

# Use of Fuzzy Cognitive Maps for Climate System Stability Analysis

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Abstract: In the present work we use fuzzy cognitive maps for the qualitative analysis of the earth's climate system dynamics. First of all, we identify the subsystems which determine, as a whole, the stability of the climatic system. Later we develop cognitive maps (knowledge networks) based on the documented relationships between the subsystems (nodes of the network). The relationships between the nodes can be precise (quantifiable) or fuzzy (not quantifiable). Once the map is built, we use the state vector and adjacent matrix technique to assess the response of the system (the system converges or diverges) to the changes in the input node values in order to identify the possible feedback. Then the *Min-Max* criteria is used to evaluate the effect of the network over the nodes, according to the fuzzy weights assigned to the edges (causal relations between nodes). Finally, we discuss some possible changes in the network in order to show how the system dynamic can be modified and can lead the system into a desired equilibrium state.

## 1 INTRODUCTION

Fuzzy cognitive maps are fuzzy-graph structures for representing causal reasoning between variable concepts (Kosko, 1986). The concepts are represented as nodes ( $C_1, C_2, \dots, C_n$ ) in an interconnected network, each node  $C_i$  represents a concept and the edges  $e_{ij}$  which connect  $C_i$  and  $C_j$  (denoted as  $C_i \rightarrow C_j$ ) are causal connections and express how much  $C_i$  causes  $C_j$ . These edges can be negatives or positives. A positive relation  $C_i \rightarrow +C_j$  states that if  $C_i$  grows also does  $C_j$ , and a negative relation  $C_i \rightarrow -C_j$  indicate that as  $C_i$  grows  $C_j$  decreases. The fuzzy cognitive maps have been used to model different kind of systems, such as control, social and economics systems, and also in robotics, computed assisted learning, expert systems, and many others. Even though cognitive maps have been applied to many areas, little has been done in atmospheric sciences. This is the importance of the present work.

As an example of cognitive maps, consider the following network of 4 nodes. Here we use positive (+) and negative (-) causality among the concepts. The system state can be represented as a state vector, which contains the values for each node at time  $t$ . For example, the row vector  $v_0 = [1, 0, 0, 0]$  at  $t_0$  indicates that only the first node in the system is ON at time zero. As we said, the adjacent matrix of the network expresses the causal relationships between nodes. In

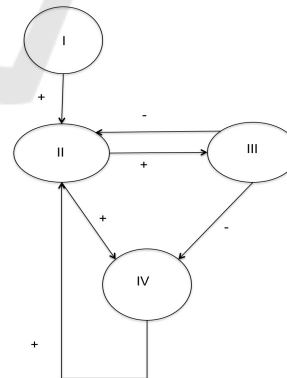


Figure 1: 4-node network.

cognitive maps the adjacent matrix is always square. In our example the matrix is:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Then the system is iterated to get  $v$  for  $t_1, t_2, \dots, t_n$ . For this, multiply the row vector by the matrix  $v * M$ , so we have:

$$v_n = v_{n-1} * M \quad (1)$$

If we consider  $v_0 = [1, 0, 0, 0]$ , and the iterations are made, we get:

$$v_1 = [0, 1, 0, 0] \text{ The node I causes II.}$$

$v_2 = [0,0,1,1]$  The node II light on the nodes III and IV.

$v_3 = [0,0,0,-1]$  The node III acts negatively over II y IV, in the node II the negative effect vanishes.

$v_4 = [0,-1,0,0]$  The node IV acts negatively on II.

$v_5 = [0,0,-1,-1]$  Node II acts negatively over III y IV.

$v_6 = [0,0,0,1]$ .

