Inverting Thanks to SAT Solving*
An Application on Reduced-step MD⋆

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Keywords: Logic, Cryptanalysis, Hash Function, MD5, Satisfiability.

Abstract: The SATisfiability Problem is a core problem in mathematical logic and computing theory. The last decade progresses have led it to be a great and competitive approach to practically solve a wide range of industrial and academic problems. Thus, the current SAT solving capacity allows the propositional formalism to be an interesting alternative to tackle cryptanalysis problems. This paper deals with an original application of the SAT problem to cryptanalysis. We thus present a principle, based on a propositional modeling and solving, and provide details on logical inferences, simplifications, learning and pruning techniques used as a preprocessor with the aim of reducing the computational complexity of the SAT solving and hence weakening the associated cryptanalysis. As cryptographic hash functions are central elements in modern cryptography we choose to illustrate our approach with a dedicated attack on the second preimage of the well-known MD⋆ hash functions. We finally validate this reverse-engineering process, thanks to a generic SAT solver achieving a weakening of the inversion of MD⋆. As a result, we present an improvement of the current limit of best practical attacks on step-reduced MD4 and MD5 second preimage, respectively up to 39 and 28 inverted rounds.

1 INTRODUCTION

The SATisfiability Problem (short for SAT) is a well-known decision NP-Complete problem (Cook, 1971). The interest in studying SAT has grown significantly over the last years because of its conceptual simplicity and ability to express a large set of various problems. Within a practical framework, a lot of works highlight SAT implications in ”real world” problems as diverse as planning (Kautz and Selman, 1996), model checking (Biere et al., 2006), VLSI design and also cryptography (Potlapally et al., 2007; Massacci and Marraro, 2000) ... In recent years, several improvements dedicated on the one hand, to the original backtrack-search DLL procedure (Davis et al., 1962), and on the other hand to the logical simplification techniques (Bacchus and Winter, 2003) have allowed SAT solvers to be very efficient in solving huge problems from industrial areas (Zhang et al., 2001). In addition cryptography remains ubiquitious and cryptographic primitives (*cryptosystems*) play a key role in computer security. Cryptanalysis usually refers to all techniques that measure the security of a primitive with a view to finding weaknesses in it that will facilitate the retrieval any of secret information. Several general approaches have been proposed over the years such as differential (Biham and Shamir, 1990) or linear (Matsui and Yamagishi, 1992) ones. This paper focuses on a particular type of algebraic cryptanalysis which consists in measuring the security of a cryptosystem thanks to a two-steps process following a boolean modeling and then a dedicated SAT solving. This was first described in (Massacci and Marraro, 2000) and named *logical cryptanalysis*. The modeling phase is to express, *in extenso* and independently from the solving phase, the algorithmic process associated to a cipher primitive, a hash function or more generally a dedicated attack, to a set of boolean equations (a SAT *formula*) describing the whole process where the sequentiality disappeared. In this way, the solving phase is to estimate the security of the SAT modeled attack by finding a solution thanks to a SAT solver.

1.1 About Logical Cryptanalysis

Recent tremendous progresses of the SAT community on practical solving allowed some promising logical cryptanalysis results. First, (Massacci and Mar-

*This work is partly supported by DGA

Legendre F., Dequen G. and Krajecki M..
Inverting Thanks to SAT Solving - An Application on Reduced-step MD*.
DOI: 10.5220/0004077603390344
In *Proceedings of the International Conference on Security and Cryptography* (SECRYPT-2012), pages 339-344
ISBN: 978-989-8565-24-2

raro, 2000) proposed a first logical cryptanalysis attack on the U.S. Data Encryption Standard algorithm. The idea was to find out the cipher key by instancing the variables corresponding of input/output plaintexts thanks to a SAT encoding of the stream cipher. Afterwards, (Mironov and Zhang, 2006) modeled a whole differential path for the best known hash functions (MD$\star$ and SHA-$\star$) into a boolean circuit and obtained conclusive results by using some of the best SAT engines. In (De et al., 2007), the authors tackled the second preimage of reduced version of MD4 and MD5. Their encoding enables them to break the second preimage of 28-step-reduced MD4 and a 26-step-reduced MD5 thanks to SAT solving. They also improved their attack against a 39-step-reduced MD4 by adding some information from the Dobbertin's attack (Dobbertin, 1996). Attacks on pseudo-preimage (Sasaki and Aoki, 2008; Aumasson et al., 2008) or partial pseudo-preimage (Leurent, 2008) are a hopeful way to weaken cryptographic functions. Finally, note to date the best results of MD$\star$ cryptanalysis remain attacks by collisions (Wang and Yu, 2005; Klíma, 2005; Yu and Wang, 2007; Wang et al., 2009).

