A General Theory of Tempo-logical Connectives and Its Application to Spatiotemporal Reasoning in Natural Language Understanding

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Abstract. Mental Image Directed Semantic Theory (MIDST) has proposed the knowledge representation language L_{md} in order to facilitate language-centered multimedia communication between ordinary people and home robots in the daily life. L_{md} has employed the 'tempo-logical connectives (TLCs)' to represent both temporal and logical relations between two events, and the 'temporal conjunctions', a subset of TLCs, have already been applied to formulating natural event concepts, namely, event concepts represented in natural language. This paper presents the theory of TLCs extended for formalizing human intuitive spatiotemporal knowledge and its application to automatic reasoning about space and time expressed in natural language.

1 Introduction

Several theories have been proposed about formalization and computation of spatial and temporal relations and a considerable number of their applications [1-6]. They, however, do not necessarily keep tight correspondence with spatiotemporal expressions in natural language reflecting human cognitive processes strongly [7, 8]. For example, consider such expressions as S1 and S1′.

(S1) It got cloudy ***and*** it rained.

(S1′) It rained ***and*** it got cloudy.

It is very natural for people to understand each expression by synthesizing the mental images evoked by its two clauses into an intuitively plausible one where spatiotemporal relations of the matters involved do not conflict with their empirical knowledge of the real world. In this case, people would make a special effort to arrange the two events, namely, 'getting cloudy' and 'raining' on the time axis adequately because the temporal relation between them is not explicit in either expression. According to the previous psycholinguistic experiments [7], people are apt to interpret the construction 'A happened ***and*** B happened' in spatiotemporal expressions as a specific event, namely, as 'A happened ***before*** B happened' (c.f., S3′). That is, people usually do not understand such expressions as S1 and S1′ in the same meaning as 'A $\wedge$ B' equivalent to 'B $\wedge$ A' in standard logic.

Consider another expression S2 below.

(S2) It gets cloudy ***before*** it rains.

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People usually interpret the construction '*A* happens *before B* happens' as a general causality, namely, as 'If *B* happens, *A* happens in advance' [7]. This is easily understood by the fact that S2 and S2′ are semantically not identical while S3 and S3′ can refer to the same compound event as S1. That is, it is not always the case that cloudiness is followed by rain.

(S2′) It rains ***after*** it gets cloudy.

(S3) It got cloudy ***before*** it rained.

(S3′) It rained ***after*** it got cloudy.

The conventional method of temporal arguments can formalize the constructions of S1 and S2 as (1) and (2), respectively, where the events *A* and *B* are unnaturally but inevitably to be provided with the time points at extra argument-places and their relations. Here and after, a time point 't_i' is represented as a real number (i.e., $t_i \in \boldsymbol{R}$).

$$(\exists t_1,t_2)A(t_1) \wedge B(t_2) \wedge t_1<t_2 \ . \qquad (1)$$

$$(\forall t_2)(\exists t_1)(B(t_2) \ .\supset. A(t_1)) \wedge t_1<t_2 \ . \qquad (2)$$

On the other hand, the conventional method of relative temporal relations can provide a counterpart for (1) as (3), possibly more naturally, but not for (2) because such a predicate as *'after', 'contains'* or so is intrinsically a conjunction (i.e., '∧') furnished with a certain ***purely*** temporal relation. That is, (3) could be formalized otherwise as (4), where *A* and *B* are parameterized with time-intervals $[t_{11},t_{12}]$ and $[t_{21},t_{22}]$, respectively, presuming that $t_{11}<t_{12}$ and $t_{21}<t_{22}$.

$$before(A,B)(\equiv after(B,A)) \ . \qquad (3)$$

$$(\exists t_{11},t_{12},t_{21},t_{22})A([t_{11},t_{12}]) \wedge B([t_{21},t_{22}]) \wedge t_{12}<t_{21} \ . \qquad (4)$$

Mental Image Directed Semantic Theory (MIDST) [8, 14] has proposed a systematic method to model human's mental images as 'loci in attribute spaces', so called, and to describe them in a formal language $\boldsymbol{L_{md}}$ (Mental-image Description Language), where a general locus is to be articulated by "Atomic Locus" over a absolute certain time-interval formulated as (5) so called "Atomic locus formula". All loci in attribute spaces are assumed to correspond one to one with movements of the Focus of the Attention of the Observer (i.e., FAO).

$$L(x,y,p,q,a,g,k) \ . \qquad (5)$$

The intuitive interpretation of (1) is given as follows (Refer to [8] for the details).

