

# Application of Evolutionary Strategies for Optimisation of Parameters during the Modelling of the Magnetic Hysteresis Loop of the Construction Steel

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**Abstract:** This paper concerns the possibility of use evolutionary strategies for optimisation of magnetic characteristics model's parameters. The Jiles-Atherton extended model was used for modelling the magnetic hysteresis loop of construction steel St10. In this model  $k$  parameters change their value during the magnetisation process. However, determination of model's parameters by gradient optimisation was not successful. Only use of evolutionary strategies for optimisation enables achievement of very good agreement with results of experimental measurements. This agreement was confirmed by high values of the  $R^2$  determination coefficient.

## 1 INTRODUCTION

Magnetic method is frequently used in non-destructive testing of ferromagnetic components (Blitz, 2007). The most important disadvantage of this method is its limitation only to the comparative measurements. For this reason, it is important to develop the material's magnetisation model, which will enable a generalised description of the characteristics of magnetisation changes and only accordant to the mechanical state of that material. This model should be based on physical principles, and to include the influence of mechanical stress or fatigue failures.

Among many models of magnetisation of ferromagnetic materials (Andrei et al., 1998), used in non-destructive testing of construction steel, the Jiles-Atherton model, seems to be the most useful (Jiles and Atherton, 1986). This model not only mathematically reproduces the magnetic hysteresis loops, but also takes into account the physical aspect of the material magnetisation process. For this reason, it is used in stress assessment of ferromagnetic construction materials and widely documented in literature (Chwastek and Szczyglowski, 2006).

However, Jiles-Atherton model has significant limitations. For one set of calculated model parameters, results of the modelling are in

accordance with results of experimental measurements, but only for one of the amplitudes of the magnetising field.

Therefore, in these studies the Jiles-Atherton extended model is used, which allows to avoid this disadvantage. In all models based on Jiles-Atherton approach, model parameters are determined on the base of minimisation of the sum of squared differences, between the hysteresis loop obtained by modelling, and the hysteresis loop resulting from experimental measurements. However, for extended Jiles-Atherton model of magnetic characteristics, the determination of model's parameters on the base of commonly used gradient optimisation is not successful, due to the presence of local minima (Szewczyk R. 1, 2007). In this case, the evolutionary strategies were proposed.

The paper presents a novel method of determination of Jiles-Atherton model parameters. The method can enable technological breakthrough in non-destructive testing of construction steels, such as *St10*.

## 2 JILES-ATHERTON EXTENDED MODEL

The Jiles-Atherton model is based on an analysis of the thermodynamic potentials (Sablík et al., 1988).

From a physical point of view these potentials characterise the thermodynamic transformations and are described by the following relations:

$$A = G + \mu_0 \cdot H \cdot M \quad (1)$$

$$G = U - T \cdot S + \frac{3}{2} \sigma \cdot \lambda \quad (2)$$

$$U = \frac{1}{2} \alpha \cdot \mu_0 \cdot M^2 \quad (3)$$

where:  $A$  - Helmholtz free energy,  $G$  - Gibbs free energy,  $U$  - materials internal energy,  $S$  - materials free entropy,  $M$  - magnetisation,  $H$  - magnetising field,  $\mu_0=4\pi 10^7$  H/m is vacuum permeability,  $T$  - materials temperature,  $\sigma$  - stress mechanics in material,  $\lambda$  - magnetostrictive strain,  $a$  - coefficient describing the coupling between the domain (according to the Bloch model) (Liorzou et al., 2000).

The original Jiles-Atherton model of magnetisation process utilizes seven parameters:  $a$  - quantifies domain walls density,  $k$  - quantifies average energy required to break pinning site,  $c$  - coupling coefficient,  $\alpha$  - is interdomain coupling,  $K_{an}$  - anisotropy energy density,  $t$  - participation of anisotropic phase,  $M_s$  - saturation magnetisation.