1.2 Our Approach

This paper deals with a logical cryptanalysis of hash functions of MD$\star$ family. The main contributions are about SAT encoding of the MD4 and MD5 primitives on which we apply some logical inference rules. We then illustrate our approach by improving, than to parallel SAT solving, the current limit of best practical attacks on step-reduced MD4 and MD5 second preimage, respectively up to 39 and 28 inverted steps.

The paper is organized as follows: In section 2, we present some definitions and outline the benefits of a SAT encoding within the context of the inversion problem. In section 3, we describe a dedicated SAT encoding of MD5 hash function. As an illustration of this technique, we describe the method to obtain an encoding of a classical adder circuit and then show how SAT solving can be useful for inverting a hash function. The section 4 presents some results about breaking reduced-step MD4 and MD5 thanks to parallel SAT solving. Finally, the section 5 concludes about our works and opens future works.

2 BACKGROUND

2.1 Brief Overview of the SAT Problem

Let $\mathcal{V} = \{ v_1, \ldots, v_n \}$ be a set of n boolean variables. A signed boolean variable is named a *literal*. We denote, v_i and $\overline{v_i}$ the positive and negative literals referring to the variable v_i respectively. The literal v_i (resp. $\overline{v_i}$) is TRUE (also said *satisfied*) if the corresponding variable v_i is assigned to TRUE (resp. FALSE). Literals are commonly associated with logical AND and OR operators respectively denoted $\wedge$ and $\vee$. A *clause* is a disjunction of literals, that is for instance $v_1 \vee \overline{v_2} \vee v_3 \vee v_4$. Hence, a clause is satisfied if at least one of its literals is satisfied. A SAT formula $\mathcal{F}$ is usually considered as a conjunction of clauses and said under *Conjunctive Normal Form (CNF)*. Consequently, $\mathcal{F}$ is satisfied if all its clauses are satisfied. SAT is the problem of determining if exists an assignment of $\mathcal{V}$ on $\{$TRUE, FALSE$\}$ such as to make the formula $\mathcal{F}$ TRUE. If such an assignment exists, $\mathcal{F}$ is said SAT and UNSAT otherwise. In the following, 1 (resp. 0) could mean TRUE (resp. FALSE).

In order to solve the SAT problem, two classes of techniques are commonly used by the community.

- *Complete* approaches guarantee an answer in a finite but exponential runtime. These methods are mainly based on the DLL (Davis et al., 1962) algorithm which consists in a systematic enumeration of truth assignments thanks to a binary search-tree. We choose to use this type of solving approach within the context of this paper.
- *Incomplete* SAT solving methods are those that cannot guarantee an answer in a finite runtime. Among the incomplete approaches to SAT solving, one of the most efficient is based on `gsat` and `walksat` algorithms which can be briefly described as noisy and greedy searches into the search-space. The reader should refer to (Biere et al., 2009) for more details.

2.2 Cryptographic Hash Functions

Cryptographic hash functions are central elements of modern cryptography. A hash function can be defined as a deterministic process that generates a fixed-length bit string, usually named *digest*, from any-length bit string also named the *message*. It is commonly used for integrity checking of files or communications but also in authentification protocols. It is interesting to notice for a given message, a cryptographic hash function computes a unique digest. On the other hand, a single digest could be associated to several messages. Two different messages hashing the same digest is a *collision*. Finally, a process that leads to yield a message from a given digest faster than exhaustive search is a *preimage* attack.

2.3 About MD5

MD5 was designed in 1991 by Ron Rivest as an evolution of MD4, strengthening its security by adding some improvements. The operating principle of this function is based on the Merkle-Damgard model (Merkle, 1989; Damgård, 1989) and consists in a hashing process where four states are initialized and then modified at each of the 64 steps.

The compression function is required to satisfy three properties : (i) Collision Resistance, (ii) Second Preimage Resistance and (iii) Preimage Resistance.

