

# Synthesis of Reuse Water Networks by PSO Approach

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**Abstract:** In the present paper the problem of reuse water networks have been modeled and optimized by the application of a modified Particle Swarm Optimization (PSO) algorithm. A proposed modified PSO method lead with both discrete and continuous variables in Non-Linear Programming (NLP) and Mixed Integer Non-Linear Programming (MINLP) formulations that represent the water allocation problems. Pinch analysis concepts are used jointly with the improved PSO method. A literature problem was solved with the developed systematic and results has shown excellent performance in the optimality of reuse water network synthesis based on the criterion of minimization of annual total cost.

## 1 INTRODUCTION

In the last decades the studies in the minimization of primary water consumption in industrial processes and in the wastewater reduction from such processes have contributed to the minimization of environmental impacts. Instead of applying graphic and algebraic technologies to solve the problems of process integration, mathematical programming has been used as a very convenient alternative method when the subject can be formulated as an optimization problem.

The problem of Water/Wastewater Allocation Planning (WAP) can also consider both discrete and continuous variables. A large group of WAP problems have Mixed Integer Non-Linear Programming (MINLP) and NLP formulations and a great variety of algorithms has been proposed, developed and improved.

The PSO algorithm was properly modified in order to satisfy the requirements of leading with discrete-type variables and other strategies were also included to solve MINLP-based models. In addition, as criteria for obtaining the synthesis of the reuse water network, the minimization of the total cost was applied.

## 2 WAP PROBLEM DEFINITION AND MODEL FORMULATION

With regard to the total possible configurations of mass transfer between the water streams and the process streams and all the possibilities for reuse water, a superstructure was built, as reported by Trigueros et al. (2012), in order to attain optimization of the mass exchange network project in a simultaneous analysis procedure. A reduction in the high contaminant loads of the process streams is essentially performed by transferring mass to a cleaner water stream, with the possibility of reusing it in the other (N-1) process units.

In this work, maximum inlet and outlet pollutant concentration data were used in the synthesis of the reuse water network, calculating the maximum water flow rate (Eq. 1) and demanding a global mass balance (Eq. 2). The superstructure was fractioned in small components corresponding to each process unit, mixing and splitting nodes in which their individual mass balances are defined by Equations (3)–(5), respectively.

In addition, pollutant mass balances in the process streams are also performed (Eqs. (6) and (7)), and the maximum allowed pollutant concentration constraints for the inlet and outlet of each process unit, two inequalities (Eqs. 8 and 9). A condition necessary to warrant no violation of the minimum  $\Delta C_i$ , (see Eq.10), was demanded in each

process unit (see Eq. 10). In addition, the pinch point freshwater flow rate value was introduced in the modeling as a physical restriction variable (Eq. 11), among other non-negativity constraints (Eqs. (12–17)). Finally, as an optimization criterion of the reuse water network synthesis the total cost of the industrial plant (Eq. (18)) was considered.

$$f_i = \frac{\dot{m}_i}{(C_j^{out\max} - C_j^{in\max})} \quad (1)$$

$$\sum_{i=1}^N f_i^{freshwater} + \sum_{i=1}^N \sum_{j=1}^J \dot{m}_{i,j} - \sum_{i=1}^N f_i^{wastewater} = 0 \quad (2)$$

$$f_i^{out} - f_i^{in} - \sum_{j=1}^J \dot{m}_{i,j} = 0 \quad \forall i \in N \quad (3)$$

$$f_i^{freshwater} + \sum_{\substack{k=1 \\ k \neq i}}^N f_{i,k} - f_i^{in} = 0 \quad \forall i \in N \quad (4)$$

$$f_i^{wastewater} + \sum_{\substack{k=1 \\ i \neq k}}^N f_{k,i} - f_i^{out} = 0 \quad \forall i \in N \quad (5)$$

$$f_i^{freshwater} C_i^{freshwater} + \sum_{\substack{k=1 \\ i \neq k}}^N f_{i,k} C_{k,j}^{out} - f_i^{in} C_{i,j}^{in} = 0 \quad (6)$$

$$\forall i \in N; \forall j \in J$$

$$f_i^{in} C_{i,j}^{in} + \dot{m}_{i,j} - f_i^{out} C_{i,j}^{out} = 0 \quad (7)$$

$$\forall i \in N; \forall j \in J$$

$$C_{i,j}^{in} \leq C_{i,j}^{in\max} \quad \forall i \in N; \forall j \in J \quad (8)$$

$$C_{i,j}^{out} \leq C_{i,j}^{out\max} \quad \forall i \in N; \forall j \in J \quad (9)$$

