X-FEM based Topological Optimization Method

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Keywords: Topology Optimization, Isoline, X-FEM, Eso.

Abstract: This study presents a new algorithm for structural topological optimization by combining the Extended Finite Element Method (X-FEM) with an evolutionary optimization algorithm. Taking advantage of an isoline design approach for boundary representation in a fixed grid domain, X-FEM can be implemented to obtain more accurate results on the boundary during the optimization process. This approach can produce topologies with clear and smooth boundaries without using a remeshing or a moving mesh algorithm. Also, reanalysing the converged solutions in NASTRAN confirms the high accuracy of the proposed method.

1 INTRODUCTION

In recent years, structural optimization has become a rapidly growing field of research with application in many areas such as mechanical, civil and automotive engineering. Topology optimization is one of the most challenging aspects of structural optimization, in which one needs to find the best topology as well as shape of a design domain. The approaches which have been proposed for the topology optimization of continuous structures fall into two categories: first, mathematical based methods such as homogenization (Bendsøe and Kikuchi, 1988), Solid Isotropic Material with Penalization (SIMP) (Bendsøe, 1989); (Zhou and Rozvany, 1991) and level set method (Wang et al., 2003); (Allaire et al., 2004); second, heuristic methods which are more intuitive and less mathematical, such as evolutionary structural optimization ESO/BESO methods (Xie and Steven 1993); (Querin et al., 1998); (Yang et al., 1999).

ESO is based on the assumption that the optimal layout of the design domain can be obtained by gradually removing inefficient material from the design domain (Huang and Xie, 2009). In the original ESO method, the elements of the design space are ranked in terms of their sensitivity, and those with lower sensitivity are removed from the design domain until a desired optimum is obtained. Bi-directional evolutionary structural optimization (BESO) is an extension of ESO in which the elements are allowed to be added and removed simultaneously. These heuristic methods are easy to program and provide a clear topology (no grey regions of intermediate densities as in SIMP) in the resulting optimal designs. Conventional ESO/BESO algorithms have been successful since they can be easily combined with the finite element model of a structure. However they suffer from a week capability of boundary representation. In these methods the geometrical information of the boundaries is not clear during the optimization process and the boundaries of the optimal solution are represented by the jagged edges of the finite elements. This limitation causes difficulties in combining these methods with CAD and the obtained solutions require post processing to manufacture a smooth design.

The fixed grid finite element method (FG-FEM) allows the boundaries of the design to cross over finite elements. This capability has been used in boundary based optimization methods such as the level set method, and element based optimization methods such as fixed grid evolutionary structural optimization (FG-ESO) method. FG-ESO or Isoline/ Isosurface approach (Victoria et al, 2009; Victoria et al, 2010) is an alternative to ESO in which the inefficient material is allowed to be removed/added within the elements of the design domain during an evolutionary process. The boundaries are defined by the intersection of Iso-line plane with the criteria distribution of the design domain. Since in this approach the boundary of the design is no longer consistent with the fixed finite elements as in ESO, a classical finite element analysis may result in poor FE approximation on the boundary. Conventionally

Abdi M., Ashcroft I. and Wildman R..
X-FEM based Topological Optimization Method.
DOI: 10.5220/0004148404660471
In *Proceedings of the 2nd International Conference on Simulation and Modeling Methodologies, Technologies and Applications* (SDDOM-2012), pages 466-471
ISBN: 978-989-8565-20-4

in the fixed grid finite element approach, the element stiffness is assumed proportional to the area fraction of the solid material within the element (also called the density scheme). Although this approach is widely accepted and implemented in many works (Allaire et al., 2004); (Victoria et al., 2009), studies have shown that it cannot provide accurate results for the boundary elements (Dunning et al., 2008); (Wei et al., 2010).The Extended finite element method (X-FEM) is another fixed grid approach which can be used to model void/solid interfaces. X-FEM extends the classical finite element approach by adding special shape functions which can represent a discontinuity inside finite elements. In this approach, the geometry of the discontinuity is often described by a level set method. Combination of the level set description of the geometry and the fixed mesh framework of X-FEM has been used in recent level set based topology optimization work (Wei et al., 2010); (Miegroet et al., 2007).

