Modelling How Students Organize Knowledge

Ismo T. Koponen
Department of Physics, University of Helsinki, Helsinki, Finland

Keywords: Knowledge Organization Patterns, Concept Maps, Directed Networks, Learning.

Abstract: We discuss here how students organize their knowledge (in physics) by connecting closely related concepts. Attention is paid on the relational structure of the ordering of concepts so that the introduction of new concepts is justified on the basis of concepts which have already been learned. Consequently, there is then direction of progress in introducing new concepts - there is ”flux of information” so that what was learned before is the basis for learning new conceptual knowledge. Such ordered and directed process of introducing the concepts can be conveniently described and analysed in the framework of directed ordered graphs. We propose here a model of knowledge organization for such concept maps. The model is based on the assumption that students use simple procedures connecting new concepts mostly to concepts introduced few steps before. On basis of the model results we suggest that the most important properties of concept maps can be understood on a basis of such simple rules for organising knowledge.

1 INTRODUCTION

Scientific knowledge is quite often described as webs or networks, where concepts are linked to other concepts and principles; concepts are thus conceived as parts of the whole system of knowledge. In such a picture of knowledge, it is evident that the existing structure and relations within it also affect how the new concepts can be introduced (as parts of the networks) and how the conceptual knowledge can be represented and transferred forward (Thagard, 1992; Novak, 2002; O'Donnell et al., 2002). Consequently, knowledge processing and acquisition have been recently discussed within the framework of network theory (Costa, 2006; Batista and Costa, 2010; Goni et al., 2010). Such an approach seems to be well adapted also in describing knowledge processing in learning, in particular how students organize their conceptual knowledge and, on the other hand, how conceptual knowledge can be approached in learning and teaching (Koponen and Pehkonen, 2010; Koponen and Nousiainen, 2012).

In teaching and learning the network of concepts is often represented by using concept maps (McClure et al., 1999; Novak, 2002; O'Donnell et al., 2002; Koponen and Pehkonen, 2010). Here we focus on the question, can we possibly understand the local and global structure of concept maps by making assumptions about the rules or strategies students may have used in construction of the maps. The empirical sample discussed here consists of concept maps made by physics teacher students for purposes of organising content knowledge for teaching. The linking of concepts is done by paying attention on how concepts are used in quantitative experiments and construction of models. As we have shown previously (Koponen and Nousiainen, 2012), such students concept maps representing their knowledge of (or about) physics concepts can be analysed conveniently and reliably within the theoretical framework based on directed ordered graphs (Karrer and Newman, 2009; Goni et al., 2010). Here we develop a phenomenological model for knowledge ordering based on directed ordered graphs (DOGs). We show that the typical structural features of students' concept maps can be modelled by assuming that concepts are mostly related to nearby concepts one or two concepts away, with few links between concepts about 10-15 steps away. When concepts are introduced in this way in ordered succession, substantial number of triangular patterns (as found in the real networks) are quite naturally generated. Comparison of the model results with the real student concept maps shows that the model reproduces the salient properties of connectedness and ordering found on students' concept maps. Consequently, the results suggest that students indeed use simple but effective strategies in ordering and processing their knowledge.

Koponen I..
Modelling How Students Organize Knowledge.
DOI: 10.5220/0004105201430148
In *Proceedings of the International Conference on Knowledge Engineering and Ontology Development* (KEOD-2012), pages 143-148
ISBN: 978-989-8565-30-3

2 CONCEPT NETWORKS

The relational structure of concepts can be thought as network-like node-link-node representations of the relations between concepts. For different types of representations there are different ways of establishing the relations and different rules in regard to linking the concepts, but the skeletal structure of the concept network is always a network of nodes (i.e. concepts) connected by links. The concept networks studied here are done by physics teacher students for purposes of representing how they think concepts can be introduced in teaching, so that each step is justified either on basis of experiments or model, which are both central procedures connected to the construction and use of knowledge. It is then natural to assume that these procedures of the experiments and modelling play an important role in conferring the structure of the concept networks. In the operationalizing experiment the concept is operationalized i.e. made measurable through the pre-existing concepts. The new concept or law is constructed sequentially, starting from the already existing ones, which provide the basis for an experiment's design and interpretation. In its most idealized form the new concept (or law) C is formed on the basis of two pre-existing concepts A and B so that the operationalization creates C on the basis of the relations $A \rightarrow C$ and $B \rightarrow C$, but which also requires that A and B can be related as $A \rightarrow B$. There is then a triangular mutual dependence $A \rightarrow B \rightarrow C \leftarrow A$.The modelling procedures, which in the simplest cases are often deductive procedures, produce very similar patterns (Koponen and Pehkonen, 2010; Koponen and Nousiainen, 2012). It is interesting to note cognitively oriented studies of knowledge formation suggest that procedures of knowledge construction and processing may be simple ones, reducible to basic patterns, even in those cases where the resulting structures are complex. In that,triangular patterns have been recognized as an essential feature not only in the case of functional knowledge but also in information acquisition as well as information processing (Kemp et al., 2007; Kemp and Tenenbaum, 2008; Duong et al., 2009).

