

FPGA Implementation of Hodgkin-Huxley Neuron Model

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Abstract: In this paper an implementation of Hodgkin-Huxley single neuron is provided. Unlike almost all of the existing implementations, the arithmetic logics are implemented with computation techniques (i.e. CORDIC) and look-up-tables (LUTs) are used only in few modules. This makes our design more robust and flexible to simulate the functionality of a large network of neurons. Most of the previous works are based on the software implementations which overshadow the parallel nature of the neural system or using LUTs for hardware implementation which needs more space and also limited flexibility. In this paper, an FPGA is selected as our hardware implementation platform to provide an appropriate reconfigurable platform for simulating the functionality of a network of neurons. We validated our design based on our high level implementation of Hodgkin-Huxley neuron in MATLAB and report our implementation results based on Xilinx SPARTAN 3 FPGA in Xilinx ISE Design Suite.

1 INTRODUCTION

There has been a recent interested in computational neuroscience, which tries to reveal how the brain processes information, based on the functionality and the relationship between different parts of the nervous system. Neurons are considered as the basic building blocks of the nervous system. They are excitatory/inhibitory cells that receive electrical signals from other neurons, combine them, and transmit them to other neurons through their short or long axons. To perform a specific function, up to thousands neurons may interconnect to each other in a neural network. It is of special interest to develop computational tools to simulate the behaviour of neurons, and verify the theories that were proposed to justify the results of experiments (Kandel et al., 2000).

Transient, as well as the steady state, response of each neuron affects the behavior of all neurons connected to it. This dynamic interaction among the neurons seems to be very important in functionality of the neural networks. Such an interaction cannot be implemented in a serial system. Therefore, modeling parallelism as well as dynamic interaction of the neurons is essential.

A neuron can be modeled in two ways: software simulation and hardware implementation. Software simulation has a serial nature (Li et al., 2010) and, as a result, simulation of transient interaction of the neurons is very difficult in software simulation. In addition, it is slower in comparison to the hardware implementation. Moreover, due to the required large number of interconnected neurons in a neural network, hardware implementation provides the results in a reasonable time. There are two different hardware implementation approaches: analog and digital implementations. Digital implementation is widely used because of its lower noise sensitivity, more flexibility, easier testability and repeatability (Muthuramalingam et al., 2008). In addition, digital implementation has lower design time and cost (Gatet et al., 2009). Nevertheless, it is easier to model dynamic interaction of the neurons in analogue systems but the main deficiency of digital implementation is the limited available area which demands more optimal designs.

There are different alternatives for digital implementation; including ASIC, DSP, and FPGA. The ASIC-based implementation is not suitable because it is not reconfigurable and it cannot be changed after manufacturing (Wanhammar, 1999). The key feature of DSP-based implementation is

that, it is sequential and the neurons cannot be easily modelled in a parallel architecture. In other words, DSP-based systems are special purpose implementation of processor-based system. FPGAs benefit from reconfigurable and parallel architecture and these properties make them the best candidate for neuron's implementation (Muthuramalingam et al., 2008).

Several FPGA implementations of biological neurons have been proposed so far (Graas et al., 2004); (Mokhtar et al., 2008); (Rice et al., 2009); (Pourhaj and Teng, 2010) that most of them implemented Izhikevich models (Mokhtar et al., 2008); (Rice et al., 2009). In this paper, we are interested to implement a more biologically plausible neuron model. For this purpose, the Hodgkin-Huxley (H-H) neuron model is chosen. In (Graas et al., 2004); (Pourhaj and Teng, 2010), the H-H neuron model has been implemented. E. L. Graas et al. (Graas et al., 2004) used look-up-table (LUT) instead of implementing equations for calculating the critical parts of the implementation. So, this type of implementation needs a large size of memory to pre-compute the required results (of course with limited number of bits) and save them in LUTs which reduces the final accuracy because of constraints on the memory size. In addition, in (Graas et al., 2004) a neuron was implemented using System Generator but it is obvious that automatic software tools cannot produce an optimal hardware. P. Pourhaj et al. (Pourhaj and Teng, 2010) used LUT to implement different parts, which has a side effect on the final accuracy of the implementation. For instance, the LUT-based implementation of some equations with the exponential terms has not (and also can not) enough accuracy.