This study presents a simple and effective evolutionary optimization approach in a fixed grid domain. The novelty of this work is to apply X-FEM to the evolutionary optimisation algorithm. The proposed method doesn't require a level set framework for geometry description in the X-FEM and the boundaries of the design can be simply represented by isolines of a desired structural performance. The algorithm is implemented in the topology optimization of two test cases and the final solutions are reanalysed with NASTRAN to evaluate the accuracy of the proposed method.

2 METHODOLOGY

2.1 Structural Optimization Problem

The topology optimisation problem where the objective is to minimize the strain energy can be written as:

$$\text{Minimize: } c = \frac{1}{2}U^T K U \quad (1)$$

$$\text{Subject to: } \frac{\sum_{e=1}^{N} v_{e,s}}{V_0} = V^* \quad (2)$$

where c is the total strain energy, and U and K are the global displacement and global stiffness matrices, respectively. N denotes the number of finite elements in the design domain, $v_{e,s}$ the volume of the solid part of the element, V_0 the design domain volume and V^* the prescribed volume fraction. While in ESO/BESO methods, the presence/absence of each element in the design domain is considered as a design variable, in our proposed method the distribution of material inside each element is considered as a design variable. In our study, we have used strain energy density (SED) as the criterion for finding the efficient material distribution within the design domain. Therefore, the solid material will be gradually removed from the low SED regions and added to the high SED regions during the evolutionary procedure. The effective removal of material can be achieved by assigning a weak material property to low SED regions (soft-kill method). The strain energy density of the elements can be calculated from

$$SED_e = \frac{1}{2} u_e^T k_e u_e / v_e \quad (3)$$

with u_e the element displacement vector and k_e the element stiffness matrix which is calculated using an XFEM scheme.

2.2 Isoline Topology Optimization

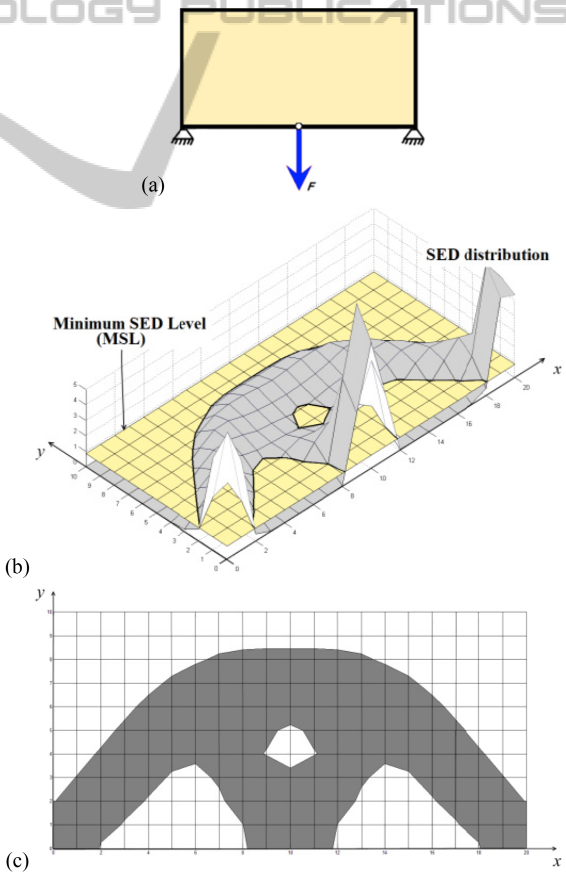

Figure 1: a- Initial design domain with boundary conditions. b- Structural boundary represented by intersection of criteria (SED) distribution and MSL. c- Final solution shown in a fixed grid domain.

The basic idea of isoline design is to represent the shape and topology of the structure using the contours of desired structural behaviour. This idea has been suggested in several studies (Maute and Ramm, 1995); (Lee et al., 2007). The isoline optimization algorithm that we use in this paper is originated from the isoline topology design (ITD) algorithm proposed by Victoria et al., 2009.

