

Clustering of Gabor Atoms Describing Event-Related Potentials *Solution for ERP Detection Algorithm based on Matching Pursuit when ERP Waveform is Approximated by Two or More Gabor Atoms*

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Abstract: In our research group, we also focus on methods for automatic detection of event-related potentials in the EEG signal. We published the algorithm for event-related potential detection based on the matching pursuit algorithm in one of our previous papers. As usual, this method does not work well under special circumstances which can occur (it is a situation when the waveform of event-related potential is approximated by more than one function from the matching pursuit base functions dictionary). This paper introduces solution of this issue which is based on the self-organizing map and the connected-component labeling algorithm (it allows to group the functions related to a one kind of event-related potential to a cluster - this should prevent the detection algorithm based on matching pursuit from the fault described above).

1 INTRODUCTION

There is a well-known phenomenon which is related to the EEG domain – event-related potentials (ERPs). ERPs are waveforms with specific frequencies, latencies, and polarities (note that there is a special family of ERPs which has alternating polarity) independent of EEG activity. Unfortunately, amplitude of the EEG signal is about ten times greater than amplitude of the strongest ERP waveform. If we assume that EEG is only a noise (as well as interferences from surrounding electromagnetic field and signals of non-cerebral origin: eye movement, muscles activity, EKG activity, etc.), then the signal to noise ratio is very low.

This is the main reason why it is not trivial for neuroscientists to recognize typical ERP waveforms which appear in the electroencephalogram. Of course, it is even more complicated for automatic detection algorithms.

There are a lot of methods which can be used for ERPs detection. One of these methods is the matching pursuit (MP) algorithm that decomposes the EEG/ERP signal to atoms – functions chosen

from a dictionary of base functions. Svoboda, Mautner and Moucek (2008) demonstrated its ability to detect the P3 waveform. When they detect an ERP component in the EEG/ERP signal, they look for an atom which looks like the ERP waveform.

However, there is a problem which can cause the detection fails. This situation occurs when the ERP waveform is decomposed into more than one atom. In this case, no of these atoms approximates the ERP waveform well enough.

In this paper, we introduce an approach to avoid of this fail in detection. The approach is based on identification of all atoms describing the ERP waveform.

The paper is organized as follows: Section 2 is a brief introduction to the ERP domain. It explains what ERPs are and which ERP is decided to be used in this paper. Section 3 introduces the MP algorithm and the principle of the ERPs detection method which was introduced by Svoboda et al. (2008) as well as the problem with decomposition of ERP waveform into multiple atoms. The method for identification of atoms which approximate the ERP waveform is described in Section 4. The methodology for evaluation its ability to identify ERP waveforms is described in Section 5. Last two

sections contain a summary of results and conclusion.

2 EVENT-RELATED POTENTIALS

An event-related potential is any measurable brain response that is a direct result of a specific sensory, cognitive, or motor event. More formally, it is any stereotyped electrophysiological response to a stimulus (Luck, 2005). This brain response is characterized by its amplitude, frequency, and latency (time between stimulus and maximum/minimum value of the waveform).

We decided to use the ERP P3 – visual event-related potential - the third positive wave (see Figure 1) in our experiment. The P3 waveform occurs only if the subject is actively engaged in the task of detecting the targets. Its amplitude varies with the improbability of the targets. Its latency varies with the difficulty of discriminating the target stimulus from the standard stimuli (Picton, 1992).

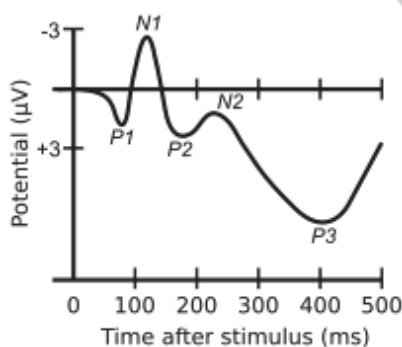


Figure 1: Typical amplitudes, frequencies, latencies, and waveforms of some ERPs (Luck, 2005). In this figure the axis of functional values grows downwards.