In this paper the H-H neuron model is used for implementation. We employ MATLAB for high level design of a single neuron's behavior and use the results as a gold standard to check our implementation on FPGA. The rest of the paper is organized as follows. In Section 2 the H-H model is briefly explained. The FPGA implementation details are discussed in Section 3. In Section 4 experimental results are provided and finally Section 5 concludes the work and provides the future works.

2 HODGKIN HUXLEY MODEL

In a series of experiments carried out on the squid giant axon, Hodgkin and Huxley introduced a model to explain the process of action potentials generation in the neuron, based on ionic current through the

membrane (Hodgkin and Huxley, 1952). Voltage-dependent ion channels for potassium and sodium control flow of these ions. All other ions (mostly chloride ions) that flow through the neuron membrane constitute the leak current. Concentration and electrical potential gradients are two forces driving ions passing through the membrane channels. The electrical potential, in which these two forces become equal and the net flowing currents becomes zero, is called equilibrium rest state. It is about -65 mV and may vary in different neurons. If the neuron is stimulated by an external current, or through other up-stream neurons, the potential of neurons increases to a positive value and after a short time falls back to the equilibrium rest potential (Izhikevich, 2007). This abrupt change in membrane voltage is called action potential. The Hodgkin & Huxley proposed circuit for squid giant axon is shown in figure 1, and equation (1) gives the relation between input current and the membrane voltage.

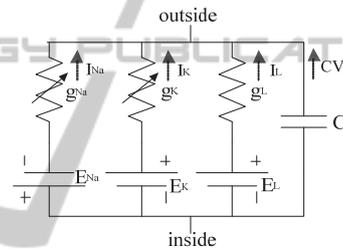


Figure 1: Hodgkin & Huxley proposed circuit for squid giant axon. g_K & g_{Na} are voltage-dependent conductance.

The complete set of H-H current's equations according to (Izhikevich, 2007) comes in (1) to (4).

$$C\dot{V} = I_{ext} - I_K - I_{Na} - I_L \quad (1)$$

Where, the potassium current, I_K is given by equation (2).

$$I_K = \bar{g}_K n^4 (V - E_K) \quad (2)$$

I_K is the potassium current with four activation gates which is shown by n^4 .

I_{Na} is the sodium current that is defined in equation (3). It has three activation gates (m^3) and one inactivation gate (h).

$$I_{Na} = \bar{g}_{Na} m^3 h (V - E_{Na}) \quad (3)$$

I_L is the ohmic leak current which is given in equation (4).

$$I_L = g_L(V - E_L) \quad (4)$$

According to (Hodgkin and Huxley, 1952), the values of activation and inactivation's parameters are updated by equation (5).

$$\dot{X} = \frac{X_\infty(V) - X}{\tau_X(V)} \quad (5)$$

Where $X \in \{m, n, h\}$ and $\tau_X(V)$ and $X_\infty(V)$ are obtained by (6).

$$\tau_X(V) = \frac{1}{\alpha_X(V) + \beta_X(V)} \quad (6)$$

$$X_\infty(V) = \frac{\alpha_X(V)}{\alpha_X(V) + \beta_X(V)}$$

α and β 's equations are shown in (7).

$$\alpha_n(V) = \frac{0.01(V + 55)}{1 - \exp[-0.1(V + 55)]}$$

$$\alpha_m(V) = \frac{0.1(V + 40)}{1 - \exp[-0.1(V + 40)]}$$

$$\alpha_h(V) = 0.07 \exp[-0.05(V + 65)] \quad (7)$$

$$\beta_n(V) = 0.125 \exp[-0.0125(V + 65)]$$

$$\beta_m(V) = 4.0 \exp[-0.0556(V + 65)]$$

$$\beta_h(V) = \frac{1}{1 + \exp[-0.1(V + 35)]}$$

According to (Izhikevich, 2007), typical values of maximal conductance and the membrane capacitance are shown in equation (8).