Finally, we obtain  $v_7 = [0,1,0,0]$ , which is equal to  $v_1 = [0,1,0,0]$ . In this case the system reaches a limit cycle. In general it can be shown that for every matrix the system converges to a point, to a limit cycle, or diverges in the space of states for the system. If we express the state vector as  $X(t) = (x_1(t), \dots, x_n(t))$ , and the signal value as  $S_i$ , we define a threshold binary function, which limits the signal value by a pre-defined threshold for each time  $t$  as:

$$S_i(x_i^t) = \begin{cases} 1 & \text{si } x_i^t > U_i \\ S_i(x_i^{t-1}) & \text{si } x_i^t = U_i \\ 0 & \text{si } x_i^t < U_i \end{cases} \quad (2)$$

In this way, the node values will be restricted between 0 and 1. Similarly, it can be defined ternary functions with values of  $\{-1,0,1\}$ .

## 2 SUBSYSTEMS OF THE CLIMATIC SYSTEM

The proposed subsystems to describe the dynamic of the climatic system are based on the article "A safe operating space for humanity" written by Johan Rockström (2009), and also in the Intergovernmental Panel on Climate Change Report (IPCC, 2007). These are:

### Industrialization.

This process comprises: social and economic development, technology tendencies and its applications, industry grown, as well as demographic changes associated with these processes (IPCC, 2007).

### Climate Change.

This concept refers to increase in the mean temperature of the earth, i.e. changes in climate variability in terms of the extreme and mean values. (IPCC, 2007). Specifically, we refer to antropogenic climate change, which is a consequence of the human activity.

### Changes in CO<sub>2</sub> Concentration.

Defined as the increase in the parts per million of

CO<sub>2</sub> molecules in the atmosphere (IPCC, 2007).

### Biodiversity Loss.

Refers to the extinction rate, the number of species loss per million per year. Mace and collaborators (2005), define biodiversity as the variability of living organisms, included terrestrial and marine ecosystems, other aquatic ecosystems and the ecological systems in which they reside. Comprises the diversity within species, among species and within ecosystems. Mace emphasizes three levels of biodiversity: genes, species, and ecosystems. Biodiversity loss during the industrial period has grown notably. The species extinction rate is estimated against the fossil record. The extinction rates per million per year varies for marine life between 0.1 and 1 and for mammals between 0.2 and 0.5.

### Phosphorus and Nitrogen Cycles.

The changes in  $P$  and  $N$  cycles are estimated with the quantity of  $P$  going to the oceans, measured in million tones per year, and with the amount of  $N_2$  removed from the atmosphere for human use, also in million tones per year.

### Ocean Acidification.

Defined as the ocean PH increase, mainly in the surface layer. The acidification process is closely related with the CO<sub>2</sub> emission level. When the atmosphere CO<sub>2</sub> concentration increases, the amount of carbon dioxide dissolved in water as carbonic acid increases, which in turn, modifies the surface PH. Normally, the ocean surface is basic with a PH of approximately 8.2. Nevertheless, the observations show a decline in PH to around a value of 8. These estimations are made using the levels of aragonite (a form of calcium carbonate) that is created in the surface layer. This concept has an important relation with biodiversity loss as many organisms (like corals and phytoplankton), basic for the food chain, use aragonite as to produce their skeletons or shells. As the aragonite value decreases, the ocean ecosystems weaken, and the fisheries production falls. (Foley *et al.*, 2010).

### Land Use (Urban Growth and Agriculture Use).

The IPCC defines the change in land use as the percentage of global land converted into cropland. A general definition of land use change includes any type of human use. This transformation, either to cropland or urban, increases the biodiversity loss, which is associated with the destruction of ecosystems. In order to establish the difference in

land use between urban growth and agriculture use, and their different consequences, we include both concepts as nodes in the map.

**Increase in Fresh Water Demand.**

Defined as an increase in its current use. Today around the globe the annual use of freshwater from rivers, lakes and groundwater aquifers is of 2,600 km<sup>3</sup>. From that, 70% is destined for irrigation, 20% for industry, and 10% for domestic use. This extraction causes the drying and reduction body waters. (IPCC, 2007).

**Stratospheric Ozone Depletion.**

O<sub>3</sub> depletion is estimated according to the ozone concentration in the atmosphere in Dobson Units.<sup>1</sup>

**Chemic Pollution.**

It refers to the emitted quantity, persistence, or concentration of organics pollutants, plastics, heavy metals, chemical and nuclear residues, etc., which affect the dynamic of ecosystems.

