p_{rau't}$: Selling price of product *r*, aged *a* and directly transported to the plant *u'* during period *t*
p_{raut}: Price of finished product *r*, aged *a* and manufactured by the sawmill *u* during period *t*
$p_{rau''t}$: Price of finished product *r*, aged *a* and produced by the paper mill *u* during period *t*
α_{ru}: Coefficient of adjustment of units: raw material; finished product *r* for the sawmill *u*
$\beta_{ru''}$: Coefficient of adjustment of units: raw material; finished product for the paper mill *u''*
θ_{rwa}: Proportion of aging product *r*, aged *a* for the source *w*
θ^{si}_{rua}: Proportion of aging product *r*, aged *a* for the initial stock of the sawmill *u*
θ^{sf}_{rua}: Proportion of aging product *r*, aged *a* for the final stock of the sawmill *u*
$\theta^{si}_{ru''a}$: Proportion of aging product *r*, aged *a* for the initial stock of the paper mill *u''*
$\theta^{sf}_{ru''a}$: Proportion of aging product *r*, aged *a* for the final stock of the paper mill *u''*
b^{hmx}_{rwt}: Maximum harvesting capacity product *r* of the source *w* during period *t*
b^{hmn}_{rwt}: Minimum harvesting capacity product *r* of the source *w* during period *t*
b^{s}_{w}: Maximum available storage capacity of the supply source *w*
b^{si}_{u}: Maximum storage capacity for raw materials from the sawmill *u*
$b^{si}_{u''}$: Maximum storage capacity for raw materials of the paper mill *u''*
b^{sf}_{u}: Maximum storage capacity of finished products from the sawmill *u*
$b^{sf}_{u''}$: Maximum storage capacity of finished products of the paper mill *u''*
b^{T}_{tw}: Maximum transport capacity from the source *w* during period *t*
b^{TMin}_{tw}: Minimum transport capacity from the source *w* during period *t*
b^{A}_{te}: Maximum supply capacity from the external source *e* during period *t*
b^{T}_{tu}: Maximum transport capacity from the sawmill *u* during period *t*
b^{f}_{ut}: Maximum processing capacity of the sawmill *u* during period *t*
$b^{f}_{u''t}$: Maximum processing capacity of the paper mill *u''* during period *t*
D_{raut}: Demand of product *r*, aged *a*, from the sawmill *u* customers during period *t*
$D_{rau''t}$: Demand of product *r*, aged *a*, from the paper mill *u''*customers during period *t*
M: Big number

Decision Variables

Y_{rwt}: Harvested volume of product *r* from the source *w* during period *t*
SW_{rawt}: Stored volume of product *r*, aged *a*, in the supply source *w* during period *t*
X_{rawut}: Transported volume of product *r*, aged *a*, from the source *w* to the sawmill *u* during period *t*

$X_{rawu't}$: Transported volume of product *r*, aged *a*, from the source *w* to the plant *u'* during period *t*
X_{raeut}: Transported volume of product *r*, aged *a*, from the external source *e* to the plant *u'* during period *t*
$X_{rauu''t}$: Transported volume of product *r*, aged *a*, from the sawmill *u* to the paper mill *u''* during period *t*
X_{raubt}: Transported volume of product *r*, aged *a*, from the sawmill *u* to the timber client *b* during period *t*
$X_{rau''ct}$: Transported volume of product *r*, aged *a*, from the paper mill *u''* to the final client *c* during period *t*
SU_{raut}: Inventory of stored raw materials r, aged *a*, in the sawmill *u* during period *t*
SF_{raut}: Stored volume of final product *r*, aged *a*, in the sawmill *u* during period *t*
$SU_{rau''t}$: Inventory of stored raw materials r, aged *a*, in the paper mill *u''* during period *t*
$SF_{rau''t}$: Stored volume of final product *r*, aged *a*, in the paper mill *u''* during period *t*
PU_{raut}: Inventory volume of transformed product r, aged *a*, in the sawmill *u* during period *t*
PF_{raut}: Quantity of available finished products *r*, aged *a*, in the sawmill *u* during period *t*
$PF_{rau''t}$: Quantity of available finished products *r*, aged *a*, in the paper mill *u* during period *t*
$PU_{rau''t}$: Inventory volume of transformed product r, aged *a*, in the paper mill *u''* during period *t*
ZU_{wt}: $\begin{cases} 1, \text{ if the source w is transported to the mill u during period t} \\ 0, \text{ otherwise} \end{cases}$
ZU'_{wt}: $\begin{cases} 1, \text{ if there is a transport from the source w to the mill u' during the period t} \\ 0, \text{ otherwise} \end{cases}$