The procedures of constructing experiments and models – connecting concepts to previously introduced concepts – then provide the context or the "affiliations" of concepts. Concept maps where these procedures are used to connect concepts represent then not only the relatedness of concepts, but they also represent how concepts are introduced in teaching so that knowledge learned earlier is the basis upon which new knowledge is built. This means that, in a sense, these networks also represent the "flux of information" which students have planned to take place in their teaching. In well-planned teaching there should naturally be a regular flux of information (for evenly paced learning of new knowledge), but no unnecessary abrupt changes in that flux (otherwise the demand to assimilate new knowledge would vary much); moreover, uncontrollable reductions in the flow should be hindered to prevent the impression that learned knowledge would not be needed in further learning (Koponen and Nousiainen, 2012).

3 THE METHOD OF ANALYSIS

The properties of ordering and information flux in the concept maps are explored by using the quantities based on theory of directed ordered networks (DOGs) (Karrer and Newman, 2009; Goni et al., 2010). Because we are interested in the connectedness and information fluxes in the maps, we use the following quantities (detailed mathematical definitions are given in Table 1):

1. The degree k_i of the node, which is the number of the incoming and outgoing links k_{in} and k_{out}, respectively. The average degree is denoted by D;
2. The clustering coefficient C_i, which is the ratio of triangles to all the triply connected neighbours around a given concept;
3. Flux into the nodes (Flux-I) Φ_i, which gives the total number of links terminating at the given node k from all levels $j < k$;
4. Flux around the nodes (Flux-A) Ψ_k, which gives the total number of links bypassing the given node k from all levels $j < k$.

In the present case, fluxes Φ and Ψ directly describe the "information" flowing from the previously introduced nodes to ones introduced later (Karrer and Newman, 2009). The most important aspect of the concept maps made by teacher students is their ordering and appreciably large clustering with $C \approx 0.2$. Both features follow from the procedures that are used to connect concepts. It is of interest to develop a simple model, which captures these features. In addition to these features the model should also reproduce the steady node-by-node information flows Ψ and Φ.

4 THE MODEL

The cases studied and modelled here consist of 8 student maps, all of which are rather rich in their structure. The number of the concepts was limited to n=34 most central concepts (in electromagnetism). Details

Table 1: Definitions of the quantities characterizing the topology of the concept networks. In the definitions a_{ij} is the element of the adjacency matrix **a** and N is the number of nodes. The quantities are defined for a given node i. The average number of links k_i per node is denoted by D.

Quantity	Definition
k_i^{in}	$\sum_j a_{ji}$
k_i^{out}	$\sum_j a_{ij}$
k_i	$\sum_i k_i^{in} + k_i^{out}$
C_i	$\sum_{j'>j} \tilde{a}_{ij}\tilde{a}_{jj'}\tilde{a}_{ij'} / \sum_{j'>j} \tilde{a}_{ij}\tilde{a}_{ij'}$
Φ_i	$\left(\sum_{j=1}^{i-1} k_j^{out} - \sum_{j=1}^{i-1} k_j^{in}\right)/D$
Ψ_i	$\left(\sum_{j=1}^{i-1} k_{out}^{j} - \sum_{j=1}^{i} k_j^{in}\right)/D$