The ITD approach can be summarized into the following steps:

1- An extended finite element analysis is performed to find the distribution of strain energy density within the design domain.

2- A minimum SED level (MSL) is determined and the new structural boundary is obtained from intersection of SED distribution and MSL.

3- The regions of the domain having the criteria level less than MSL are not included in the design domain. Therefore their material property is set to the weak material. The regions where the criteria level is more than MSL are inside the design domain and their material property is set to the solid material.

4- Steps 1-3 are repeated by gradually increasing the MSL until a desired optimum is obtained.

2.2.1 Integration Scheme

In a conventional fixed grid approach, the stiffness matrices of the boundary elements are approximated by a density scheme in which the stiffness of an element is proportional to the area ratio of the solid part of the element. The material is considered to be uniformly distributed through the whole element and the variations in material distribution in an element are not taken into account in calculating the element stiffness matrix. For example, figure 2 shows three different shapes for a boundary element where the area fraction of solid material within the element is 0.50. Using density method the same stiffness is calculated for all three elements. This issue may cause errors near the boundary of the design during the optimization process.

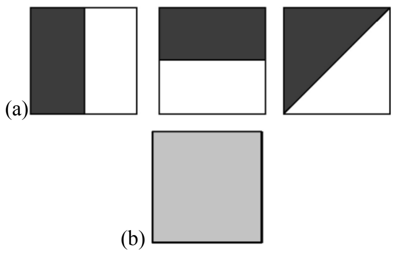

Figure 2: a- Typical boundary elements for area ratio=0.50. b- Their density scheme equivalent solid element with 50% density.

The extended finite element method (X-FEM) is an alternative fixed grid approach proposed by Moës et al in 1999. It was originally developed to represent crack growth in a fixed grid domain without meshing the internal boundaries. X-FEM has also been implemented for other kinds of discontinuities such as fluid structure interaction (Gerstenberger and Wall 2008) and modelling holes and inclusions (Sukumar et al 2001). In our case, the X-FEM scheme for modelling holes and inclusions can be implemented for modelling the boundary of the design (weak/solid material interfaces) during the optimization process. In this approach, the displacement field is approximated by the following equation:

$$u(x) = \sum_i N_i(x)\, H(x) u_i \tag{4}$$

where N_i are the classical shape functions associated to degree of freedom u_i, and the Heaviside function *H(x)* has the following properties:

$$H(x) = \begin{cases} 1 \ \ if\ x\ \in \Omega_S \\ 0 \ \ if\ x\ \notin \Omega_S \end{cases} \tag{5}$$

where Ω_S is the solid sub-domain. Since there is no enrichment in the displacement approximation equation of X-FEM in modelling holes and inclusions, there will be no augmented degrees of freedom during optimization. Equation 5 defines a zero displacement field for the void part of the element, which means that only the solid part of the element contributes to the element stiffness matrix. Thus we can use the same displacement function as FEM and simply remove the integral in the void sub-domain of the element.

$$K_e = \int_{\Omega_S} B^T D_S B t d\Omega \tag{6}$$

with B the displacement differentiation matrix, D_S the elasticity matrix for the solid material and t the thickness of the element. When an element is cut by the boundary, the remaining solid sub-domain is no longer the reference rectangular element. So we partition the solid part of the boundary element into several sub-triangles (figure 3) and use Gauss quadrature to calculate the integral given by equation 6.

2.2.2 Combining X-FEM and the Optimization Algorithm

Figure 5 illustrates the topology optimization procedure used which in general consists of initialization, X-FEM structural analysis, and isoline update scheme. In initialization, the initial material distribution within the design domain and the descretization of the design domain, as well as the necessary parameters for the isoline topology design are defined.