The P1, N1, P2, and N2 waveforms are related to sensory perception of stimuli and they are not important for this paper. A detailed description of ERPs was given by Luck (2005).

3 MATCHING PURSUIT ALGORITHM

The MP algorithm decomposes any signal to atoms, which are selected from a dictionary. The atom that approximates the input signal most closely is chosen during each iteration. This atom is subtracted from the input signal and the residue enters the next

iteration of the algorithm. The total sum of atoms selected successively in algorithm iterations is an approximation of the original signal – the more iterations we do, the more accurate approximation we get (Rondik and Ciniburk, 2011). The difference between the input signal and its approximation converges to zero with the increasing number of MP iterations.

The MP algorithm is most often associated with the Gabor atoms dictionary. Gabor atoms are defined as the Gaussian window:

$$g(t) = e^{-\pi t^2} \quad (1)$$

modulated using the cosine function as follows:

$$g_{\gamma} = g_{(s,u,v,w)}(t) = g\left(\frac{t-u}{s}\right) \cdot \cos(vt + w) \quad (2)$$

Each atom is uniquely defined by the ordered quadruple (s,u,v,w) , where s means scale, u is shift, v is frequency, and w is the phase shift. In our experiment, we used the MP algorithm implemented according to Ferrando et al., (2002).

For visualization of Gabor atoms the Wigner-Ville transform is often used – it shows time-frequency energy density. A detailed description was given by Durka (2007).

3.1 Using MP Algorithm ERP Detection

The idea published by Svoboda et al. (2008) is composed of two basic steps:

1. Decomposition of the input EEG/ERP signal into a few Gabor atoms.
2. Selection of the atom (from a set of Gabor atoms) which corresponds to the detected ERP.

It means that correlation between the atom and the input EEG/ERP signal (the value of correlation is often called modulus) must be higher than a threshold.

There are two different examples of the P3 component detection in Figure 2. On the left half of the figure, the favorable situation is shown - the P3 waveform is approximated by one Gabor atom only and the value of the correlation between this atom and the input signal is high enough to pass the threshold. On the right half of the figure, the unfavorable situation is shown. The P3 waveform is partially approximated by the first and third Gabor atom. The value of the correlation between the EEG/ERP signal and both the first and third Gabor atom is not high enough to pass the threshold. It leads to a false negative result during detection.

If we could select all atoms which partially approximate the ERP waveform, calculate the vector sum of these atoms and consider this vector sum as a new atom, we would be able to detect the ERP waveform successfully.

The solution is to use an algorithm which categorizes Gabor atoms into groups in such a way that atoms in each group are similar to each other. Once we have these groups, we can manually mark the groups which contain atoms which can approximate (or partially approximate) ERP waveforms.



Figure 2: The favorable decomposition is shown on the left half and the unfavorable decomposition is shown on the right. In order from top to bottom: the input signal with the P3 waveform; the first, second, and third Gabor atom; visualization of Gabor atoms by the Wigner-Ville transform.

4 WAVEFORMS CATEGORIZATION

We decided to solve the issue shown on Figure 2 with self-organizing map (SOM), because:

- According to Wann and Thomopoulos (1997), SOM is a suitable neural network for data clustering.
- It has an unsupervised learning algorithm. This method of learning is exactly what we need because unsupervised learning clustering methods are applied when the classification of a given set of

sample patterns is unknown (Wann and Thomopoulos, 1997).

4.1 SOM Topology

There are N kinds of SOM topology where N is a dimension of a space where neurons are equidistantly placed. N is an integer value from the interval $(1; \infty)$. We decided to choose a two-dimensional organization of neurons – the most common solution. In the following text, we consider the SOM as a two-dimensional map of neurons.