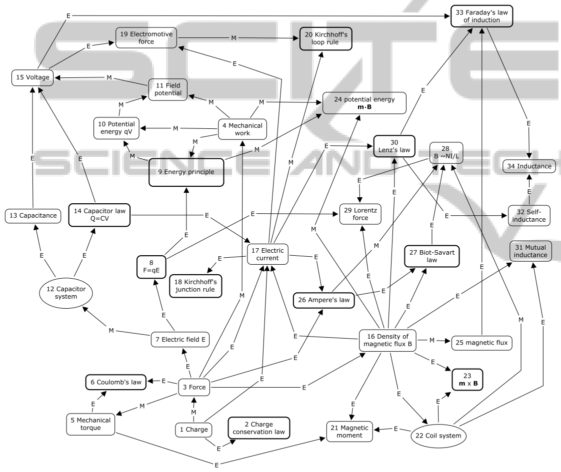

Figure 1: An example of students' concept maps (Gs) for n = 34 concepts in electromagnetism. The map shows concepts (boxes), laws and principles (boxes with thick borders). Links are either operationalising experiments (E) or modelling procedures. The nodes are numbered in the order in which they are introduced through experiments.

of the maps are not of interest here and are discussed in detail elsewhere (Koponen and Nousiainen, 2012). An example of the original concept maps made by students is shown in Fig.1

For purposes of comparison and reference, we have also constructed a "master map", where all well-motivated and well-justified connections that are found in the student maps are collated into one map. For purposes of analysis it is convenient to use for all maps the so called "spring-embedding" (Kolaczyk, 2009), which brings about the most important nodes as clusters of links. Examples of spring-embedded maps (one student map and the master map) are shown in Fig. 2.

Basic assumptions we have made about how the learners process and represent knowledge are as follows:

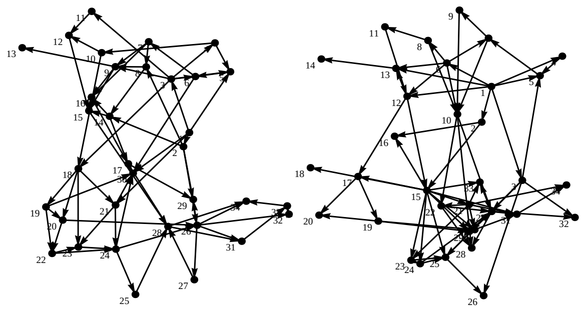

Figure 2: The "master map" (Gm) and one students' map (G1) in spring-embedded form.

1) New concepts are introduced on basis of old ones so that relatively few (from two to four) of them are used as basis of introducing the new ones; 2) The procedures provide the context or the "affiliation" for making the connections. 3) Concepts are recognized on basis of the phenomenological meaning. This gives rise to the modularity.

On the basis of above notions 1-3 we suggest a model, where nodes $1,2,\ldots i-1,i,i+1,\ldots n$ are introduced sequentially so that each node $i+1$ is connected in directed way to some of the preceding nodes $1,2,\ldots i$. The directionality is defined from ancestor nodes to new nodes. The probability distribution function (PDF) that $i+1$ connects to the given ancestor i' which is j steps away from it i.e. to node $i' = i+1-j$ is assumed to follow a gamma-distribution

$$f_{i,j}(\alpha,\lambda) = \frac{1}{Z_i(\alpha,\lambda)}\, j^{\alpha-1} \exp\left[-\lambda j\right], \qquad (1)$$

where parameters α and λ control the form of the distribution. The normalization $Z_i(\alpha,\lambda)$ is obtained in closed form in terms of the Lerch' transcendental Phi-function $\phi_{\kappa,i}(z) \equiv \phi(z,\kappa,i)$

$$Z_i(\alpha,\lambda) = \phi_{1-\alpha,1}(e^{-\lambda}) - e^{-\lambda(\alpha-1)}\phi_{1-\alpha,1+i}(e^{-\lambda}) \quad (2)$$

In practice, the detailed functional form of the distribution is not crucial, given it is peaked. Gamma-distribution is chosen because it is flexible and the cumulative distribution function (CDF) for PDF in Eq. (1) can be given in form

$$F(i,j) = \sum_{j'=1}^{j} f_{i,j'} = \frac{Z_i(\alpha,\lambda)}{Z_j(\alpha,\lambda)} \qquad (3)$$

The nodes are connected on basis of CDF in Eq. (3) by using event based Monte-Carlo method. First, the number N of connection attempts is selected, then for each attempt a random number $r \in [0,1]$ is generated and the new node is connected to ancestor node at a distance j^*, obtained from $r = F(i,j^*)$ by inversion. In practice, values of j^* corresponding different r are tabulated in advance for each i, so that the

Table 2: Simulation parameters for models m-G1, m-G2 and m-G3. Subscripts A an B refer to values for modules A and B separately, subscript AB to values between modules A and B.