$$\bar{g}_K = 36 \text{mS/cm}^2, \bar{g}_{Na} = 120 \text{mS/cm}^2, \quad (8)$$

$$g_L = 0.3 \text{mS/cm}^2, C = 1 \mu\text{F/cm}^2$$

The equilibrium potentials are shown in equation (9).

$$E_L = -54.402 \text{mV}, E_K = -77 \text{mV}, E_{Na} = 50 \text{mV} \quad (9)$$

For digital implementation of these calculations, we need to convert these floating point values and computations to specific fixed point values and then perform these computations with specific accuracy in fixed point arithmetic. In the following section, the details of our fixed point design and digital implementation is expressed.

3 FPGA IMPLEMENTATION

According to the equations (1) - (7), the proposed architecture which is implemented in this paper is shown in figure 2. Each box corresponds to one or more components. For implementing these components, the required functionalities and modules are addition, subtraction, multiplication, division and exponential function. The main constraint which should be considered in hardware implementation is the parallelism and also optimal hardware resource usage. In other words, the implemented system should respond in real-time and also have comparable accuracy with software implementation. For this reason, addition and multiplication are implemented by FPGA's special circuits (DSP Cores). We also designed the division modules using shift registers and addition to save the area.

One of the most important and critical parts of our hardware implementation is exponential function. The accuracy, speed and also logic resource usage of this module has a direct impact on the final result. For this reason we used the hyperbolic CORDIC algorithm to estimate the exponential function more accurately. According to (Ercegovac and Lang, 2003), we used the following equations in our design and implementation (equation 10).

$$\begin{aligned} x[j+1] &= x[j] - \sigma_j 2^j y[j] \\ y[j+1] &= y[j] + \sigma_j 2^{-j} x[j] \\ z[j+1] &= z[j] - \sigma_j \tan^{-1}(2^{-j}) \end{aligned} \quad (10)$$

As shown in equation (10), shifter and adder are needed for hardware implementation. The number of bits used for intermediate value representation should be selected carefully to provide the required accuracy of the neuron. On the other hand we should find the minimum number of bits to minimize the FPGA resources. In other words, there is a trade-off between accuracy and FPGA resources. We determined the optimal representing bits for implementation via high level simulation in MATLAB and comparing the final results with low level HDL simulations.

The other important part of equation (10) is $\tan^{-1}(2^{-j})$ function. It is implemented by static ROMs or LUTs with limited rows. There exist more accurate implementations of this function using more hardware resources comparing the LUT