4.2 Neighborhood Radius during Learning

In the SOM, the winner weight vector and the weight vectors of all neurons in its neighborhood are modified during the learning process. In our implementation, we used square neighborhood which radius is defined as follows:

$$radius = \left\lceil \frac{\alpha \cdot \sqrt{\text{number of neurons}}}{b^p} \right\rceil \quad (3)$$

where α is a learning rate parameter, b is the base of exponential lost (the radius decreases with each next training pattern exponentially), and p is a current learning progress ($p = \frac{done}{all}$ where *done* is the number of training patterns which were already used and *all* is the number of all training patterns).

4.3 Merging Neurons into Clusters

As a result of the learning algorithm we get a specific weight vector for each neuron. It would be worth to have the set of weight vectors such that only one neuron would be marked as a winner for all similar atoms. Unfortunately, it doesn't work in the case of the SOM because the selected atom weight vector and also weight vectors of all atoms in its neighborhood are updated during the learning process (see Section 4.2).

If we want to have all neurons with the similar weight vector in one cluster, we need a method to recognize these neurons and consider them as one cluster. At first, it is necessary to choose a metric for weight similarity. We decided to use a well-known method for measuring signal similarity – correlation. Equation (4) shows computing of correlation between two signals x and y :

$$corrln(x, y) = \frac{\sum_{i=0}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=0}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=0}^n (y_i - \bar{y})^2}} \quad (4)$$

where $\bar{x} = \frac{1}{n} \sum_{i=0}^n x_i$ and $\bar{y} = \frac{1}{n} \sum_{i=0}^n y_i$.

4.3.1 Weight Vectors Similarity Visualization

For better understanding, look at visualization of similarity between neuron weights where each neuron is shown as a 3x3 matrix. On the index $[i-1, j-1]$ is the value of correlation between the neuron on the index $[i, j]$ and the index $[i-1, j-1]$, etc. Note that there is always zero on the index $[i, j]$. For visualization, the values of the correlation result are recalculated from the interval of real values $<-1, 1>$ to the interval of integer values $<0, 255>$ (the gray scale).

$i-1, j-1$	$i-1, j$	$i-1, j+1$
$i, j-1$	i, j	$i, j+1$
$i+1, j-1$	$i+1, j$	$i+1, j+1$

Figure 3: Mask for visualization of weights similarity between neurons.

According to the description given above, visualization of weights similarity of neurons looks as shown in Figure 4.

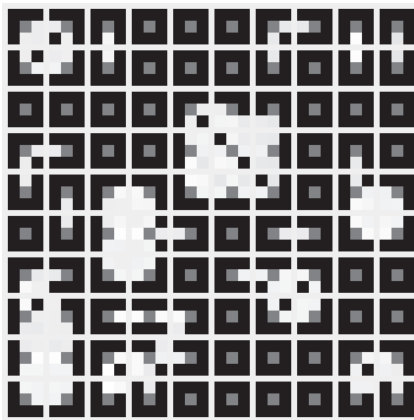


Figure 4: Neuron weights similarity in a two-dimensional map with 100 neurons.

It is easy to see clusters in Figure 4, but we need to implement an algorithm which is able to find these clusters as well. As a solution, we used an algorithm which is well-known in computer vision – connected-component labeling.

4.3.2 Connected-Component Labeling

Connected-component labeling (CCL) is a two-pass

algorithm. It uses a map of neurons as an input. In the first pass, CCL iterates through each neuron by row. The neighboring neurons are given by a mask:

$i-1, j-1$	$i-1, j$	$i-1, j+1$
$i, j-1$	i, j	

Figure 5: Mask which defines neighboring neurons during the first pass of the CCL algorithm.

According to correlation between the weight vector of the neuron $[i, j]$ and weight vectors of neighboring neurons three situations can occur:

1. If correlation with all neighboring neurons is too low to be in the same cluster with neuron on the index $[i, j]$ then a new cluster number is set to neuron on the index $[i, j]$.
2. If correlation of just one of neighboring neurons is high enough to be in the same cluster as neuron on the index $[i, j]$ then the neuron on position $[i, j]$ is put to the same cluster.
3. If correlation of more than one of neighboring neurons is high enough to be in the same cluster as neuron on the index $[i, j]$ one of them is randomly selected and the neuron on position $[i, j]$ is put to the same cluster. If neighboring neurons with high enough correlation value belong to different clusters, save that these clusters are equivalent to a special data structure.