The Jiles-Atherton model should include the anisotropy of the material (Szewczyk 2, 2007). Anisotropy can be caused by stress arising in the material. The total magnetisation  $M$  is given as the sum of reversible magnetisation  $M_{rev}$  and irreversible magnetisation  $M_{irr}$ . Reversible magnetisation  $M_{rev}$  are calculated from the equation (Jiles and Atherton, 1986):

$$M_{rev} = c(M_{an} - M_{irr}) \quad (4)$$

where  $M_{an}$  is the anhysteretic magnetisation, which should be calculated as a weighted sum of the anisotropic magnetisation  $M_{aniso}$  and isotropic magnetisation  $M_{iso}$  (Jiles et al., 1997).

$$M_{an} = tM_{aniso} + (1-t)M_{iso} \quad (5)$$

where  $t$  - weight coefficient, describing participation of anisotropic phase in the material.

Isotropic magnetisation  $M_{iso}$  in material is given by the equation (Jiles and Atherton, 1986):

$$M_{iso} = M_s \left[ \operatorname{cath} \left( \frac{H_{eff}}{a} \right) - \left( \frac{a}{H_{eff}} \right) \right] \quad (6)$$

where  $H_{eff} = H + \alpha M$  - effective magnetising field, where  $\alpha$  represents interdomain coupling.

Anisotropic magnetisation  $M_{aniso}$  in material is given by the equation (Ramesh et al., 1996):

$$M_{aniso} = M_s \frac{\int_0^\pi e^{E(1)+E(2)} \sin \theta \cos \theta d\theta}{\int_0^\pi e^{E(1)+E(2)} \sin \theta d\theta} \quad (7)$$

where  $E(1)$  and  $E(2)$  are energies and are given by the equation:

$$E(1) = \frac{H_{eff}}{a} \cos \theta - \frac{K_{an}}{M_s \mu_0 a} \sin^2(\psi - \theta) \quad (8)$$

$$E(2) = \frac{H_{eff}}{a} \cos \theta - \frac{K_{an}}{M_s \mu_0 a} \sin^2(\psi + \theta) \quad (9)$$

where  $K_{an}$  - anisotropic energy density,  $\psi$  - angle between the easy axis of the material and the magnetising field direction.

Equation for the anisotropic magnetisation can be calculated only by numerical methods, because the primary functions of the integral functions are not known.

The original model allows to model hysteresis loops only for one value of the magnetising field. Extended model can be used to model the hysteresis loop for different values of the magnetising field. This is possible, because the model's parameters are dependent on change of the value of magnetisation.

In the Jiles-Atherton extended model parameter  $k$  is connected with magnetisation  $M$  in material and is given by the equation (Szewczyk, 2009):

$$k(|M|/M_s) = g_0 + \frac{e^{g_2 \cdot (1-|M|/M_s)} - 1}{e^{g_2} - 1} \cdot (g_1 - g_0) \quad (10)$$

where:  $g_0$  - defines the value  $k$  in demagnetized state,  $g_1$  - defines the value  $k$  of magnetic saturation,  $g_2$  - factor of the waveform functions  $k(|M|/M_s)$ , where  $M_s$  is saturation magnetisation.

### 3 METHODOLOGY OF MEASUREMENTS

Experimental measurements of magnetic characteristics of steel were made for the ring-shaped core made of construction steel *St10*. This core has the following dimensions: inner diameter 30,9 mm, outer diameter 44,9 mm, height 9,55 mm.

Experimental setup for measurements of  $B(H)$  characteristics of ring-shaped samples is presented in figure 1. The measuring setup is controlled by PC. The hysteresis loop was measured using HBP 2.0

hysteresismeter. Measurement was carried out for the speed of gain of the magnetising field  $H$  of 150 A/m/s. Hysteresis loops were determined for increasing amplitude of magnetic field intensity in the range from 20 to 1142 A/m. Also the initial magnetisation curves were measured. Between the measurements of magnetic hysteresis loop the core was demagnetised with sinusoidal waveform of the exponentially decreasing amplitude. Frequency of this waveform was 10 Hz, initial amplitude was 1142 A/m, ratio of successive amplitudes was 1,03.