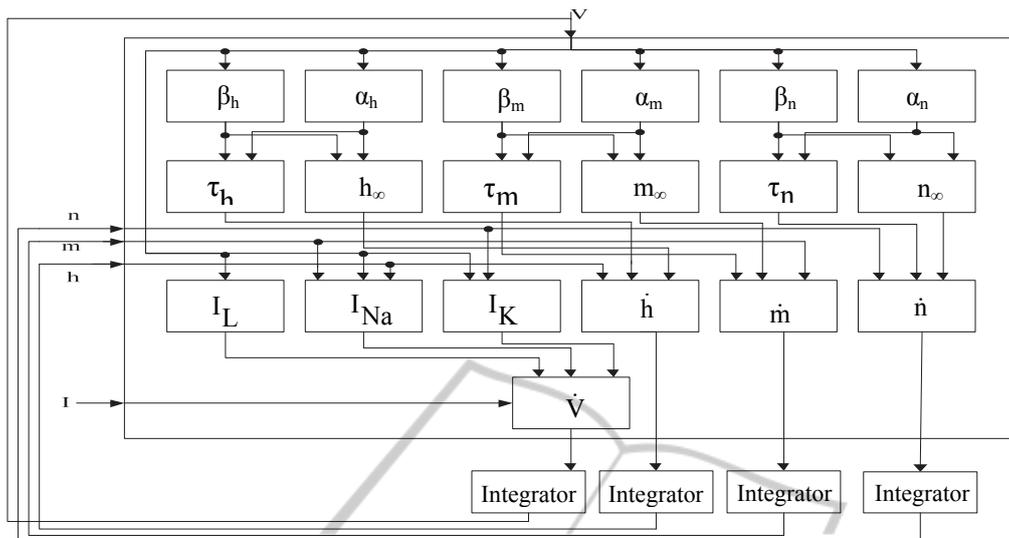


Figure 2: The proposed architecture for H-H neuron model implementation.

implementation. As we need limited accuracy, we used LUT-based implementation to minimize the required FPGA resources. The initial value of $z[j]$ is used as an argument to calculate the exponential function. Simulation results show that 32 bit words are enough to represent all intermediate values. As CORDIC algorithm is defined for the limited inputs, we separate the inputs into two parts. The most 12 significant bits are used as integer part, and the remaining 20 bits show the fractional part of the number. The higher 12 bits that presents the integer part are used as LUT's input, in which the exponential value of this integer part is saved. In other words, for integer numbers, the results of the computation with adequate accuracy is computed before, and saved in a LUT. In run-time a simple search is performed instead of calculating its value. As a result, the time complexity to compute this function is $O(1)$ and the fractional parts is given to CORDIC algorithm. According to equation (11) the output of the LUT and the output of the CORDIC algorithm should be multiplied to produce the exponential function results for 32-bit input.

$$\begin{aligned} \theta &= X + Y \\ \exp(\theta) &= \exp(X).\exp(Y) \end{aligned} \tag{11}$$

Based on equation (7) we use the same architecture with different constant values to calculate α_n and α_m (see figure 3). The selected constant's values used in figure 3 are shown in Table 1.

According to equation (7), α_h , β_n and β_m have similar architecture which is shown in figure 4. Table 2 shows the selective constant's values used in figure 4.

Table 1: Constant value for figure 3.

	C1	C2	C3
α_n	55	-0.1	0.01
α_m	40	-0.1	0.1

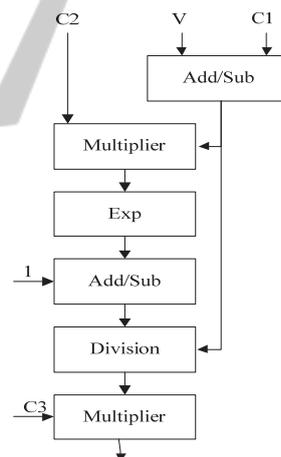


Figure 3: α_n and α_m architecture.

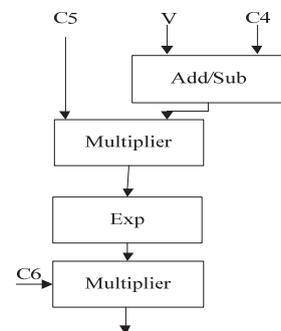


Figure 4: α_h , β_n and β_m architecture.

Table 2: Constant values for figure 4.

	C4	C5	C6
α_h	65	-0.05	0.07
β_n	65	-0.0125	0.125
β_m	65	-0.0556	4

We used a similar architecture to calculate β_h (See figure 5).

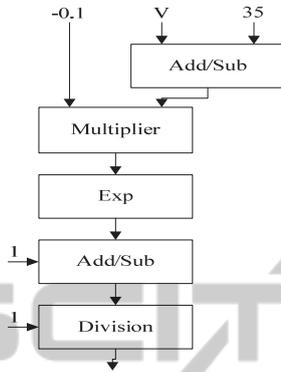


Figure 5: Proposed architecture to compute β_h .

First of all, the results of the equation (7) have to be computed, and the outputs should be produced. So α_x and β_x are ready to be used as inputs in equation (6). The high level implementation of equation (6) that produces τ_x is shown in figure 6. Inputs and output of this module are given in Table 3.