	α_A	λ_A	α_B	λ_B	α_{AB}	λ_{AB}
m-G1	2	2/3	2	2/3	2	1/3
m-G2	5/2	2/5	5/2	2/5	7	1/3
m-G3	2	2/5	2	2/5	5	1/3

repeated inversion of $r = F(i, j^*)$ during the simulation is avoided. When connection attempt is repeated N times same attempts to connect already connected nodes may occur, in which case no multiple connections are allowed.

In the simulation model we have two modules A and B (corresponding two modules in maps, for two topical areas) and within the both modules we use the same above explained method to connect the nodes, but the values of the parameters α, λ, and N can be different for modules A and B. Connection between the nodes in different modules is also made on same basis. The parameters for models are given in Table 2. The networks based on these simulations are visually very similar to students' networks. Examples to be compared with student networks are shown in Fig. 3.

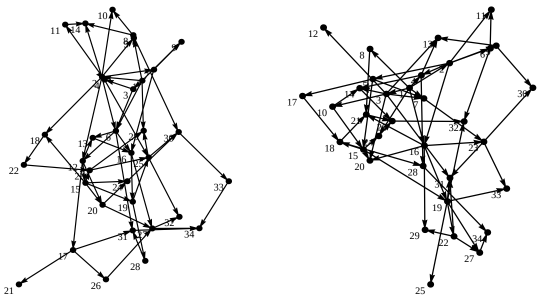

Figure 3: The model graphs m-G1 and m-G2 in spring-embedded form.

The analysis of the networks is carried out on basis of their adjacency matrices **a**, where the variables a_{ij} indicate the connections between nodes i and j so that if nodes are connected, then $a_{ij} = 1$ and if there is no connection, then $a_{ij} = 0$. All quantities of interest can now be calculated from the matrix **a** as they are defined in Table 1.

5 RESULTS

The student maps have relatively high clustering and connectedness (Koponen and Nousiainen, 2012). On average the clustering attains values around 0.10-0.25, which is common to networks designed for purposes of passage of information (Kolaczyk, 2009). Large values of clustering indicate that there are appreciable connections also between concepts connected to a given concept, i.e. an abundance of the nearest neighbour connections. High connectivity (on average 3-4 links per node) means that the information fluxes are also rather large. The fluxes are given as a total flux per expected number of links (total flux divided by average value D of links per node, see Table 1 for definition. The results reveal that typically, per one link connected to a given node, there are from three to four links coming from the lower levels. This means that each node is rather well supported by the many previous nodes - the meaning content of the concept (node) is supported or backed up by knowledge contained on the network existing before the introduction of the new node. In general, just these properties must be reproduced by model which attempts to capture the essential features of the maps.

Running the simulation for different choices of parameters shows that there is a range of parameters, where it is possible to obtain networks very similar to the empirical ones. It should be noted that it is not of interest (or even possible) to try to optimize parameters so that for a given empirical network exactly similar network is found in simulations. Instead, Simulations are used to explore the ensemble of possible networks and how the measurable properties of the networks are distributed within these ensembles and how the values of degree D, clustering C, and fluxes Φ and Ψ compare with the empirical observations, as is shown by results in Figs. 4 (empirical results) and 5 (model results). The average values of the clustering and fluxes are given in Table 3 for some of the students' maps and model maps. For comparison, some other values of students' maps are given in Table 4. From the results it seen that the model reproduces the most important qualitative features of the maps and, in addition, quantitative agreement is satisfactory taken into account the variation of measured values within the student maps (see Table 4).

6 CONCLUSIONS

The relational structure of concepts in the concept networks made by physics teacher students have revealed that properties of such networks are dominated by triangular knowledge organisation patterns so that the structure has thus relatively high clustering of nodes but yet performs well in passing the information i.e. information fluxes provided the connections are high (Koponen and Nousiainen, 2012). It is assumed that these properties can be traced back to