In the second pass, CCL iterates through each neuron by row and gets rid of equivalent cluster numbers for one cluster using the special data structure from the first pass. After the second pass, each cluster is signed with just one cluster number (even if the cluster consists of one neuron only).

4.4 Suitable Feature Vector

Selection of a suitable feature vector is a critical decision for further successful clustering. There are no exact or universal rules to identify an optimal feature vector (see Lotte et al., (2007); Pradhan et al., (1996) and Gotman and Wang (1991) for examples of feature vectors used in the EEG domain).

We decided to use the following feature vectors based on Gabor atoms (the result of the MP algorithm):

- Functional values of the Gabor atom (full length).
- Functional values of the Gabor atom subsampled to 32 samples.

5 EXPERIMENT METHODOLOGY

5.1 Test Data

Our test data were obtained during the experiment based on oddball paradigm (originally designed by Squires et al. (1975), where the white *O* character on a black background shown on the full screen was the non-target stimulus and the white *Q* character on a black background shown on the full screen was the target stimulus. Brain activity for the Fz, Cz, and Pz electrode (see 10-20 system in Niedermeyer and Lopes da Silva (2004)) was sampled with 1 kHz frequency (this frequency is sufficient according to Nyquist-Shannon sampling theorem). Frequencies higher than 45 Hz were cut off automatically. Then the electroencephalogram was split into epochs (each epoch starts with stimulus onset and takes 512 ms (because of used MP implementation) which means – according to sampling frequency - 512 samples per epoch), the baseline was corrected, and epochs were divided into target and non-target ones.

We decomposed all target epochs to five Gabor atoms using the MP algorithm. According to Rondik and Ciniburk (2011), five atoms are sufficient for our purposes. The whole test set was composed of 359 Gabor atoms.

5.2 Initial Setup of SOM

We tested all feature extraction methods based on Gabor atoms described in Section 4.4. We used the following initial SOM setting:

- A two-dimensional neural network with 100 neurons
- learning rate is equal to 0.7 (see Section 4.2)
- initial values of weight vectors were randomly chosen from the interval of real values $(0, 1)$

5.3 SOM Learning

The neural network was learned with all feature vectors. At the end of the learning procedure, we obtained a neuron index for each feature vector. Because we know which feature vector is related to which Gabor atom, we can assign each Gabor atom to a specific neuron.

5.4 Clusters Definition

At this point, we assign neurons to clusters using the CCL algorithm (the clustering threshold was set to

0.8). These clusters are inputs for the clustering quality evaluation method. We need to evaluate quality of clusters to be sure that clustering via SOM and CCL works well.

5.5 Clustering Quality Evaluation

For evaluation of clustering quality we need a measure which can compute similarity between Gabor atoms in each cluster. For measuring the similarity of all signals in one cluster we used:

$$sim(C_i) = \frac{1}{n(n-1)} \sum_{j=0}^n \sum_{k=0}^n crrltn(x_j^i, x_k^i); j \neq k \quad (5)$$

where C_i is i^{th} cluster and x^i is the set of all signals in the cluster C_i .

6 RESULTS

Actually we could assume that we are interested in clustering quality for clusters which contain waveforms which approximate (or partially approximate) P3 waveforms only. The clustering quality of other waveforms which do not describe the P3 waveform is – from the MP algorithm detection issue point of view - not significant.

Looking at average similarities in Table 1 it can be easily seen that there is no significant difference between clustering quality of P3 waveforms clusters and all clusters.

Table 1: Comparison of clusters quality with respect to used feature vector.

Feature vector	All clusters	ERP clusters only	
	Average similarity per cluster	Number of averaged clusters	Average similarity per cluster
512 samples	0.5804	6	0.4851
32 samples	0.5694	6	0.5567

In Figure 6 the clusters which are related to Gabor atoms which approximate (or partially approximate) P3 waveform are highlighted (the highlighting was done manually). Other clusters belong to waveforms which are not related to P3 waveforms.