$$f_i^{in} - f_i^{\max} \leq 0 \quad \forall i \in N \quad (10)$$

$$\sum_{i=1}^N f_i^{freshwater} - f^{Pinch} \leq 0 \quad (11)$$

$$f_i^{freshwater}, f_i^{wastewater}, f_i^{in}, f_i^{out}, f_{i,k}, f_{k,j} \geq 0 \quad (12-17)$$

$$\forall i \in N$$

$$\begin{aligned} Min(z) = & AB \sum_{k=1}^N \left( \sum_{i \neq k}^N f_{i,k} + f_i^{freshwater} \right)^\alpha + \\ & CD \sum_{k=1}^N \left( \sum_{i \neq k}^N f_{i,k} + f_i^{freshwater} \right) + CE \sum_{i=1}^N f_i^{freshwater} + \\ & + F \left( \sum_{k=1}^N \sum_{i=1}^N y_{i,k} + \sum_{i=1}^N y_i^{freshwater} + \sum_{i=1}^N y_i^{wastewater} \right) \end{aligned} \quad (18)$$

## 2.1 PSO Proposed Algorithm

A PSO algorithm that was earlier reported by Kennedy and Eberhart (2001) applied by Trigueros et al. (2010) to solve problems with continuous variables, was modified to consider also discrete variables. A numeric generator function in the 0-1 range and a cut-off value condition were introduced in the PSO algorithm (Eqs. 19-20), including a complementary binary attribution test (Eqs. 21 and 22). Within the modified PSO algorithm, the possible interference between discrete and continuous variable positions as well as their respective velocities was avoided by creating another search space where binary particles are moving with other velocity ranges. To avoid non-viable solutions, the original objective function was penalized by adding the inequality and equality constraints that were previously violated as well as assigning weights to each type of violation, according to Eqs. (23-25).

$$sig(\mathbf{x}_i^k) = \frac{1}{1 + e^{(-\mathbf{x}_i^k)}} \quad (19)$$

$$\text{If } \mathbf{x}_i^k > sig(\mathbf{x}_i^k) \text{ then } \mathbf{x}_i^k = 1 \text{ else } \mathbf{x}_i^k = 0 \quad (20)$$

$$\text{If } y_{i,k} = 0 \text{ then } f_{i,k} = 0 \text{ else } f_{i,k} = rand() \quad (21)$$

$$\text{If } f_{i,k} = 0 \text{ then } y_{i,k} = 0 \text{ else } y_{i,k} = 1 \quad (22)$$

$$\mathbf{H}(\mathbf{x}, \mathbf{y}) = \begin{cases} \mathbf{h}(\mathbf{x}, \mathbf{y}) & \text{if } |\mathbf{h}(\mathbf{x}, \mathbf{y})| - \varepsilon > 0 \\ 0 & \text{if } |\mathbf{h}(\mathbf{x}, \mathbf{y})| - \varepsilon \leq 0 \end{cases} \quad (23)$$

$$\mathbf{G}(\mathbf{x}, \mathbf{y}) = \begin{cases} \mathbf{g}(\mathbf{x}, \mathbf{y}) & \text{if } \mathbf{g}(\mathbf{x}, \mathbf{y}) > 0 \\ 0 & \text{if } \mathbf{g}(\mathbf{x}, \mathbf{y}) \leq 0 \end{cases} \quad (24)$$

$$\mathbf{z}_p = \mathbf{z} + \left[ \sum_{i=1}^m b_i \mathbf{G}_i + \sum_{j=1}^n d_j \mathbf{H}_j \right] \quad (25)$$

In order to avoid the constrain search space and the increasing computational time, two strategies were considered: dependent and independent variables were defined in the mathematical model and adopting the fundamental concepts of pinch analysis in order to achieve feasible or very near feasible solutions.

## 3 CASE STUDY

An early proposition of Olsen and Polley (1997),

summarized in Table 1, was used as a modified PSO-method testing system in the optimization procedure. Firstly, the pinch point flow rate was estimated (157.14 ton/h) and required as a physical criterion in the optimization procedure. A set of 51 equations (38 equality constraints and 12 inequality constraints and one objective function), 73 continuous and 48 binary decision variables are required to represent the WAP problem. By redefining some variables as dependent in the PSO algorithm, decision variables were reduced. All financial parameter (A,  $\alpha$ , B, C, D, E and F) values were obtained from Wang and Smith (1994). As some situations that are expected for the minimization of the total cost, two strategies were applied, being a fixed pinch point flow rate as first strategy, whereas no constraint on the consumption of freshwater was considered as second strategy.