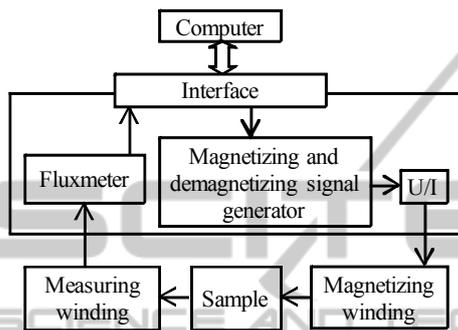


Figure 1: Schematic block diagram of the measuring setup.

#### 4 METHOD OF OPTIMISATION

The optimisation process bases on the minimisation of target function, which is given by the equation (Szewczyk, 2009):

$$F = \sum_{i=1}^n (B_{J-A-S}(H_i) - B_{pom}(H_i))^2 \quad (11)$$

where:  $n$  - number of measurement points,  $H_i$  - magnetic field,  $B_{J-A-S}(H_i)$  - results of the modelling,  $B_{pom}(H_i)$  - results of the experimental measurements.

In presented case, the best is to use a two-stage optimisation. In the first step, the evolutionary strategies ( $\mu+\lambda$ ) (Schwefel, 1995), combined with simulated annealing (Schwefel, 1981), (Wilson et al., 2001), should be used. In the second step, the gradient optimisation should be used, for the 20 best results obtained after the first step.

The evolutionary strategies ( $\mu+\lambda$ ) are the heuristic optimisation methods, based on adaptation and evolution. In evolutionary strategies, the population of vectors, which contain parameters of the Jiles-Atherton extended model, is subjected to the three operators. First - mutation operator, which randomly changes the value of the parameter of the model. Second - crossover operator, which

exchanges values between the two vectors. And third - selection operator, which to select the best value of the target function  $F$ .

From the population of  $\mu$  individuals (parents), population of  $\lambda$  individuals (descendants) is created. During this process, copies of randomly selected  $\mu$  individuals are parents. Then, on the base of  $\mu$  parents, the population of  $\lambda$  descendants is created randomly, using the operators of mutation and crossover. Population of descendants is combined with the parents population, creating a population of  $\mu+\lambda$  individuals. The best  $\mu$  individuals from the  $\mu+\lambda$  population gives the new population for the next iteration.

During the optimisation process, physical limits of the Jiles-Atherton model have to be strictly observed. If physical limits are exceeded (e.g. value of anisotropic energy density  $K_{an}$  is lower than zero) the value of the target function  $F$  is significantly increased. As a result, the optimisation process is carried out within physical limits.

In the minimisation process a population of 900 vectors was used. The crossover operator of a group of  $\mu=3$  vectors (parents) generated  $\lambda=12$  vectors (descendants). Then the descendants vectors were subjected to the mutation. The distribution of value changes of the parameters during the process of mutation was a normal distribution, of which standard deviation was equal to 3% of the initial value of the modified parameter. In every iteration, in accordance with the simulated annealing, the standard deviation decreased by 5%.

#### 5 RESULTS

The target function  $F$  was calculated for 3 hysteresis loops (measured for different magnetising fields) at the same time.

The figure 1 below shows the changes in the value of the target function  $F$  during the optimisation process by using the evolutionary strategies.

Because the functions  $F_{min}$  (for best vector of the population, calculated during the the optimisation process) and  $F_{max}$  (for worst vector of the population, calculated during the the optimisation process) decreases monotonically in the next iterations, the optimisation process can be regarded as convergent.

The next figure 2 shows the results of experimental measurement of hysteresis loop  $B(H)$  (marked with •) and modelling results (marked with •)

—). The obtained parameters of the Jiles-Atherton model for steel *St10* are shown in table 1.