"Matter 'x' causes Attribute 'a' of Matter 'y' to keep (*p=q*) or change (*p ≠ q*) its values temporally (*g=Gt*) or spatially (*g=Gs*) over an absolute time-interval, where the values 'p' and 'q' are relative to the standard 'k'."

The formal language $\boldsymbol{L_{md}}$ is employed for many-sorted predicate logic provided with 'tempo-logical connectives (TLCs)' with which to represent both temporal and logical relations between two loci over certain time-intervals. Therefore, TLCs are for interval-based time theories with relative temporal relations but are generalized for all the binary logical connectives (i.e., conjunction '∧', disjunction '∨', implication '⊃' and equivalence '≡') unlike the conventional ones exclusively for the conjunction [1, 9-13]. This paper presents a general theory of TLCs intended to formulate human empirical knowledge expressed in spatiotemporal language and its application to automatic reasoning about space and time.

2 Tempo-logical Connectives

The definition of a tempo-logical connective K_i is given by **D1**, where τ_i, χ and K refer to one of ***purely*** temporal relations indexed by an integer '*i*', a locus, and an ordinary binary logical connective such as the conjunction '$\wedge$', respectively.

The definition of each τ_i is provided with Table 1 implying the theorem **T1,** where the durations of χ_1 and χ_2 are $[t_{11}, t_{12}]$ and $[t_{21}, t_{22}]$, respectively. This table shows the complete list of temporal relations between two intervals, where 13 types of relations are discriminated by the suffix '*i*' ($-6 \leq i \leq 6$). This is in accordance with the conventional notation [1, 9-13] which, to be strict, is for 'temporal conjunctions ($=\wedge_i$)' but not for pure 'temporal relations ($=\tau_i$)'.

D1. $\chi_1 \; K_i \; \chi_2 \Leftrightarrow (\chi_1 \; K \; \chi_2) \wedge \tau_i(\chi_1, \chi_2)$

T1. $\tau_{-i}(\chi_2, \chi_1) \equiv \tau_i(\chi_1, \chi_2)$ $(\forall i \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6\})$

(Proof) Trivial in Table1. [**Q.E.D.**]

Table 1. List of temporal relations.

Definition of τ_i		Allen's notation
$t_{11}=t_{21} \wedge t_{12}=t_{22}$	$\tau_0(\chi_1, \chi_2)$	equals(χ_1,χ_2)
	$\tau_0(\chi_2, \chi_1)$	equals(χ_2, χ_1)
$t_{12}=t_{21}$	$\tau_1(\chi_1, \chi_2)$	meets(χ_1, χ_2)
	$\tau_{-1}(\chi_2, \chi_1)$	met-by(χ_2, χ_1)
$t_{11}=t_{21} \wedge t_{12}<t_{22}$	$\tau_2(\chi_1, \chi_2)$	starts(χ_1, χ_2)
	$\tau_{-2}(\chi_2, \chi_1)$	started-by(χ_2, χ_1)
$t_{11}>t_{21} \wedge t_{12}<t_{22}$	$\tau_3(\chi_1, \chi_2)$	during(χ_1, χ_2)
	$\tau_{-3}(\chi_2, \chi_1)$	contains(χ_2, χ_1)
$t_{11}>t_{21} \wedge t_{12}=t_{22}$	$\tau_4(\chi_1, \chi_2)$	finishes(χ_1, χ_2)
	$\tau_{-4}(\chi_2, \chi_1)$	finished-by(χ_2, χ_1)
$t_{12}<t_{21}$	$\tau_5(\chi_1, \chi_2)$	before(χ_1, χ_2)
	$\tau_{-5}(\chi_2, \chi_1)$	after(χ_2, χ_1)
$t_{11}<t_{21} \wedge t_{21}<t_{12} \wedge t_{12}<t_{22}$	$\tau_6(\chi_1, \chi_2)$	overlaps(χ_1, χ_2)
	$\tau_{-6}(\chi_2, \chi_1)$	overlapped-by(χ_2, χ_1)

As easily understood, the properties of a TLC depend on those of the purely logical connective (K) and the temporal relations (τ_i) involved. By the way, there are a considerable number of trivial theorems concerning temporal relations such as (6)-(13) below. All the possible cases of transitivity between two temporal relations are listed up in Table 2. This table shows that the transitivity is not always determined uniquely as easily calculated.