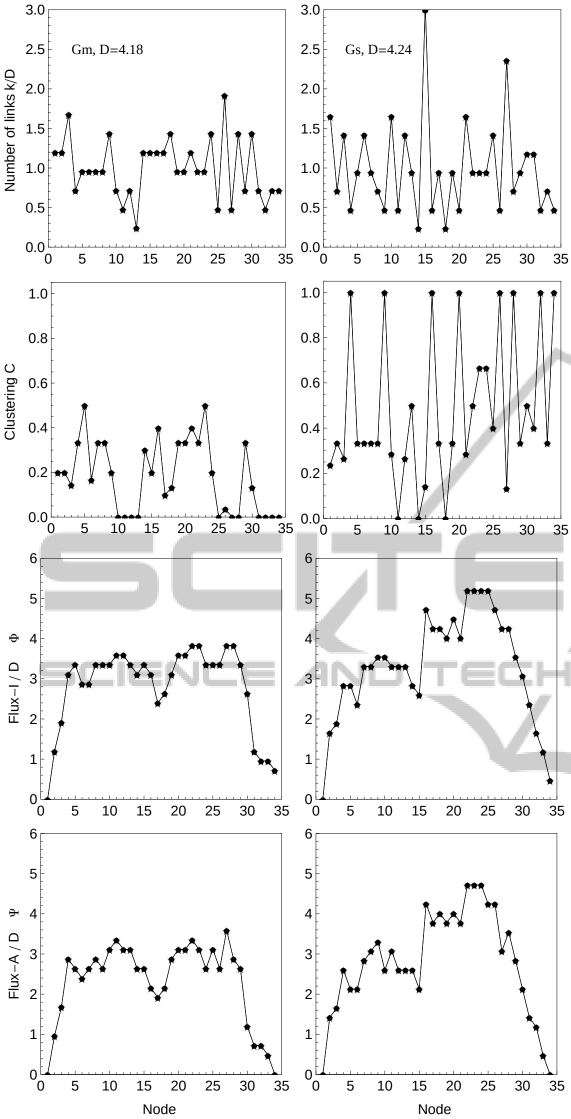

Figure 4: Node-by-node (nodes 1-34) values of degree *D*, clustering *C*, and fluxes Φ and Ψ. The first column shows the values for master map Gm and the second column for the student map G1.

Table 3: Average degree *D*, clustering *C* and information fluxes Ψ and Φ for master map Gm and student maps G1 and G2 and for models maps m-G1 - m-G3.

	Gm	G1	G2	m-G1	m-G2	m-G3
D	4.18	3.47	4.24	3.71	4.47	3.58
C	0.18	0.25	0.48	0.19	0.33	0.24
Φ	2.82	3.00	3.31	2.73	3.20	3.09
Ψ	2.32	2.51	2.81	3.06	2.81	3.01

students' systematic use of simple procedures (here quantitative experiments and modelling) which they use to add new concepts on the existing concept network.

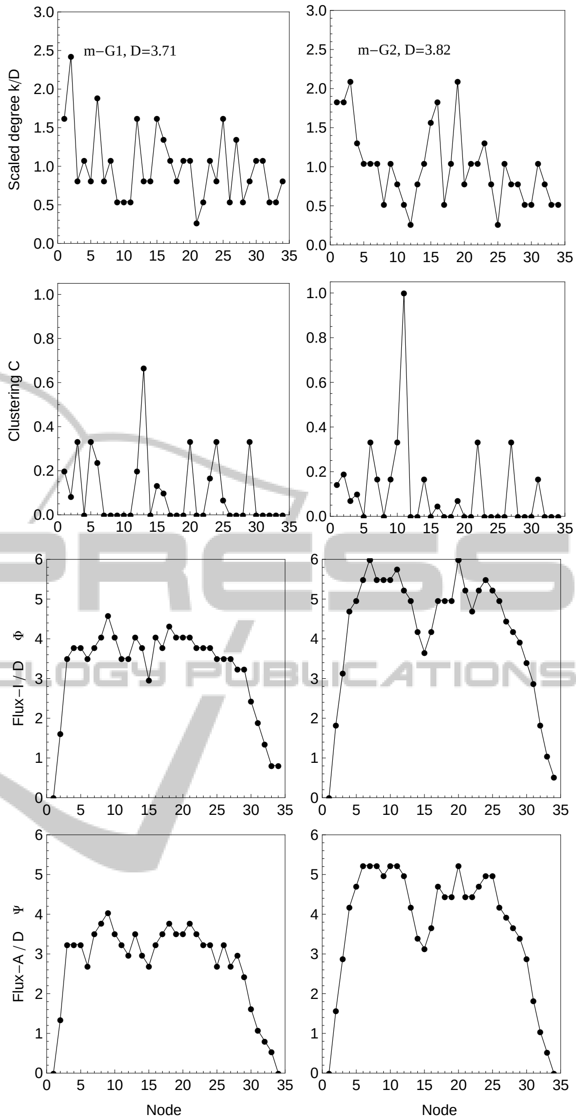

Figure 5: Node-by-node (nodes 1-34) values of degree *D*, clustering *C*, and fluxes Φ and Ψ. The first column shows the values for model graph m-G1 and the second column for the graph m-G2.