**Aerosol Loading.**

Referred as the concentration of particles in the atmosphere. These can be lead, copper, magnesium, iron, traces of fire, ashes, etc.

With the use of these definitions we create a cognitive map establishing and weighting the relationships among the concepts (different subsystems). Through the analysis of the map we create a qualitative global vision of the climatic system dynamics.

**3 THE MODEL**

Using the subsystems we build the cognitive map shown below. The nodes and the relations among them are represented in it. These relations had been established with the consulted references. As many relations between concepts can not be quantifiable, i.e. can not be expressed numerically, we have used linguistic variables. These variables will be later used with the *Min-Max* criteria. To analyze the system we start with the adjacent matrix and the state vector, considering values of 0 and 1 (positive causality and no causality) and a binary function.

<sup>1</sup>Dobson unit is a measure of the ozone layer thickness, equal to 0,01 mm of thickness in normal conditions of pressure and temperature (1 atm and 0 C respectively), express as the molecule number. DU represents the existence of 2.69 x 10<sup>16</sup> molecules per square centimeter.

It is important to note that in the map all the relations (edges) are positives, so the associated matrix is:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

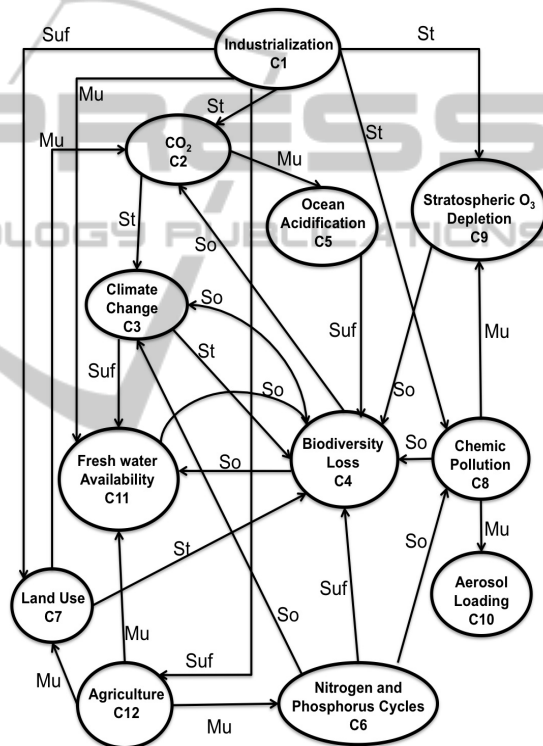


Figure 2: Cognitive map with fuzzy edges. We have used four causal quantities: Strong (St), Much (Mu), Sufficient (Suf), and, and Some (So).

First we turn on the first node in the state vector  $a_0 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$ , then we iterate  $a * M$ , and reset to 1 the first entrance of the vector  $a$  after each iteration i.e. ( $a[1] = 1$ ). In the network the *shortest way* between the first node (C<sub>1</sub>) and any other consists of two steps, so when we turn on the node C<sub>1</sub> the system converges to the equilibrium state “ $a_2$ ” in two steps, with:

$$a_2 = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$$

This vector state represents the increase of each node

due to the causal action of  $C_1$  but it does not say anything more, i.e. the action of  $c_1$  causally increases the other nodes of the network when is forced with ( $a[1] = 1$ ). When the first node is not forced ( $a[1] = 1$  only in  $t_0$ ), the system reaches an equilibrium among the nodes which have *feedback*. In this case we get:

$$a_0 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

$$a_1 = [0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1]$$

The nodes that are connected with  $C_1$  turn on.