$$\tau_i(\chi_1,\chi_2) \wedge \tau_0(\chi_2,\chi_3) \;.\supset.\; \tau_i(\chi_1,\chi_3) \;. \tag{6}$$

$$\tau_1(\chi_1,\chi_2) \wedge \tau_1(\chi_2,\chi_3) \;.\supset.\; \tau_5(\chi_1,\chi_3) \;. \tag{7}$$

$$\tau_1(\chi_1,\chi_2) \wedge \tau_3(\chi_2,\chi_3) \;.\supset.\; \tau_5(\chi_1,\chi_3) \;. \tag{8}$$

$$\tau_1(\chi_1,\chi_2) \wedge \tau_4(\chi_2,\chi_3) \;.\supset.\; \tau_5(\chi_1,\chi_3) \;. \tag{9}$$

$$\tau_1(\chi_1,\chi_2)\wedge\tau_5(\chi_2,\chi_3) .\supset. \tau_5(\chi_1,\chi_3) . \quad (10)$$

$$\tau_1(\chi_1,\chi_2)\wedge\tau_6(\chi_2,\chi_3) .\supset. \tau_5(\chi_1,\chi_3) . \quad (11)$$

$$\tau_2(\chi_1,\chi_2)\wedge\tau_1(\chi_2,\chi_3) .\supset. \tau_5(\chi_1,\chi_3) . \quad (12)$$

$$\tau_5(\chi_1,\chi_2)\wedge\tau_1(\chi_2,\chi_3) .\supset. \tau_5(\chi_1,\chi_3) . \quad (13)$$

Table 2. List of all possible values of ‘k’ when $\tau_i(P_1,P_2) \wedge \tau_j(P_2,P_3) .\supset. \tau_k(P_1,P_3)$ (‘∀’ denotes that k=0,±1,±2,±3,±4,±5, ±6).

i \ j	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-6	-6 -5 -1	-5	-6 -3 -2	-6 -5 -3 -2 -1	-6 -5 -1	-5	-6	-4 -3 6	-6 3 4	-6 3 4	-6	-4 -3 1 5 6	-6 -4 -3 -2 0 2 3 4 6
-5	-5	-5	-5	-5	-5	-5	-5	-6 -5 -1 3 4	-6 -5 -1 3 4	-6 -5 -1 3 4	-5	-6 -5 -1 3 4	-6 -5 -1 3 4
-4	-6 -3 -2	-6 -5 -3 -2 -1	-4	-3	-3	-6 -3 -2	-4	1	6	2 3 6	-4 0 4	5	6
-3	-6 -3 -2	-6 -5 -3 -2 -1	-3	-3	-3	-6 -3 -2	-3	-4 -3 6	-4 -3 6	-6 -4 -3 -2 0 2 3 4 6	-6 -3 -2	-4 -3 1 5 6	-4 -3 6
-2	-6	-5	-3	-3	-2	-1	-2	-4 -3 6	-2 0 2	-6 3 4	-6	-4 -3 1 5 6	-4 -3 6
-1	-5	-5	-1	-5	-5	-5	-1	-2 0 2	-6 3 4	-6 3 4	-1	-4 -3 1 5 6	-6 3 4
0	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
1	2 3 6	-6 -5 -3 -2 -1	5	5	1	-4 0 4	1	5	1	2 3 6	2 3 6	5	5
2	-6 3 4	-5	1 5 6	-3 1 4 5 6	-2 0 2	-1	2	5	2	3	3	5	1 5 6
3	-6 -5 -1 3 4	-5	1 2 3 5 6	∀	-6 -5 -4 -1 3	-5	3	5	3	3	3	5	1 2 3 5 6
4	-6 -5 -1	-5	-4 0 4	-6 -5 -3 -2 -1	-6 -5 -1	-5	4	1	3	3	4	5	2 3 6
5	1 2 3 5 6	∀	5	5	5	1 2 3 5 6	5	5	5	1 2 3 5 6	1 2 3 5 6	5	5
6	-6 -4 -3 -2 0 2 3 4 6	-6 -5 -3 -2 -1	1 5 6	-4 -3 1 5 6	-4 -3 6	-2 -3 -6	6	5	6	2 3 6	2 3 6	5	1 5 6

In order for explicit indication of ***absolute*** time elapsing, ‘Empty Event’ denoted by ‘ε’ is introduced as **D2** with the attribute ‘Time Point (A_{34})’ and the Standard of absolute time ‘T_a’. Usually people can know only a certain ***relative*** time point by a clock that is seldom exact and that is to be denoted by another Standard in $\boldsymbol{L_{md}}$ [8, 14]. Hereafter, $\boldsymbol{\Delta}$ denotes the total set of absolute time intervals. According to this scheme, the suppressed absolute time-interval $[t_a, t_b]$ of a locus χ can be indicated as (14).