Table 4: Average degree *D*, clustering *C* and information fluxes Ψ and Φ for six student maps G3-G8.

	G3	G4	G5	G6	G7	G8
D	2.85	3.06	3.18	3.18	3.72	4.07
C	0.24	0.15	0.25	0.16	0.21	0.17
Φ	1.84	2.76	2.97	3.12	3.19	2.70
Ψ	1.34	2.26	2.47	2.65	2.62	2.21

In order to confirm these expectation, a simple phenomenological model simulating the process of making the connections was introduced. In the model, there are two assumed modules of concepts. Within the module closely located nodes have higher probability to connect, but there is appreciably smaller probability for connection between the modules (on the average 15-20 % of connections between modules). These simple rules seem enough to generate networks with very similar properties as those found empirically.

The conclusion we can draw from the results with some confidence is that learners handle the knowledge so that they process the relational aspect in rather small pieces, finding the connections on basis of "affiliation" of concepts in the procedures (experiments and models), where they are used. In the context studied here - making plans for teaching - this is natural and desired aspect. In more general (and speculative level) the results support the assumption that processing of knowledge is based on simple affiliation schemes. In certain context of description or prediction few known concepts are used, and new concept (knowledge) or new generalization is introduced on basis of the already known concepts (knowledge). The fact that in each context (affiliation) only few known concepts is used tells probably something about the: 1) human capability to process and handle knowledge, 2) human capability to infer dependencies. In short, there seems to be preference for certain parsimony in handling the knowledge. Of course, this finding is not very unexpected, but nicely confirmed here through structural analysis of knowledge representations. These notions encourage thinking that the methods developed here provide a fruitful starting point for monitoring learning outcomes and can give insight to the ways knowledge is processed and represented.

ACKNOWLEDGEMENTS

This work has been supported by the Academy of Finland through grant SA133369.

REFERENCES

Batista J. B. and Costa, L. da F. (2010). Knowledge Acquisition by Networks of Interacting Agents in the Presence of Observation Errors. *Physical Review E*, 82, 016103.

Costa, L. da F. (2006). Learning About Knowledge: A Complex Network Approach. *Physical Review E*, 74, 026103.

Duong, T. H., Jo, G. S., Jung, J. J. and Nguyen, N. T. (2009). Complexity Analysis of Ontology Integration Methodologies: A Comparative Study. *Journal of Universal Computer Science*, 15, 877–897.

Goni, J., Corominas-Murtra, B., Solé, R. V. and Rodríguez-Caso,C. (2010). Exploring the Randomness of Directed Acyclic Networks. *Physical Review E*, 82, 066115.

Karrer, B. and Newman, M. E. J. (2009). Random Graph Models for Directed Acyclic Networks. *Physical Review E*, 80, 046110.

Kemp, C., Perfors, A. and Tenenbaum, J. B. (2007). Learning Overhypotheses with Hierarchical Bayesian Models. *Developmental Science*, 10, 307-321.

Kemp, C. and Tenenbaum, J. B. (2008). The Discovery of Structural Form. PNAS, 105, 10687–10692.

Kolaczyk, E. D. (2009). *Statistical Analysis of Network Data*. New York: Springer.

Koponen, I. T. and Pehkonen M. (2010). Coherent Knowledge Structures of Physics Represented as Concept Networks in Teacher Education. *Science & Education*, 19, 259–282.

Koponen, I. T. and Nousiainen M. (2012). Pre-service physics teachers understanding of the relational structure of physics concepts. *International Journal of Science and Mathematics Education*, in print, DOI 10.1007/s10763-012-9337-0.

McClure, J. R., Sonak, B. and Suen, H. K. (1999) Concept Map Assesment of Classroom Learning: Reliability, Validity, and Logistical Practicality. *Journal of Research in Science Teaching*, 36, 475–492.

Novak, J. (2002). Meaningful Learning: The Essential Factor for Conceptual Change in Limited or Inappropriate Propositional Hierarchies Leading to Empowerment of Learners. *Science Education*, 86, 548–571.

O'Donnell, A. M., Dansereau, D. F. and Hall, R. H. (2002) Knowledge Maps as Scaffolds for Cognitive Processing. *Educational Psychology Review*, 14, 71–86.

Thagard, P. (1992). *Conceptual Revolutions*. Princeton NJ:Princeton University Press.