$$a_2 = [0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 0]$$

The nodes 3, 4, 5, 6, 10 and 11 turn on. These are the nodes connected in two steps with  $C_1$ . While node 8 turn off (no node activates it in this iteration). In the fifth iteration the system converges to vector  $a_5$ :

$$a_5 = [0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0]$$

Another approximation is to start the first node with an intermediate value in  $[0,1]$ , which represents the case where the industrialization node decreases from the current level but is not zero. In this case we choose the value 0.5 i.e. assuming an intermediate industrialization activity. We first iterate the system turning on the first node after each iteration. In the following experiment we start the system without turning on the node. In the first case, the system converges in six iterations to  $a_6 = [0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0]$ , and in the second case the system converges in four iterations to  $a_4 = [0.5, 1, 1, 1, 1, 0.5, 1, 1, 1, 1, 0.5]$ . In the first case, we found that the system reaches an equilibrium among nodes 2, 3, 4, 5, and 11, which we will discuss in the next section. In the second case, we found a new state of equilibrium corresponding to the value applied to  $C_1$ . These approximations help us to figure out how policies can affect the equilibrium states of the system when they are applied to a specific node.

### 3.1 Feedback

The equilibrium is reached among nodes 2, 3, 4, 5, and 11 where it exists feedback. They remain ON, even though the other nodes are OFF. This fact says that once these nodes are turn on, they remain ON, and this does not depend on the rest of the system. If we analyzed the subsystem of nodes 2, 3, 4, 5 and 11 (that represent  $CO_2$  growth, Climate Change, Biodiversity Loss, Ocean Acidification, and Fresh Water Availability, respectively) renamed as 1, 2, 3, 4, and 5 we have:

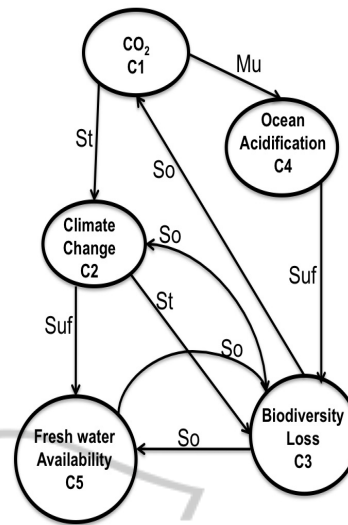


Figure 3: System of nodes 2, 3, 4, 5, and 11 renamed as 1, 2, 3, 4, and 5 respectively.

$$M_{subsystem} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

If we force the first node in each step, in two iterations we get  $a = [1, 1, 1, 1, 1]$ . And when we activate  $C_1$  only in ( $a_0$ ) the system also converges to  $a = [1, 1, 1, 1, 1]$  in five steps. This means that whenever the node  $C_1$  is activated, at any time, the system converges into a point, and the feedback remains. To analyze the system behavior we add fuzzy weights to the edges. We consider (in this case) four quantifiers for each edge value *some*, *sufficient*, *much*, *strong*, and associate the values in  $[0, 1]$  as follows *some* = 0.25, *sufficient* = 0.5, *much* = 0.75 and *strong* = 1. These values correspond to the maximum membership values for each linguistic set. This is shown in figure 4.

With these values the system matrix is expressed as:

$$M_{subsystem} = \begin{bmatrix} 0 & 1 & 0 & 0.75 & 0 \\ 0 & 0 & 1 & 0 & 0.5 \\ 0.25 & 0.25 & 0 & 0 & 0.25 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 \end{bmatrix}$$

Turning ON  $C_1$  and keeping  $a[1] = 1$  after each iteration the system converges to the vector:

$$a_3 = [1.00, 1.00, 1.00, 0.75, 0.75]$$

When we turn on the first node only at the first iteration the state vector shows the following behavior:

$$a_0 = [1, 0, 0, 0, 0]$$

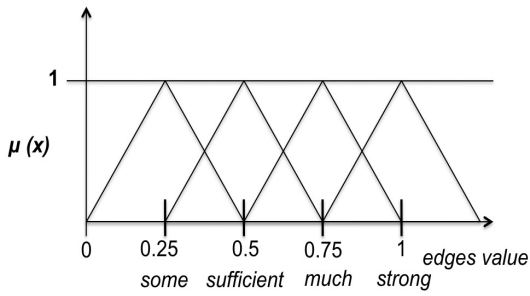


Figure 4: The x-axis represents the values of the causality between nodes divided in four sets. The y-axis represents the membership degree of each value in the x-axis for each fuzzy set. In this manner, 0.25 belongs to the set of *some* causality with a membership degree  $\mu(0.25) = 1$  represented in the y-axis. The value 0.75 belongs to the set *sufficient* with a membership degree of 1 and so on.