Mathematical Model

Objective Function

$$Max \sum_{r,a,u,t} p_{raut} * X_{raubt} + \sum_{r,a,w,u',t} p_{rau't} * X_{rawu't} + \sum_{r,a,u'',t} p_{rau''t} * X_{rau''ct} - \sum_{r,w,t} C^{H}_{rwt} * Y_{rwt} - \sum_{r,e,u',t} C_{reut} * X_{raeut} - \sum_{r,a,w,t} SW_{rawt} * C^{SW}_{rwt} - \sum_{r,a,w,u,t} X_{rawut} * C^{T}_{rwut} - \sum_{r,a,w,u',t} X_{rawu't} * C^{T}_{rwu't} - \sum_{r,a,u,u'',t} X_{rauu''t} * C^{T}_{ruu''t} - \sum_{r,a,u,b,t} X_{raubt} * C^{T}_{rubt} - \sum_{r,a,u,t} SU_{raut} * C^{SI}_{rut} - \sum_{r,a,u'',t} SU_{rau''t} * C^{SI}_{ru''t} - \sum_{r,a,u,t} PU_{raut} * C^{PU}_{raut} - \sum_{r,a,u'',t} PU_{rau''t} * C^{PU}_{rau''t} - \sum_{r,a,u,t} SF_{raut} * C^{SF}_{rut} - \sum_{r,a,u'',t} SF_{rau''t} * C^{SF}_{ru''t} \quad (1)$$

Constraints

- Demand constraints

$$\sum_{b \epsilon B} X_{raubt} \leq D_{raut} \quad \forall\ r \epsilon R, a \epsilon A, u \epsilon U \ et\ t \epsilon T \quad (2)$$

$$\sum_{c \epsilon C} X_{rau''ct} = D_{rau''t} \quad \forall\ r \epsilon R, a \epsilon A, u'' \epsilon U''\ et\ t \epsilon T \quad (3)$$

- Production constraints

$$PF_{raut} = \alpha_{ru} * PU_{raut} \quad \forall\ r \epsilon R, a \epsilon A, u \epsilon U\ et\ t \epsilon T \quad (4)$$

$$PF_{raut} = \alpha_{ru} * PU_{raut} \quad \forall\ r \epsilon R, a \epsilon A, u \epsilon U\ et\ t \epsilon T \quad (5)$$

$$SW_{rawt} = \theta_{rwa} * [Y_{rwt} - (\sum_{u \epsilon U} X_{rawut} + \sum_{u' \epsilon U} X_{rawu't})] + (1 - \theta_{rwa}) * SW_{raw(t-1)} \quad \forall\ r \epsilon R, a \epsilon A, t \epsilon T, w \epsilon W et\ a = 1 \quad (6)$$

- Conservation flows and aging constraints

$$SW_{rawt} = \theta_{rw(a-1)} * SW_{r(a-1)w(t-1)} + (1 - \theta_{rwa}) * SW_{raw(t-1)} - (\sum_{u \epsilon U} X_{rawut} + \sum_{u' \epsilon U} X_{rawu't}) \quad \forall\ r \epsilon R, a \epsilon A, t\ \epsilon T, w\ \epsilon W\ et\ a > 1 \quad (7)$$

$$SU_{raut} = \theta^{si}_{rua} [(\sum_{w \epsilon W} X_{rawut} + \sum_{e \in E} X_{raeut}) - PU_{raut}] + (1 - \theta^{si}_{rua}) * SU_{rau(t-1)} \quad \forall\ r \epsilon R, a \epsilon A, t \epsilon T, u \epsilon U\ et\ a = 1 \quad (8)$$

$$SF_{raut} = \theta^{sf}_{rua} [PF_{raut} - (\sum_{u'' \epsilon U} X_{rauu''t} + \sum_{b \in B} X_{raubt})] + \left(1 - \theta^{sf}_{rua}\right) * SF_{rau(t-1)} \quad \forall\ r \epsilon R, a \epsilon A, t \epsilon T, u \epsilon U\ et\ a = 1 \quad (9)$$

$$SU_{raut} = \theta^{si}_{ru(a-1)} * SU_{r(a-1)u(t-1)} + \left(1 - \theta^{si}_{rua}\right) * SU_{rau(t-1)} + (\sum_{w \epsilon W} X_{rawut} + \sum_{e \in E} X_{raeut}) - PU_{raut} \quad \forall\ r \epsilon R, a \epsilon A, t \epsilon T, u \epsilon U\ et\ a > 1 \quad (10)$$

$$SF_{raut} = \theta^{sf}_{ru(a-1)} * SF_{r(a-1)u(t-1)} + \left(1 - \theta^{sf}_{rua}\right) * SF_{rau(t-1)} - \left(\sum_{u'' \epsilon U} X_{rauu''t} + \sum_{b \in B} X_{raubt}\right) \quad \forall\ r \epsilon R, a \epsilon A, t \epsilon T, u \epsilon U\ et\ a > 1 \quad (11)$$

$$SU_{rau''t} = \theta^{si}_{ru''a} * \left(\sum_{u \epsilon U} X_{rauu''t} - PU_{rau''t}\right) + \left(1 - \theta^{si}_{ru''a}\right) * SU_{rau''(t-1)} \quad \forall\ r \epsilon R, a \epsilon A, t \epsilon T, u'' \epsilon U'' et\ a = 1 \quad (12)$$