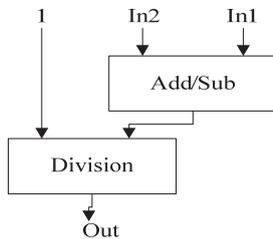


Figure 6: A typical implementation of τ_X .

Table 3: The output and input's values used in figure 6.

Out	In1	In2
τ_n	α_n	β_n
τ_m	α_m	β_m
τ_h	α_h	β_h

A similar architecture is used to implement X_∞ , but the constant value 1 in division's input is replaced by the first input (In1). The outputs of these modules which are the outputs of equation (6) applied as inputs to equation (5). Implementation of equation (5) is same as the architecture in figure 6,

but the first input (In1) is connected to X, and the second input (In2) is connected to X_∞ to make the subtraction's result and τ_x is connected to the division's input which is 1 in figure 6.

The high level circuit of I_K implementation is shown in figure 7.

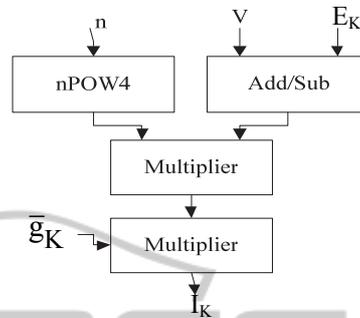


Figure 7: The architecture of I_K .

Another required component which is used to calculate I_K is n^4 . This function is implemented using three multipliers (shown as nPOW4 in figure 7).

A similar circuit is designed and implemented to compute I_{Na} . The mPOW3 (shown in figure 8) computes m^3 .

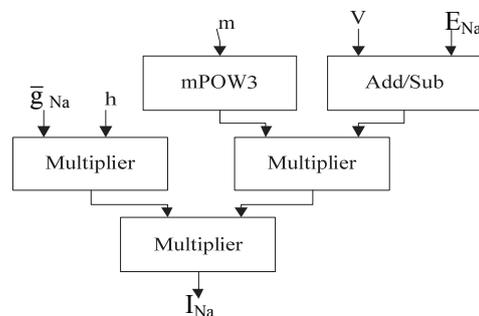


Figure 8: The architecture of I_{Na} .

The last required component is I_L . The architecture of this component is shown in figure 9.

Membrane voltage in equation (1) is calculated by the last module that is an adder with four inputs, called I_K , I_{Na} , I_L and I_{ext} . Input I_{ext} is an external current that stimulates the neuron.

The new values of the n, m, h and V are calculated using integrators with \dot{n} , \dot{m} , \dot{h} and \dot{V} as inputs, respectively. To implement the integrator, equation (12) is used by step time of 0.01 ms. This step time is selected based on our design and also

our evaluation board which is used for prototyping.

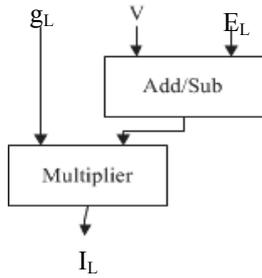


Figure 9: The architecture of I_L .

$$X(t + \Delta t) = X(t) + \Delta t * \dot{X}(t) \quad (12)$$

Equation (12) is implemented using a simple MAC (multiply and accumulate) and rounded to 32 bits. In the following section, the experimental setup, validation and evaluation of the proposed architecture is described.

4 RESULTS

To validate the FPGA implementation results, the implemented bit level simulation is compared with MATLAB simulation system. For high level simulation we use Simulink, and all hardware components are designed and implemented using VHDL modelling language. The main design objective of the low level implementation is the output accuracy. Due to the implementation constraints, we should use the minimum number of bits in all module implementations. The details of implementation are given in Tables 4 and 5. We tested the behavior of both systems for different values of parameters, initial values, and external currents. Here, the results for two important cases are shown; i) no stimulation is applied ($I_{ext} = 0$), and the neuron goes to the rest state after a transition time (Figure 10), ii) a strong stimulation ($I_{ext} = 30\text{mA}$) is applied, and thus the neuron exhibits periodic (tonic) spiking (Figure 11). In figure 10, and 11, the membrane voltage (V) of the neuron from MATLAB simulation (solid blue line), and from FPGA implementation (dashed red line) is shown. There are only very little differences between the two waveforms, which are due to the rounding error, due to the limitation of the number of bits to 32 in FPGA implementation. This error is in an acceptable range, and can be reduced, by extending this number representation system.