$$\begin{aligned}
 a_1 &= [0, 1, 0, 0.75, 0] \\
 a_2 &= [0, 0, 1, 0, 0.5] \\
 a_3 &= [0.25, 0.25, 0.25, 0, 0.25] \\
 a_4 &= [0.03, 0.28, 0.31, 0.18, 0.15] \\
 a_5 &= [0.07, 0.10, 0.41, 0.02, 0.21] \\
 a_6 &= [0.10, 0.18, 0.17, 0.05, 0.15] \\
 a_7 &= [0.04, 0.14, 0.25, 0.07, 0.13] \\
 &\vdots \\
 &\vdots \\
 &\vdots \\
 a_{20} &= [0.01, 0.02, 0.04, 0.01, 0.02]
 \end{aligned}$$

Which means that if we force the system only at  $t_0$  the system tends to the equilibrium vector  $a_{eq} = [0, 0, 0, 0, 0]$ . In these case the matrix coefficients represent the system damping to the initial perturbation. The system behavior given this particular structure can be interpreted in terms of the scenarios and mitigation actions, e.g. can the matrix weights be modified by specific actions or strategies?

### 3.2 Fuzzy Weighted System

The matrix associated with the fuzzy system is shown in figure 5.

Turning on the node  $C_1$  we have ( $a_0 = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$ ), and iterating, the system converges in two steps into vector:  $a_2^* = [1.00, 1.00, 1.00, 1.00, 0.75, 0.37, 0.87, 1.00, 1.00, 0.75, 1.00, 0.5]$ . Here we have forced  $C_1$ , i.e. at each iteration  $a[1] = 1$ . We found that the nodes 3, 5, 6, 7, 8, 10, and 12, reach stable values, while the nodes 2, 4, 9, and 11, have been limited to 1 by the threshold function. This offers important information about the network. As nodes 2, 4, 9, and 11 diverge, they could be interpreted as sensitive nodes, i.e. these are the nodes that can not reach an equilibrium with the present

conditions. Then, for example, is possible to think about short-term actions for them in order to avoid unwanted scenarios. Furthermore we can say that even though the forcing node is industrialization, mitigation actions may be focused in more than one node. When we restart  $C_1$  only at  $t_0$  the system is damping again by the matrix coefficients.

$$a_1 = [0, 1.00, 0, 0, 0, 0, 0, 0.5, 1.00, 1.00, 0, 0.75, 0.5]$$

$$a_2 = [0, 0.37, 1.00, 1.00, 0.75, 0.37, 0.37, 0, 0.75, 0.75, 0.37, 0]$$

In  $a_2$  the node 4 has been thresholded from 1.18 to 1, these is because this node has the greatest causality in the network.

$$a_3 = [0, 0.53, 0.71, 1.00, 0.28, 0, 0, 0.09, 0, 0, 0.75, 0]$$

In  $a_3$  the node 4 has been thresholded again.

$$a_4 = [0, 0.25, 0.78, 1.00, 0.39, 0, 0, 0, 0.07, 0.07, 0.60, 0]$$

In  $a_4$  we also threshold node 4 from 1.07 to 1

$$a_5 = [0, 0.25, 0.5, 1, 0.18, 0, 0, 0, 0, 0, 0.64, 0]$$

In  $a_5$  node 4 is threshold from 1.15 to 1

$$a_6 = [0, 0.25, 0.5, 0.75, 0.18, 0, 0, 0, 0, 0, 0.5, 0]$$

In step 6 node 4 need no threshold. From these point the system converges to zero.

$$a_7 = [0, 0.18, 0.43, 0.71, 0.18, 0, 0, 0, 0, 0, 0.43, 0]$$

$\vdots$   
 $\vdots$   
 $\vdots$

$$a_{14} = [0, 0.08, 0.17, 0.29, 0.07, 0, 0, 0, 0, 0, 0.18, 0]$$

We can see how the subsystem with feedback remains, but converges to zero as iterations progress. The fact that the node 4 diverges give us important information about the sensibility of this node to the network conditions, i.e. node 4 has strong sensibility.