D2. $\varepsilon([t_i,t_j])\Leftrightarrow(\exists x,y,g)L(x,y,t_i,t_j,A_{34},g,T_a)$,

where $[t_i, t_j]\in\Delta$ $(=\{[t_1, t_2] \mid t_1<t_2\ (t_1, t_2\in\mathbf{R})\})$.

$$\chi\Pi\varepsilon([t_a,t_b]) . \quad (14)$$

A locus corresponding directly to the live image of a specific phenomenon outside is called ‘Perceptual Locus’ and can be formulated with atomic locus formulas and temporal conjunctions such as SAND ($\wedge_0$ or Π) and CAND ($\wedge_1$ or •). This is not necessarily the case for the other type of locus, so called, ‘Conceptual Locus’ that does not correspond directly to such a live image but to such a generalized mental image or knowledge piece as is conventionally represented by (2) with logical connectives other than conjunctions also involved. This is essentially due to no interpreting a negated atomic locus formula as a locus with ***a unique time-interval***. That is, **D1** is exclusively for perceptual loci so far as it is. Whereas, such a theorem as ‘A⊃B.≡.~A∨B’ in standard logic can give us a good reason for the identity of a locus formula with its negative in absolute time-interval, that is, negation-freeness of absolute time passing under a locus referred to by its suppressed absolute time-interval. Therefore, in order to make **D1** valid also for conceptual loci, we introduce a meta-function δ defined by **D3** and its related postulates **P1** and **P2** as follows, where δ is to extract the ***suppressed*** absolute interval of a locus formula χ.

D3. $\delta(\chi)\in\Delta$

P1. $\delta(\sim\alpha)=\delta(\alpha)$, where α is an atomic locus formula.

P2. $\delta(\chi)=[t_{min}, t_{max}]$, where t_{min} and t_{max} are respectively the minimum and the maximum time-point included in the absolute time-intervals of the atomic locus formulas, either positive or negative, within χ.

These postulates lead to **T2** (Theorem of negation-freeness of a suppressed absolute time-interval) below.

T2. $\delta(\sim\chi)=\delta(\chi)$

(Proof) According to **P1** and **P2**, the time-interval of each atomic locus formula involved in $\sim\chi$ is negation-free and therefore so are t_{min} and t_{max} in $\delta(\sim\chi)$. **[Q.E.D.]**

The counterpart of the contrapositive in standard logic (i.e., $A\supset B.\equiv.\sim B\supset\sim A$) is given as **T3** (Tempo-logical Contrapositive) whose rough proof is as follows immediately below, where the left hand of ':' refers to the postulates or theorems (e.g., **PL** is a subset of those in pure predicate logic) employed at the process indicated by the conventional meta-symbol '$\leftrightarrow$' for bidirectional deduction.

T3. $\chi_1\supset_i\chi_2.\equiv.\sim\chi_2\supset_{-i}\sim\chi_1$

(Proof)

D1: $\chi_1\supset_i\chi_2\leftrightarrow(\chi_1\supset\chi_2)\wedge\tau_i(\chi_1,\chi_2)$

PL: $\leftrightarrow(\sim\chi_2\supset\sim\chi_1)\wedge\tau_i(\chi_1,\chi_2)$

T2: $\leftrightarrow(\sim\chi_2\supset\sim\chi_1)\wedge\tau_i(\sim\chi_1,\sim\chi_2)$

D1: $\leftrightarrow(\sim\chi_2\supset\sim\chi_1)\wedge\tau_{-i}(\sim\chi_2,\sim\chi_1)$

D1: $\leftrightarrow\sim\chi_2\supset_{-i}\sim\chi_1$ **[Q.E.D.]**

By the way, an empty event can be generated by **T4**, whose proof is trivial.