7 CONCLUSIONS

Let us note that this is the first experimental result. Taking this into account, the results cannot be

considered as statistically significant. However, it shows us, that the way we decided to solve the problem with ERPs detection which affects method described in Svoboda et al. (2008), may be right.

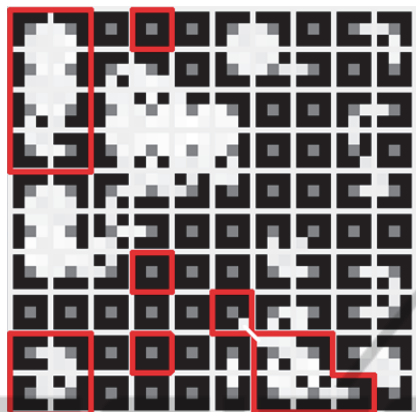


Figure 6: Neuron weights similarity in a two-dimensional map with 100 neurons with manually highlighted clusters which are related to Gabor atoms which approximate ERP P3 waveform.

Looking at results given in Table 1, it does not matter which of two feature vectors presented in this paper will be used. The only difficulty which affects the described method is that clusters which approximate (or partially approximate) ERP waveforms must be marked manually by an expert.

In the future, we will use the proposed method in ERP detection algorithm based on MP to prove, that this method can improve the reliability of ERPs on a statistically significant level.

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REFERENCES

- Luck, S. J., (2005). *An Introduction to the Event-Related Potential Technique*. Cambridge: The MIT Press.
- Rondik, T. and Ciniburk, J., (2011). Comparison of Various Approaches for P3 Component Detection Using Basic Methods for Signal Processing. *4th International Conference on Biomedical Engineering and Informatics (BMEI)*, 700 - 704, New York: IEEE.

- Picton, T. W., (1992). The p300 wave of the human event-related potential. *Journal of Clinical Neurophysiology*, 9, 456-479.
- Svoboda, J., Mautner, P. and Moucek, R., (2008). Detection of ERP using Matching Pursuit Algorithm. *Xth International conference on cognitive neuroscience*, 226, Istanbul.
- Ferrando, S. E., Kolasa, L. A. and Kovacevic, N., (2002). Algorithm 820: A Flexible Implementation of Matching Pursuit for Gabor Functions on the Interval. *ACM Transactions on Mathematical Software*, 28(3), 2002, 337-353.
- Wann, C. D. and Thomopoulos, S. C. A., (1997). A Comparative Study of Self-organizing Clustering Algorithms Dignet and ART2. *Neural Networks*, 10(4), 737-753.
- Lotte, F., Congedo, M., Lecuyer, A., Lamarche, F. and Arnaldi, B., (2007). A Review of Classification Algorithms for EEG-based Brain-Computer Interfaces. *Journal of Neural Engineering*, 4.
- Pradhan, N., Sadasivan, P. K. and Arunodaya, G. R., (1996). Detection of seizure activity in EEG by an artificial neural network: A preliminary study. *Computers and Biomedical Research*, 29, 303-313.
- Gotman, J. and Wang, L., (1991). State-dependent spike detection: Concepts and preliminary results. *Electroencephalography and Clinical Neurophysiology*, 79(1), 11-19.
- Squires, N. K., Squires, K. C. and Hillyard, S. A., (1975). Two varieties of long-latency positive waves evoked by unpredictable auditory stimuli in man. *Electroencephalography and Clinical Neurophysiology*, 38(4), 387-401.
- Niedermeyer, E. and Lopes da Silva, F., (2004). *Electroencephalography: Basic Principles, Clinical Applications, and Related Fields*. Philadelphia: Lippincott Williams & Wilkins.
- Durka, P. J., (2007). *Matching pursuit*. Retrieved April 22, 2010, from: http://www.scholarpedia.org/article/Matching_pursuit.