Another important analysis is what happens when the network is at equilibrium and then the conditions change? In order to analyze this we consider the vector  $a_2^*$  that we obtained while maintaining forcing. Then turn off the first node and calculate  $a * M_{sist}$ .

$$a_0 = [1.00, 1.00, 1.00, 1.00, 0.75, 0.37, 0.87, 1.00, 1.00, 0.75, 1.00, 0.5]$$

$$a_1 = [0, 1.90, 1.34, 3.18, 0.75, 0.37, 0.87, 1.09, 1.75, 0.75, 1.87, 0.5]$$

using threshold to limit the values of nodes 2, 3, 4, 8, 9, and 11

$$a_1 = [0, 1.00, 1.00, 1.00, 0.75, 0.37, 0.87, 1.00, 1.00, 0.75, 1.00, 0.5]$$

in the second iteration we have:

$$a_2 = [0, 0.90, 1.00, 1.00, 0.75, 0.37, 0.37, 0.09, 0.75, 0.75, 1.00, 0]$$

$$M_{system} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0 & 0.75 & 0.5 \\ 0 & 0 & 1 & 0 & 0.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.25 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0.5 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0.75 & 0.75 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.75 & 0.75 & 0 & 0 & 0 & 0.75 & 0 \end{bmatrix}$$

Figure 5: Weighted system.

where we also used the *threshold* function for nodes 3, 4 and 11. At iteration 10 we have:  
 $a_{10} = [0, 0.13, 0.29, 0.49, 0.12, 0, 0, 0, 0, 0, 0.34, 0]$

Which shows that once forcing is removed, the system reaches the equilibrium at  $a_n = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$ , where the matrix coefficients remain as damping (*Liapunov*) coefficients. Nevertheless, it is important to characterize the feedback processes in order to know if the threshold function is changing (or not) the system's behavior.

### 3.3 Indirect and Total Effects (Min-Max)

When we assign *fuzzy weights* we can use the Indirect and total effect operators (*I* and *T*) to analyze the causal action among nodes. Let us analyze, for example, the causality between industrialization and biodiversity loss. To go from  $C_1$  to  $C_4$  we have: (1,2,3,4), (1,7,4), (1,12,11,4), (1,11,4), (1,8,4), (1,9,4), and (1,12,6,4),(1,12,6,8,4), and (1,12,6,3,4), which are denoted here as  $I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}$  respectively. If we analyze the linguistic quantifiers among them:  $I_1(C_1, C_4) = \min\{e_{12}, e_{23}, e_{34}\} = \min\{strong, strong, strong\} = strong$ . and so:  $I_2(C_1, C_4) = \min\{e_{17}, e_{74}\} = \min\{sufficient, strong\} = sufficient$ .  
 $I_3(C_1, C_4) = \min\{sufficient, much, much, sufficient\} = sufficient$ .  
 $I_4(C_1, C_4) = \min\{sufficient, much, some\} = some$ .  
 $I_5(C_1, C_4) = \min\{much, some\} = some$ .  
 $I_6(C_1, C_4) = \min\{strong, some\} = some$ .  
 $I_7(C_1, C_4) = \min\{strong, some\} = some$ .  
 $I_8(C_1, C_4) = \min\{sufficient, much, sufficient\} = sufficient$ .  
 $I_9(C_1, C_4) = \min\{sufficient, much, some, some\} =$

*some*.  
 $I_{10}(C_1, C_4) = \min\{sufficient, much, some, strong\} = some$ .