T4. $\chi.\equiv_0.\chi\,\Pi\varepsilon(\delta(\chi))$

3 Knowledge Representation with TLCs

Perceptual loci are inevitably articulated by tempo-logical conjunctions. For example, (3) or (4) is represented as (15).

$$A\wedge_5 B\ (\equiv B\wedge_{-5}A)\ . \quad (15)$$

As easily understood, any pair of loci temporally related in certain attribute spaces can be formulated as (16)-(20) in exclusive use of SANDs, CANDs and empty events.

$$\chi_1\wedge_2\chi_2\ .\equiv.\ (\chi_1\bullet\varepsilon)\Pi\chi_2\ . \quad (16)$$

$$\chi_1 \wedge_3 \chi_2 \ .\!\equiv\!. \ (\varepsilon_1 \bullet \chi_1 \bullet \varepsilon_2) \Pi \chi_2 \ . \tag{17}$$

$$\chi_1 \wedge_4 \chi_2 \ .\!\equiv\!. \ (\varepsilon \bullet \chi_1) \Pi \chi_2 \ . \tag{18}$$

$$\chi_1 \wedge_5 \chi_2 \ .\!\equiv\!. \ \chi_1 \bullet \varepsilon \bullet \chi_2 \ . \tag{19}$$

$$\chi_1 \wedge_6 \chi_2 \ .\!\equiv\!. \ (\chi_1 \bullet \varepsilon_3) \Pi (\varepsilon_1 \bullet \chi_2) \Pi (\varepsilon_1 \bullet \varepsilon_2 \bullet \varepsilon_3) \ . \tag{20}$$

Consider such somewhat complicated sentences as S4 and S5. The underlined parts are deemed to refer to some events neglected in time and in space, respectively. These events correspond with skipping of FAOs and are called 'Temporal Empty Event' and 'Spatial Empty Event', denoted by ε_t and ε_s as empty events with g=G_t and g=G_s at **D2**, respectively. The images evoked by S4 and S5 can be formalized as (21) and (22) in $\boldsymbol{L_{md}}$, respectively. A_{15} and A_{17} represent the attributes 'Trajectory' and 'Mileage', respectively, whose vales are relative to certain Standards (Refer to [8] for the details).

(S4) The *bus* runs 10km straight east from A to B, and <u>*after a while*</u>, at C it meets the street with the sidewalk.

$$\begin{gathered}(\exists x_1,x,y,z,p,q,k,k_1,k_2,k_3)(L(x_1,x,A,B,A_{12},G_t,k)\Pi \\ L(x_1,x,0,10km,A_{17},G_t,k_1)\Pi L(x_1,x,\mathit{Point},\mathit{Line},A_{15},G_t,k_2) \\ \Pi L(x_1,x,\mathit{East},\mathit{East},A_{13},G_t,k_3))\bullet\varepsilon_t\bullet(L(x_1,x,p,C,A_{12},G_t,k) \\ \Pi L(x_1,y,q,C,A_{12},G_s,k)\Pi L(x_1,z,y,y,A_{12},G_s,k)) \\ \wedge bus(x)\wedge street(y)\wedge sidewalk(z)\wedge p\neq q \ .\end{gathered} \tag{21}$$

(S5) The *road* runs 10km straight east from *A* to *B*, and <u>*after a while*</u>, at *C* it meets the street with the sidewalk.

$$\begin{gathered}(\exists x_1,x,y,z,p,q,k,k_1,k_2)(L(x_1,x,A,B,A_{12},G_s,k)\Pi \\ L(x_1,x,0,10km,A_{17},G_s,k_1)\Pi L(x_1,x,\mathit{Point},\mathit{Line},A_{15},G_s,k_2) \\ \Pi L(x_1,x,\mathit{East},\mathit{East},A_{13},G_s,k_3))\bullet\varepsilon_s\bullet(L(x_1,x,p,C,A_{12},G_s,k) \\ \Pi L(x_1,y,q,C,A_{12},G_s,k)\Pi L(x_1,z,y,y,A_{12},G_s,k)) \\ \wedge road(x)\wedge street(y)\wedge sidewalk(z)\wedge p\neq q \ .\end{gathered} \tag{22}$$

From the viewpoint of cross-media reference, the formula (22) can refer to such a spatial event depicted as the still picture in Fig.1 while (21) can be interpreted into a motion picture.

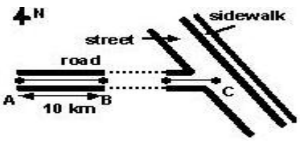


Fig. 1. Pictorial interpretation of the formula (22).