Results from the use of the developed PSO algorithm for the reuse water network synthesis are shown in Fig1. When applying the first strategy, a minimum water flowrate of 157.16 ton/h and an annual total cost of US\$ 2,217,101.70 were attained. The network contains 5 fresh water, 4 reuse water and 5 wastewater streams (see Fig. 1a). By considering a variation on the fresh water flowrate near to the Pinch point, other reuse network synthesis were obtained with different annual total cost as shown in Fig. 2, where its behavior is depending on the total fresh water (see Fig. 2a) and reuse water flowrate (see Fig. 2b).

### 4 CONCLUSIONS

A modified PSO algorithm was proposed and tested to optimize a WAP problem. It is possible to achieve

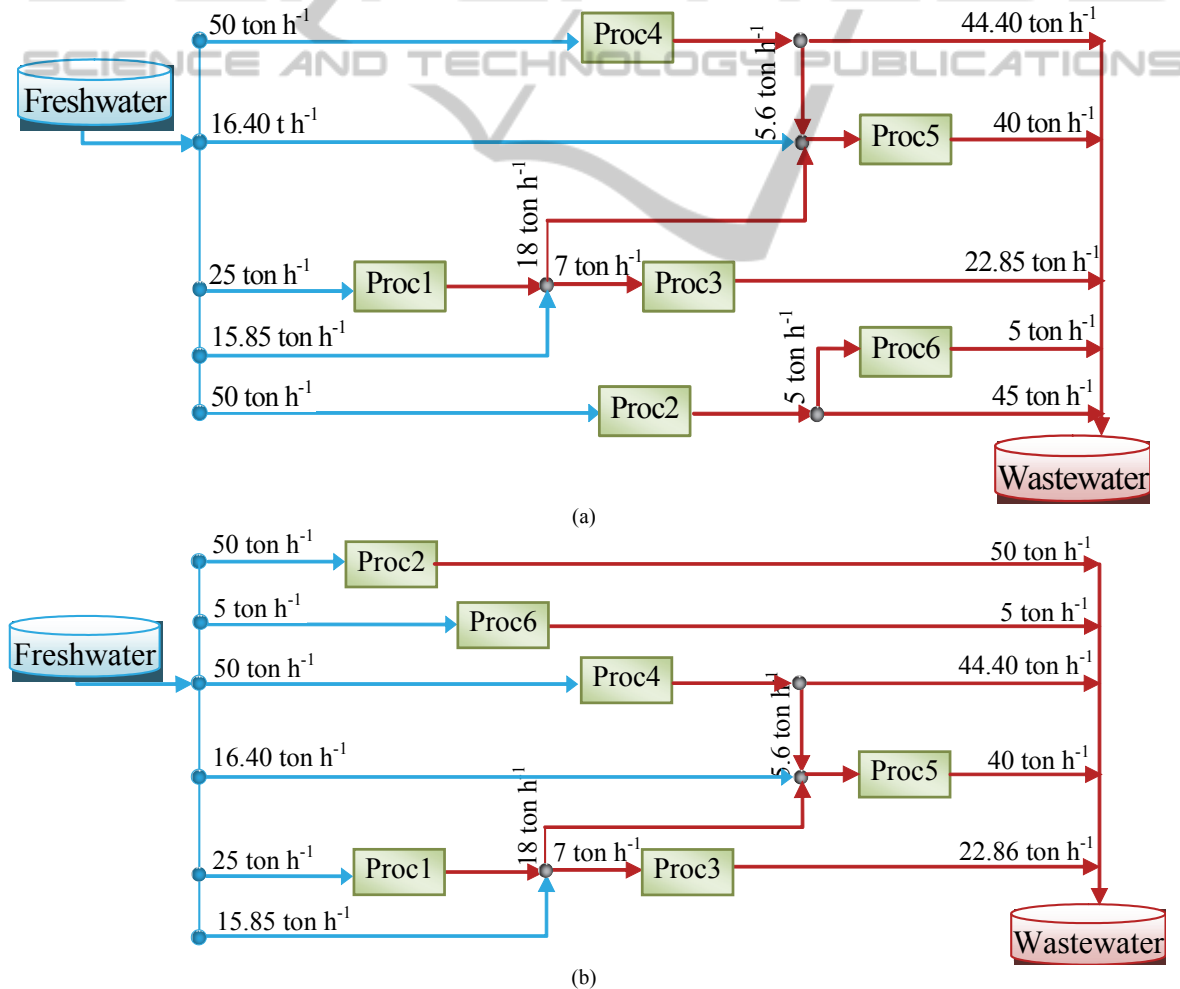


Figure 1: Reuse network synthesis by the PSO method for the minimization of total cost, considering the (a) first and (b) second strategies.