$T(C_1, C_4) = \max\{I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}\} = \max\{strong, sufficient, sufficient, some, some, some, some, sufficient, some, some\} = strong$ . Which means that  $C_1$  "strongly causes"  $C_4$ . The (*Min-Max*) analysis give us important information about how causality is operating in the system, and which relation or relations among subsystems exert more influence over one specific node. In the example, we found that  $C_1$  strongly causes  $C_4$ , however, the only path that posses strong causality as indirect effect is  $I_1(C_1, C_4)$ . We can use this information for strategic planning, i.e. to modify the action that  $C_1$  exerts over  $C_4$  (the total effect *T*), we must focus on the relations  $e_{12}$ ,  $e_{23}$ , and  $e_{34}$  rather than the other edges. Changing only one of these relations, the total effect of  $C_1$  over  $C_4$  will change to *sufficient*.

## 4 CONCLUSIONS

Throughout this work we build a model for the earth climate system by relating earth subsystems recognized by experts as those which determine the climate system stability. In the analysis we found that with the established network conditions, and considering only positive causality, the system diverges, which means that all nodes increase without limit leading the system into an undesired state. Then, we turn off the system's forcer in order to identify the feedback processes. Nodes with feedback are specially important for us as the causality persists among them, even though the forcing has been turned off. So, these nodes need special attention because the system behavior depends on the edges weight, once a

node is activated. When we focus on the subsystem with feedback, we discover that without the forcer the system returns to the equilibrium state, even without threshold. Still, we use the threshold function when working with the complete system. It is necessary to explore in greater depth the connection edges and their weights to know if the system is really modeling these interactions.

The system behaviour starting in a particular state exhibits all the behaviours mentioned above, however the threshold use is stronger in this case, which also suggests the necessity of refining the weights among subsystems.

With respect to the *Min/Max* criteria, we found that the trajectory ( $I_1$ ) with more weight between the forcer ( $C_1$ ) and the node with more entries in the net ( $C_4$ ) is the one that goes from 1 to 2, from 2 to 3 and from 3 to 4. The analysis of all other possible trajectories shows that the indirect effects of each one are less than those of  $I_1$ . To modify the causality of  $C_1$  over  $C_4$ , we have to focus only on the indicated trajectory, i.e. if we change the other trajectories we would not change the weight over the  $C_4$  node. This is one of the examples where the cognitive map allow us to make strategic decisions about a net.

Finally, the use of cognitive maps in climate systems allows the creation of knowledge networks that integrate systems that under any other circumstance would be impossible to integrate to obtain qualitative information of the system that can be easily adapted to decision making schemes.

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## REFERENCES

- Foley, J. A. (2010). Boundaries for a healthy planet. *Scientific American*.
- Kosko, B. (1986). Fuzzy cognitive maps. *Man-Machine Studies*, 24:65–75.
- Mace, G., Masundire, H., Baillie, J., Ricketts, T., and Brooks, T. (2005). *Ecosystems and human well-being: Current state and trends*, chapter Chapter 4: Biodiversity. Island Press USA.
- on Climate Change, I. I. P. (2007). Cambio climático 2007: Informe de síntesis. contribución de los grupos de trabajo i, ii y iii al cuarto informe de evaluación del

grupo intergubernamental de expertos sobre el cambio climático. IPCC, Suiza.

Rockström, J., Steffen, W., Noone, K., Persson, Chapin, F. S., Lambin, E. F., Lenton, T. M., Scheffer, M., Folke, C., Schellnhuber, H. J., Nykvist, B., Wit, C. A., T. H., Van der Leeuw, S., Rodhe, H., Srin, H., Snyder, P. K., Costanza, R., Svedin, U., Falkenmark, M., Karlberg, L., Corell, R. W., Fabry, V. J., Hansen, J., Walker, B., Liverman, D., Richardson, K., Crutzen, P., and Foley, J. A. (2009). A safe operating space for humanity. *Nature*, 461.

Stylios, C. D., Georgopoulos, V. C., and Groumpos, P. P. (1997). The use of fuzzy cognitive maps in modeling systems. In *5th IEEE Mediterranean Conference on Control and Systems MED5 paper*, volume 67, page 7.