On the other hand, the causality represented by (2) can be formulated as (23) by employing the temporal implication '$\supset_5$' or as its equivalent (24) with '$\supset_{-5}$'**.** As easily understood, these formulas are equivalent to such ones using temporal disjunctions as parenthesized. By the way, (24) can be verbalized as S6.

$$B.\!\supset_{-5}\!.A(\equiv\sim B\vee_{-5}A) \ . \tag{23}$$

$$\sim A. \supset_5. \sim B\ (\equiv A \vee_5 \sim B)\ . \qquad (24)$$

(S6) Unless it gets cloudy, it does not rain later.

Without proper treatment of temporal relations, especially in Japanese [7], such a somewhat quire contrapositive S8 would be yielded from S7.

(S7) The student does not study unless he is scolded.

(S8) The student is scolded if he studies.

Tempo-logical conjunctions are also applied to formulating event patterns involved in such verb concepts as 'carry', 'return' and 'fetch' [8, 14] and temporal implications are often employed for formalizing miscellaneous tempo-logical relations between event concepts as knowledge pieces without explicit indication of time-intervals. For example, an event 'fetch(x,y)' is necessarily *finished by* an event 'carry(x,y)' [8, 14]. This fact can be formulated as (25), which is not an axiom but a theorem deducible from the definitions of event concepts here. Similarly, the tempo-logical relation between 'fetch(x,y)' and 'return(x)' can be theorematized as (26). Furthermore, if necessary, these can be temporally quantified as (27) and (28), respectively, where d_1, $d_2 \in \boldsymbol{\Delta}$.

$$(\forall x,y)\mathrm{fetch}(x,y)\ .\supset_{-4}.\ \mathrm{carry}(x,y)\ . \qquad (25)$$

$$(\forall x,y)\mathrm{fetch}(x,y)\ .\supset_{0}.\ \mathrm{return}(x)\ . \qquad (26)$$

$$(\forall x,y)\ (\forall d_1)\ (\exists d_2)\ \mathrm{fetch}(x,y)\Pi\varepsilon(d_1)\ .\supset_{-4}.\ \mathrm{carry}(x,y)\Pi\varepsilon(d_2)\ . \qquad (27)$$

$$(\forall x,y)\ (\forall d_1)\ (\exists d_2)\ \mathrm{fetch}(x,y)\Pi\varepsilon(d_1)\ .\supset_{0}.\ \mathrm{return}(x)\Pi\varepsilon(d_2)\ . \qquad (28)$$

The postulate of reversibility of spatial events (PRS) [8] can be formulated as **P1** using '$\equiv_0$', where χ and χ^R is a perceptual locus and its 'reversal' for a certain spatial event, respectively. These loci are substitutable with each other because of the property of '$\equiv_0$'.

P1. $\chi^R.\equiv_0.\chi$

The recursive operations to transform χ into χ^R are defined by **D4**, where the reversed values p^R and q^R depend on the properties of the attribute values p and q. For example, at (22), $p^R = p$, $q^R = q$ for A_{12}; $p^R = -p$, $q^R = -q$ for A_{13}.

D4. $(\chi_1 \bullet \chi_2)^R \Leftrightarrow \chi_2^R \bullet \chi_1^R$

$$(\chi_1 \Pi \chi_2)^R \Leftrightarrow \chi_1^R\, \Pi \chi_2^R$$

$$(L(x,y,p,q,a,G_s,k))^R \Leftrightarrow L(x,y,q^R,p^R,a,G_s,k)$$

By employing **D4**, (22) is transformed into (29) as its reversal and equivalent in PRS to be verbalized as S9. That is, PRS is very helpful for paraphrasing of spatial events variously expressed.

$$\begin{aligned}
&(\exists x_1,x,y,z,p,q,k,k_1,k_2,k_3)(L(x_1,x,C,p,A_{12},G_s,k)\Pi \\
&L(x_1,y,C,q,A_{12},G_s,k)\Pi L(x_1,z,y,y,A_{12},G_s,k)) \\
&\bullet\varepsilon_s\bullet(L(x_1,x,B,A,A_{12},G_s,k)\Pi L(x_1,x,0,10\mathrm{km},A_{17},G_s,k_1) \\
&\Pi L(x_1,x,\mathit{Point},\mathit{Line},A_{15},G_s,k_2) \\
&\Pi L(x_1,x,\mathit{West},\mathit{West},A_{13},G_s,k_3)) \\
&\wedge \mathrm{road}(x)\wedge \mathrm{street}(y)\wedge \mathrm{sidewalk}(z)\wedge p\neq q\ .
\end{aligned} \qquad (29)$$

(S9)The road separates at *C* from the street with the sidewalk and, after a while, runs 10km straight west from *B* to *A*.