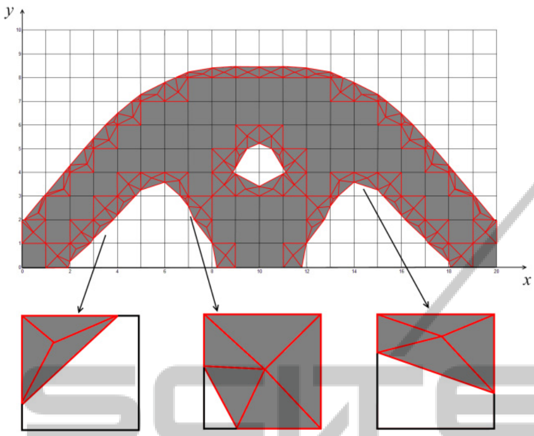

Figure 3: The solid sub-domain of the boundary elements are partitioned into several sub-triangles.

In our study, the second order Gauss rule with 3 midline Gauss points was implemented (figure 4).

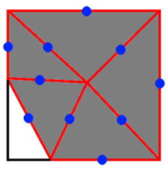

Figure 4: X-FEM integration scheme.

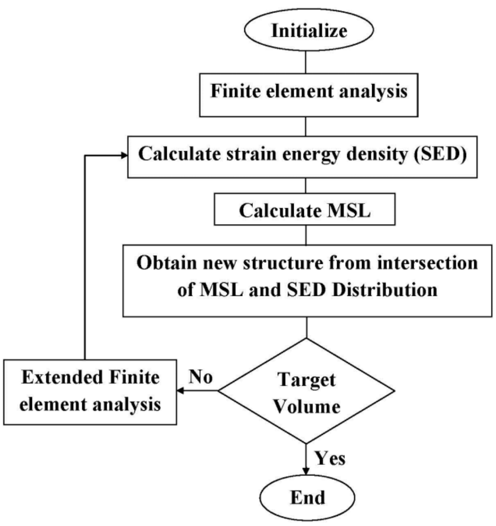

Figure 5: Flowchart of optimization algorithm.

In the X-FEM structural analysis, by using nodal criteria numbers, the elements are categorized into three groups: solid, void and boundary elements. Solid and void elements are treated using classical finite element approximation. The stiffness matrix of the boundary elements are calculated by partitioning the solid sob-domain into several sub-triangles and applying the Gauss quadrature integration scheme described in the previous section.

The minimum SED level (MSL) is calculated by increasing the value from the last iteration. The new structure is obtained from the intersection of the MSL and current criteria distribution. The process is continued until the target volume is achieved.

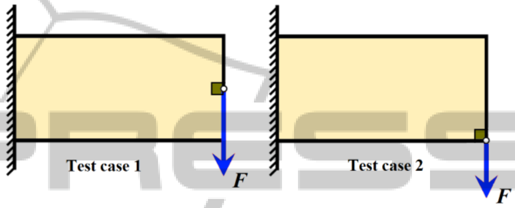

Figure 6: The two test cases.

3 TEST CASES

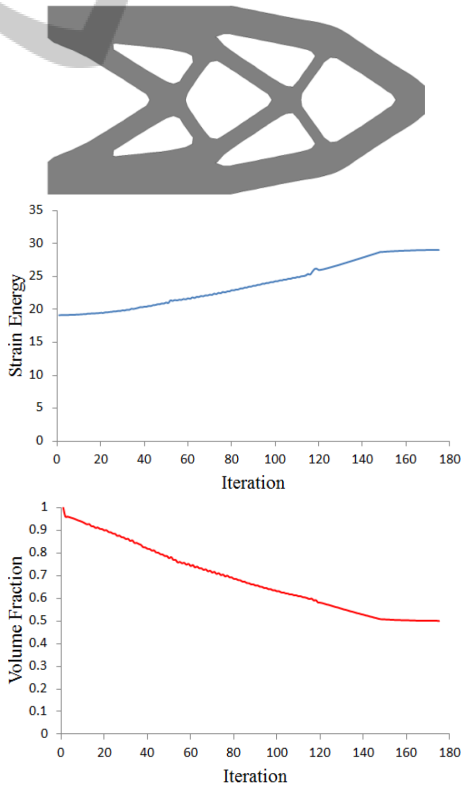

Figure 7: Optimized final design and iteration histories of objective function and volume fraction for test case 1.