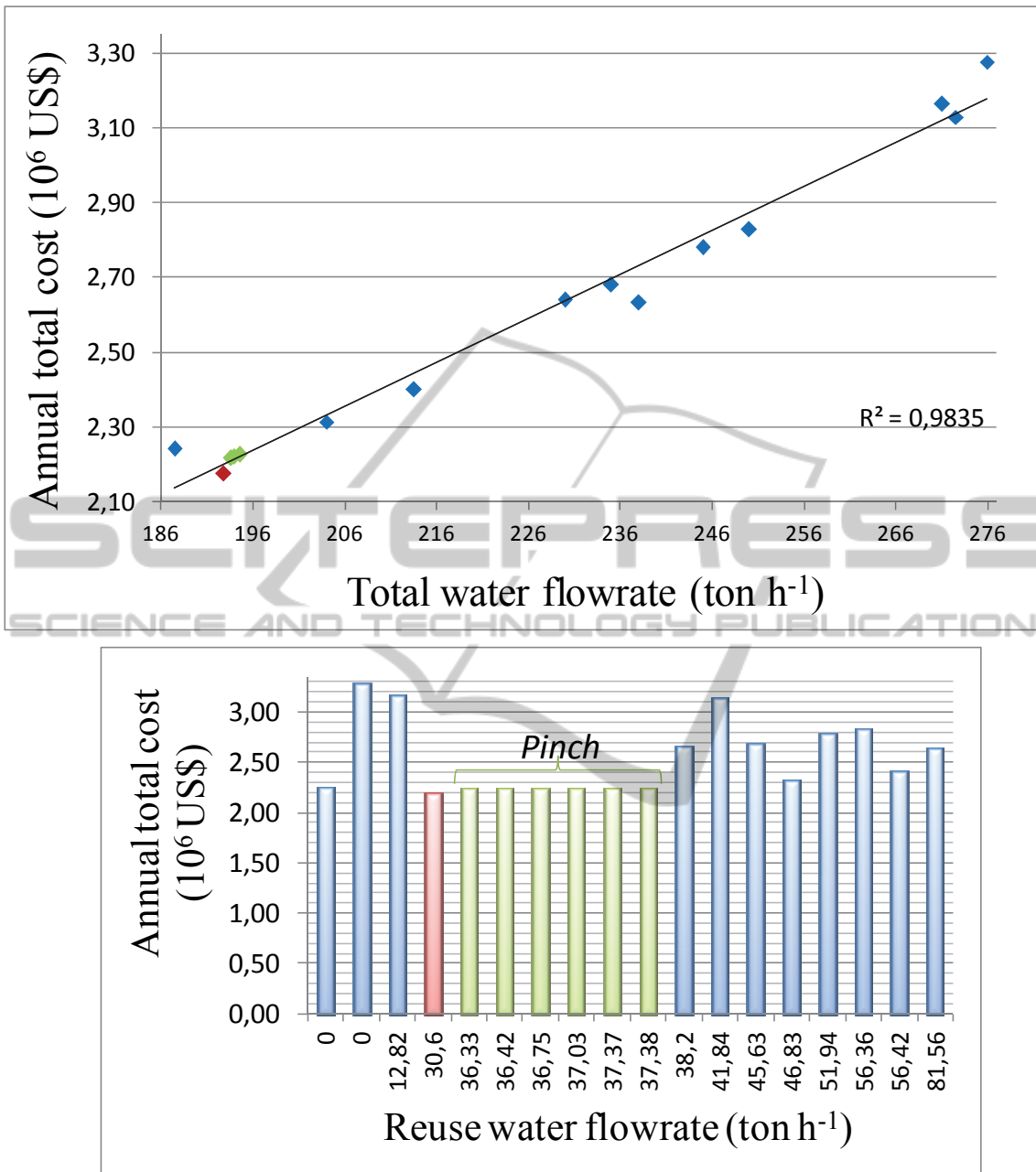


Figure 2: Behavior of the total cost as a function of (a) total water flow rate and (b) reuse water flow rates.

Table 1: Problem data - Olesen and Polley (1997).

Process	$C_{\max}^{in}$ (ppm)	$C_{\max}^{out}$ (ppm)	$m$ (g h <sup>-1</sup> )
1	25	80	2000
2	25	100	5000
3	25	200	4000
4	50	100	5000
5	50	800	30000
6	400	800	4000

different reuse water network synthesis by demanding the minimum annual total cost as criterion and requiring on fixing or assigning values near the Pinch point for the fresh water flowrate, and without any constraint in the fresh water flowrate. In conclusion, the modified PSO algorithm has shown high flexibility and capability to provide optimal results for the reuse water network synthesis, being independent of initial estimative for the decision variables.

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