4 Tempo-logical Deduction with TLCs

Here is focused on **tempo-logical syllogism** as is formalized by (30), where logical and temporal relations are calculated simultaneously in context of multiple tempo-logical implications.

$$P_1 \supset_i P_2,\ P_2 \supset_j P_3\ \ |\!-\ \ P_1 \supset_k P_3, \quad \text{where } \tau_i(P_1,P_2) \wedge \tau_j(P_2,P_3)\ .\!\supset\!.\ \tau_k(P_1,P_3)\ . \qquad (30)$$

The value of 'k' above is determined by the ordered pair (i,j) as shown in Table 2 and the proof of such a proposition as (30) is given by a set of deductions formulated as (31). The proof of such a formula as (31) is given by a set of deductions denoted as (32) with the conventional symbol of deduction '→' furnished with temporal relations.

$$P \supset_n Q\ . \qquad (31)$$

$$X \rightarrow_{i\,(j)} Y,\ \text{ where } \tau_i(X,Y) \text{ and } \tau_j(P,Y), \text{ and } j=n \text{ when } Y=Q. \qquad (32)$$

For example, consider the propositions A-F below and we can understand that F can be deduced from D and E.

A='Tom studies'

B='Tom is scolded'

C='Tom is given candies'

D='Tom does not study unless he is scolded in advance'

E='Tom studies immediately before he is given candies'

F= 'Tom is not given candies unless he is scolded in advance',

where D, E and F are formulated as (33)-(35), respectively.

$$D\ .\!\equiv\!.\ \sim B \supset_5 \sim A\ . \qquad (33)$$

$$E\ .\!\equiv\!.\ C \supset_{-1} A\ . \qquad (34)$$

$$F\ .\!\equiv\!.\ \sim B \supset_5 \sim C\ . \qquad (35)$$

The proof is as follows.

(Proof)

E, T3 :	$\sim A \supset_1 \sim C$.	(C1)
D :	$\sim B \rightarrow_{5(5)} \sim A$.	
C1, Table 2 :	$\rightarrow_{1(5)} \sim C$ (See Table2 at (i,j)=(5,1)) .	
	$\therefore \sim B \supset_5 \sim C$. **[Q.E.D]**	

5 Application to Natural Language Understanding

The intelligent system IMAGES-M [8] can perform text understanding based on word meaning descriptions as follows. Firstly, a text is parsed into a surface dependency structure (or more than one if *syntactically* ambiguous). Secondly, each surface dependency structure is translated into a conceptual structure (or more than one if *semantically* ambiguous) using word meaning descriptions. Finally, each conceptual structure is semantically evaluated.

The fundamental semantic computations on a text are to detect semantic anomalies, ambiguities and paraphrase relations.

Semantic anomaly detection is very important to cut off meaningless computations. Consider such a conceptual structure as (36), where 'A_{39}' is the attribute 'Vitality'. This locus formula can correspond to the English sentence 'The desk is alive', which is usually semantically anomalous because a 'desk' does never have vitality in the real world projected into the attribute spaces.

$$(\exists x,y,k)L(y,x,\mathit{Alive},\mathit{Alive},A_{39},G_t,k)\wedge \mathrm{desk}(x)\ . \tag{36}$$

This kind of semantic anomaly can be detected in the following process.

Firstly, assume the concept of 'desk' as (37), where 'A_{29}' refers to the attribute 'Taste'. The special symbols '*' and '/' are defined as (38) and (39) representing 'always' and 'no value', respectively.

$$(\lambda x)\mathrm{desk}(x) \Leftrightarrow (\lambda x)(\ldots L^*(y_1,x,/,/,A_{29},G_t,k_1)\wedge\ldots \wedge L^*(y_n,x,/,/,A_{39},G_t,k_n) \wedge \ldots)\ . \tag{37}$$

$$X^* \Leftrightarrow (\forall d\in\Delta)X\ \Pi\ \varepsilon(d)\ . \tag{38}$$

$$L(\ldots,/,\ldots) \Leftrightarrow \sim(\exists p)\ L(\ldots,p,\ldots)\ . \tag{39}$$

Secondly, the postulates (40) and (41) are utilized. The formula (40) means that if one of two loci exists every time interval, then they can coexist. The formula (41) states that a matter never has different values of an attribute with a standard at a time.