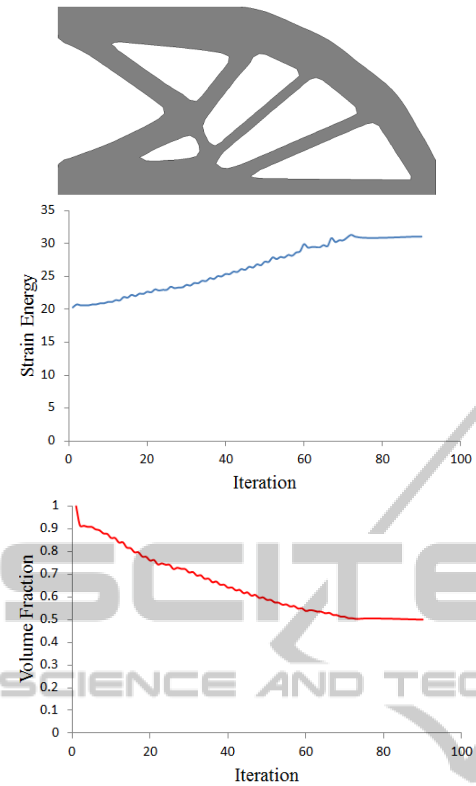

Figure 8: Optimized final design and iteration histories of objective function and volume fraction for test case 2.

The proposed method of combining X-FEM and evolutionary optimization algorithm was implemented in a MATLAB code to present the topology optimization of 2D rectangular domains. Two test cases are used in this study (figure 6). First a short cantilever beam having length 60, height 30 and thickness 1 where a unit concentrated load is applied in the middle of the free end. The second test case was a cantilever beam having the same dimensions as test case 1 but with the load applied at the bottom of the free end. A 60x30 mesh was used for both cases to discretize the design domain. To avoid singularity issues with the concentrated loading, the loading region was treated as a non-design domain.

The optimized final design, as well as the iteration histories of the objective function and volume fraction for the test cases 1and 2, are shown in figures 7 and 8, respectively. It can be seen that the strain energy increases, as material is gradually removed from the design domain, then reaches a constant value at convergence.

3.1 A Methodology for Evaluating X-FEM Solutions

To evaluate the performance of the final solutions and the accuracy of the proposed method, the obtained solutions were discretized by a very fine structured mesh and imported to NASTRAN to perform a classical finite element analysis (figure 9).

Table 1 compares the X-FEM solutions and the regenerated NASTRAN structures in terms of their strain energies and tip displacements. It can be seen that the X-FEM solutions are very close to the regenerated NASTRAN solutions. The slight difference in the X-FEM and NASTRAN results may be attributed to the different mesh size used in the two approaches.

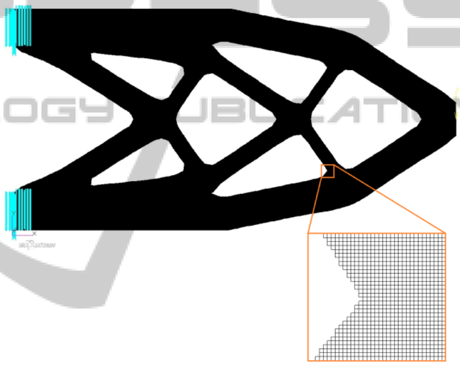

Figure 9: XFEM solution discritized by a very fine mesh and imported to NASTRAN.

Table 1: Comparison of X-FEM solutions and regenerated NASTRAN structures.

Test case 1	Strain Energy	Tip Displacement
X-FEM	29.04	54.00
NASTRAN	29.21	54.39

Test case 2	Strain Energy	Tip Displacement
X-FEM	31.08	57.54
NASTRAN	31.24	57.85

4 CONCLUSIONS

In this study, X-FEM and Isoline design are implemented for the topology optimization of 2D continuum structures. By applying the proposed X-FEM scheme there is no need to use the time consuming remeshing and moving mesh approaches to improve the FE solution. The generated structures have smooth boundaries which need no further interpretation and post-processing. The numerical examples presented in this paper show the accuracy and efficiency of the proposed algorithm.

ACKNOWLEDGEMENTS

The authors are grateful for the funding provided by Loughborough University.

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