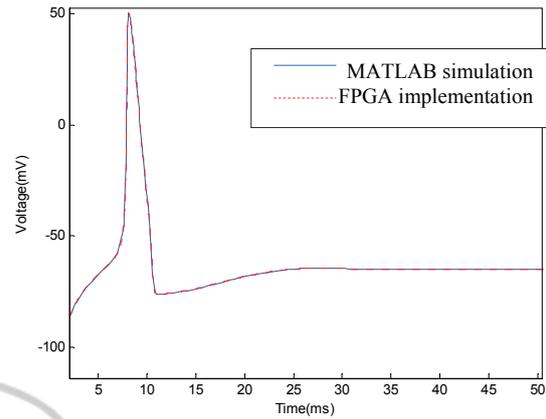


Figure 10: Membrane voltage of neuron for $I_{ext} = 0\text{mA}$.

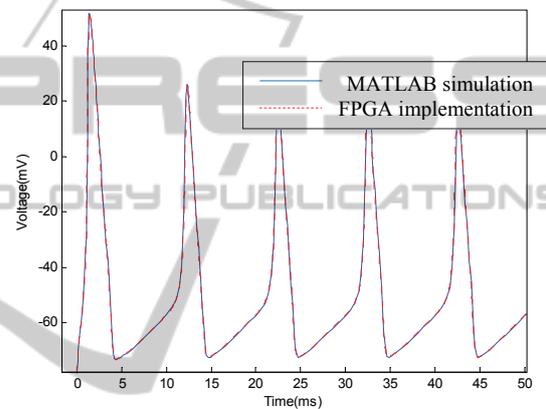


Figure 11: Membrane voltage of neuron for $I_{ext} = 30\text{mA}$.

We implemented the neuron on a Xilinx FPGA (Spartan3). The resource usages and critical path delay are reported in Table 4 and Table 5. According to the reported experimental results we are able to implement one neuron in a Spartan FPGA working with frequency of 37.563MHz. Also we can use larger devices to implement more neurons in one device.

Table 4: Device utilization summary.

Number of Slices	13273 out of 23872
Number of Slice FFs	7231 out of 47744
Number of 4 input LUTs	23514 out of 47744
Number of IOs	292
Number of bonded IOBs	292 out of 469
Number of GCLKs	24 out of 24
Number of DSP48s	99 out of 126

Table 5: Timing summary.

Minimum period	26.622ns
Minimum input arrival time before clock	10.982ns
Maximum output required time after clock	6.068ns

5 CONCLUSIONS AND FUTURE WORKS

In this paper the Hodgkin-Huxley model of a single biological neuron has been designed and implemented on an FPGA. Unlike previous approaches, we used the CORDIC algorithm for implementing the exponential functions and other arithmetic parts. So our used logic is more compact than previous ones. The accuracy and performance of our proposed approach is validated by MATLAB high level implementation. Because of establishing trade-off between used area and frequency, the number (and also format) of representing bits of our arithmetic parts were selected carefully and validated and verified by high level simulation. For instance, it was shown that the neuron spiking frequencies in MATLAB simulation and in FPGA implementation almost are the same. It is a very important parameter because it codes the information that a neuron transmits. The hierarchical proposed design and implementation allows simple modification of it to an equivalent small pipeline system, which is useful in implementing a large neural network. We plan to optimize our hardware to make it smaller and finding the optimal bit length of each parameter separately. Moreover, the behaviour of the implemented neuron will be benchmarked against the behaviour of a natural one. Furthermore, implementing a neural network of competing minicolumns (Bakhtiari. et al., 2012) in FPGA is the next target of this research.

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