$$X \wedge Y^* .\supset. X\ \Pi\ Y\ . \tag{40}$$

$$(\forall x,y,z, p_1,q_1, p_2,q_2,a,g,k)\ L(x,y,p_1,q_1,a,g,k)\Pi L(z,y,p_2,q_2,a,g,k) .\supset. p_1=p_2 \wedge q_1=q_2\ . \tag{41}$$

Lastly, the semantic anomaly of 'alive desk' is detected by using (36)-(41). That is, the formula (42) below is finally deduced from (36)-(40) and violates the commonsense given by (41), that is, " $\mathit{Alive} \neq /$ ".

$$(\exists x,y,z,k_1,k_2)L(y,x,\mathit{Alive},\mathit{Alive},A_{39},G_t,k_1)\Pi\ L(z,x,/,/,A_{39},G_t,k_2)\ . \tag{42}$$

This 15 at the insect on the desk, which is still alive.

If a text has multiple plausible interpretations, it is semantically ambiguous. For example, S11 alone has two plausible interpretations (43) and (44) different at the underlined parts, implying 'Tom with the stick' and 'Jim with the stick', respectively.

(S11) Jim follows Tom with the stick.

$$(\exists x,k)(L(Tom,Tom,p,q,A_{12},G_t,k) \underline{\Pi L(Tom,x,Tom,Tom,A_{12},G_t,k)})\bullet L(Jim,Jim,p,q,A_{12},G_t,k)\wedge p\neq q \wedge \mathrm{stick}(x)\ . \tag{43}$$

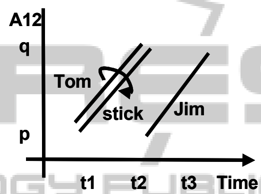
$$(\exists x,k)L(Tom,Tom,p,q,A_{12},G_t,k)\bullet(L(Jim,Jim,p,q,A_{12},G_t,k)\ \underline{\Pi}\ \underline{L(Jim,x,Jim,Jim,A_{12},G_t,k)})\wedge p\neq q\wedge stick(x)\ . \quad (44)$$

Among the fundamental semantic computations, detection of paraphrase relations is the most essential because it is for detecting equalities in semantic descriptions and the other two are for detecting inequalities in them. In our system, if two different texts are interpreted into the same locus formula, they are paraphrases of each other.

The understanding process above is completely reversible except that multiple paraphrases can be generated by tempo-logical reasoning as shown in Fig.2-a because event patterns are sharable among multiple word concepts. Fig.2-b shows the graphical interpretation of the kernel structure of the input sentence, namely, "with stick Tom precedes Jim", whose formulation in $\boldsymbol{L_{md}}$ is the same as (43) (Refer to [8] for the details).

(Input)
With the long red stick Tom precedes Jim.
(Output)
Tom with the long red stick goes before Jim goes.
Jim goes after Tom goes with the long red stick.
Jim follows Tom with the long red stick.
Tom carries the long red stick before Jim goes.
The stick moves simultaneously when Tom goes.
…………………

(a)



(b)

Fig. 2. (a) Text paraphrasing by tempo-logical reasoning, and (b) Graphical interpretation of "with stick Tom precedes Jim".

6 Conclusions

The theory of tempo-logical connectives introduced in MIDST was extended so as to be applicable to intuitive spatiotemporal knowledge expressed in natural language in order to facilitate intuitive human-robot interaction much better. This extension was concentrated on providing the theory with tempo-logical connectives other than tempo-logical conjunctions, where the principal definitions and postulates have been induced from several psycholinguistic experiments [7]. To remark reversely, they have been already psycho-linguistically validated.

MIDST is intended to provide a formal system represented in $\boldsymbol{L_{md}}$ for natural or intuitive semantics of spatiotemporal language [14]. This formal system is one kind of applied predicate logic consisting of axioms and postulates subject to human perception of space and time while the other similar systems in Artificial Intelligence [1-6, 9-13] are intended to be objective, namely, independent of human perception process and do not necessarily keep tight correspondences with natural language. For example, the postulates **P1** and **P2** of human perception of time and TLCs have brought the tempo-logical contrapositive, which leads to the naturalness of tempo-logical syllogism without explicit indication of time points. Furthermore, such paraphrasing based on spatiotemporal reasoning as in Fig.2-a shows that explicit description of word concepts grounded in loci in attribute spaces can simulate mental image processing in humans well enough for text-picture cross-reference both in spatial and temporal extents [8], which is very essential for ordinary people to have intuitive access to intelligent multimedia systems. Our future work will include further explication of potential expressiveness of TLCs.

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