The figure 3 shows the changes in the permeability for experimental measurement (marked with •) and modelling results (marked with —).

Results of the modelling utilising evolutionary strategies are correspond to results of experimental measurements. The  $R^2$  determination coefficient exceeds 99% for each of the magnetic hysteresis loop  $B(H)$  in the amplitude of the magnetising field below 1000 A/m.

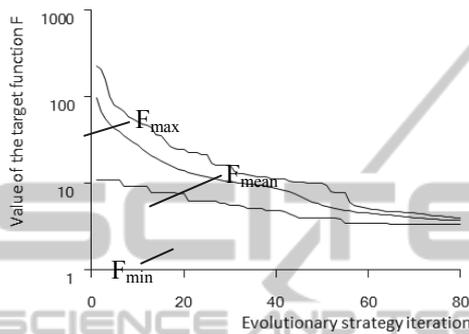


Figure 2: The changes of the target function ( $F_{min}$  for best vector,  $F_{max}$  for worst vector,  $F_{mean}$  for average) during the optimisation process.

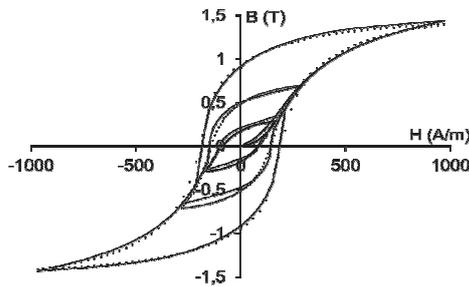


Figure 3: Results of the experiment (•) and results of modelling (—) quasistatic magnetic hysteresis loop  $B(H)$  for steel *St10*.

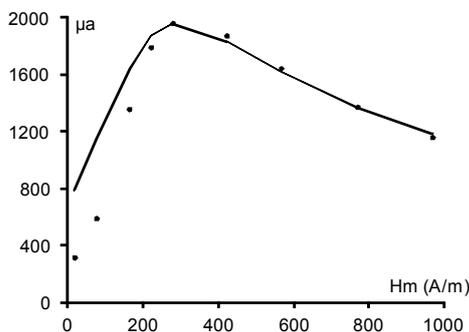


Figure 4: Results of the experiment (•) and results of modelling (—) dependence of amplitude magnetic permeability  $\mu_a$  on amplitude of magnetic field  $H$  for steel *St10*.

The obtained parameters  $g_0$ ,  $g_1$  and  $g_2$  indicate that the value of the parameter  $k$  decreases rapidly as a function of the magnetisation  $M$ . The obtained parameters are consistent with the results of physical measurements for steel. Particularly, the saturation magnetisation  $M_s$  amount 1 788 100 A/m is close to typical values for low carbon steel.

Table 1: Jiles-Atherton model parameters after optimisation.

Parameter	Value
$A$	819 A/m
$g_0$	404 A/m
$g_1$	216 A/m
$g_2$	13,9
$c$	0,491
$M_s$	1 788 100 A/m
$a$	0,00131

## 6 CONCLUSIONS

The application of evolutionary strategies to optimisation process decreased sensitivity to local minima. This optimisation not only allows to calculate the model's parameters, but also allows to obtain very good results of the modelling. These results are correspond to results of experimental measurements. This was confirmed by the high values of the  $R^2$  determination coefficient.

For this reason Jiles-Atherton extended model may be suitable for determination of stresses in *St10* construction steel during non-destructive testing. This model enables modelling of the magnetisation characteristics of *St10* steel in a wide range of amplitude of the magnetising field.

The presented application of evolutionary strategies will be particularly useful in developing methods for assessing the state of stress in the material, by measuring the magnetic hysteresis loop. Calculations were made in the Interdisciplinary Centre for Mathematical and Computational Modelling of Warsaw University, grant G36-10.

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