$$SF_{rau''t} = \theta^{sf}_{ru''a} * (PF_{rau''t} - \sum_{c \epsilon C} X_{rau''ct}) + \left(1 - \theta^{sf}_{ru''a}\right) * SF_{rau''(t-1)} \quad \forall\ r \epsilon R, a \epsilon A, t \epsilon T, u'' \epsilon U''\ et\ a = 1 \quad (13)$$

$$SU_{rau''t} = \theta^{si}_{ru''(a-1)} * SU_{r(a-1)u''(t-1)} + (1-\theta^{si}_{ru''a}) * SU_{rau''(t-1)} - PU_{rau''t} \quad \forall \quad r\epsilon R, a\epsilon A, t\epsilon T, u''\epsilon U'' \ et \ a > 1 \tag{14}$$

$$SF_{rau''t} = \theta^{sf}_{ru''(a-1)} * SF_{r(a-1)u''(t-1)} + (1-\theta^{sf}_{ru''a}) * SF_{rau''(t-1)} - \sum_{c\in C} X_{rau''ct} \quad \forall \quad r\epsilon R, a\epsilon A, t\epsilon T, u''\epsilon U'' \ et \ a > 1 \tag{15}$$

- Capacity constraints

$$\sum_{r\epsilon R}\sum_{a\epsilon A} PU_{raut} \leq b^{f}_{ut} \quad \forall \ u \ \epsilon U \ et \ t\epsilon T \tag{16}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A} PU_{rau''t} \leq b^{f}_{u''t} \quad \forall \ u'' \ \epsilon U'' \ et \ t\epsilon T \tag{17}$$

$$b^{hmn}_{rwt} \leq Y_{rwt} \leq b^{hmx}_{rwt} \quad \forall \ t \ \epsilon T \ ; \ w \ \epsilon W \tag{18}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A} SU_{raut} \leq b^{si}_{u} \quad \forall \ t \ \epsilon T \ ; \ u \ \epsilon U \tag{19}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A} SU_{rau''t} \leq b^{si}_{u''} \quad \forall \ t \ \epsilon T \ ; \ u'' \epsilon U \tag{20}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A} SF_{raut} \leq b^{sf}_{u} \quad \forall \ t \ \epsilon T \ ; \ u \ \epsilon U \tag{21}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A} SF_{rau''t} \leq b^{sf}_{u''} \quad \forall \ t \ \epsilon T \ ; \ u'' \epsilon U \tag{22}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A} SW_{rawt} \leq b^{s}_{w} \quad \forall \ t \ \epsilon T \ ; \ w \ \epsilon W \tag{23}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A}\sum_{u\in U} X_{raeut} \leq b^{A}_{et} \quad \forall \ \ t \ \epsilon T; \ e \ \epsilon E \tag{24}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A}\sum_{u\in U} X_{rawut} \leq M * ZU_{wt} \quad \forall \ \ t \ \epsilon T; \ w \ \epsilon W \tag{25}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A}\sum_{u'\in U'} X_{rawu't} \leq M * ZU'_{wt} \quad \forall \ \ t \ \epsilon T; \ w \ \epsilon W \tag{26}$$

$$b^{TMin}_{tw} * ZU_{wt} \leq \sum_{r\epsilon R}\sum_{a\epsilon A}\sum_{u\in U} X_{rawut}) \leq b^{T}_{tw} \quad \forall \ \ t \ \epsilon T; \ w \ \epsilon W \tag{27}$$

$$b^{TMin}_{tw} * ZU'_{wt} \leq \sum_{r\epsilon R}\sum_{a\epsilon A}\sum_{u'\in U'} X_{rawu't}) \leq b^{T}_{tw} \quad \forall \ \ t \ \epsilon T; \ w \ \epsilon W \tag{28}$$

$$\sum_{r\epsilon R}\sum_{a\epsilon A} (\sum_{u''\in U''} X_{rauu''t} + \sum_{b\in B} X_{raubt} \leq b^{T}_{tu}) \quad \forall \ \ t \ \epsilon T; \ u \ \epsilon U \tag{29}$$

- Non-negativity constraints

$$SW_{rawt}, Y_{rwt}, X_{rawut}, X_{rawu't}, X_{rauu''t}, X_{raeut}, X_{raubt}, X_{rau''ct}, X_{rau''ct}, SU_{raut}, SU_{rau''t}, PU_{raut}, PF_{raut}, PF_{rau''t}, PU_{rau''t}, SF_{raut}, SF_{rau''t} \geq 0 \quad \forall \ (r, a, w, u, u', u'' \ et \ t) \tag{30}$$

$$ZU_{wt}, ZU'_{wt} \in \{0,1\} \quad \forall \ (w \ et \ t) \tag{31}$$