A step is determined as follows :

$$S_i \leftarrow S_{i-4} + f(S_{i-1}, S_{i-2}, S_{i-3}) + W[j] + T[i]$$
$$S_i \leftarrow S_i \lll r_i$$
$$S_i \leftarrow S_i + S_{i-1}, i \in \{ 1, \ldots, 64 \}$$

where :

- S_i is the current state. $S_{-3}, S_{-2}, S_{-1}, S_0$ are the IV.
- $W[j]$ is the j^{th} word of 32 bits, $j \in \{ 0, 1, \ldots, 15 \}$, from the input message.
- $T[i]$ among 64 predefined constants
- f a non-linear function $\in \{$ F, G, H, I $\}$
- $\lll r_i$ the circular shifting to the left(rotating) by r_i bits position.

The non-linear functions are defined by:

$$F(x,y,z) = (x \wedge y) \vee (\overline{x} \wedge z) \text{ ; } G(x,y,z) = F(z,x,y)$$
$$H(x,y,z) = x \oplus y \oplus z \text{ ; } I(x,y,z) = y \oplus (x \vee \overline{z})$$

More generally, a step computation merely contains an addition of four operands, a circular shifting to the left and an addition of two operands (See Fig.1). At the end of the process, an ultimate addition between the last states and the initial values is computed. From this results the MD5 digest.

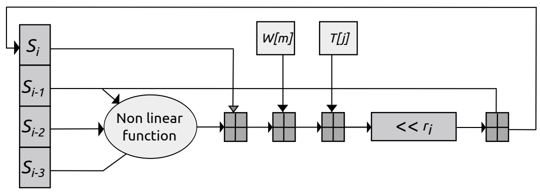


Figure 1: Hashing process of MD5.

3 ABOUT SAT ENCODING OF MD5

Even, if few works exists on this subject, a good modeling can be crucial to decrease the runtime of a SAT instance. This section is about the SAT encoding of the well-known MD5 function.

3.1 Some Logical Simplifications

SAT can be seen as a tool allowing to express any problem thanks to boolean equations. The solving of such an instance is achieved with a dedicated SAT engine (also named *solver*) that deals with reasoning techniques from I.A. Within this framework, information is treated by adding pertinent clauses and removing redundant information thanks to logical simplifications. Consider the formula $\mathcal{F}$ having the following clauses, and look at three interesting logical simplifications:

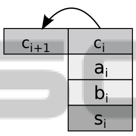
$$\begin{array}{ll} c_1 = (a \vee \overline{b}) & c_2 = (b \vee \overline{c}) \\ c_3 = (c \vee \overline{d}) & c_4 = (a \vee d \vee \overline{e}) \\ c_5 = (b \vee \overline{c} \vee f) & c_6 = (e \vee f \vee \overline{g}) \\ c_7 = (e \vee f \vee g) & \end{array}$$

- Observe that c_5 is equal to $(c_2 \vee f)$. In this case, c_5 contains as much information as c_2 and if c_2 is satisfied, necessarily c_5 is satisfied too. This well-know process is called a *subsumption*, and since c_2 *subsume* c_5, c_5 is withdrawn.
- Now focus on c_6 and c_7 and note these clauses differ only in the signedness of the variable g. This scheme is known as *resolution*. From this special scheme, a new clause named *resolvent* can be generated and consists of all the variables of the two previous clauses except the one that differs in signedness. Hence, we can add the clause $c_8 = (f \vee g)$. c_8 contains as much information as in c_6 and c_7 . Moreover, in that case, c_8 *subsumes* both c_6 and c_7 and could be helpful within the solving.
- Finally, note if $a =$FALSE then by propagation $b = c = d = e =$FALSE. In this sense, we have $(\overline{a} \Rightarrow \overline{e})$ and its corresponding clause $(a \vee \overline{e})$. This is a pertinent clause to add, because it represents also the relation $(e \Rightarrow a)$ and before adding it, from e it was impossible to deduce something about a.

Regarding MD5, the hashing process is composed of three operations: an addition of four operands, a circular shifting and an addition of two operands. We describe an encoding of each of these components into the most compressive expression. A noteworthy point by modeling MD5 into a SAT formula is that the generated instance keeps starting features from the initial cipher function, especially its behavior, its statistical data and its cryptanalytic weaknesses.

3.2 The Addition of Two Operands

To implement the addition of two operands, we directly express into SAT clauses the logical rules associated to the classical arithmetical addition. In this sense, and considering a simple adder circuit, this can be seen as two operations of implications: (operands) $\Rightarrow$ (sum) and (operands) $\Rightarrow$ (carries). A modeling of this simple adder is presented on Fig.2, where s_i corresponds of the sum of a_i and b_i, and c_{i+1} is the ouput carry generated by this addition. Arrows represent generation of carries, from one adding column to the corresponding next one. Based on this model, we set up the associated boolean truth table. In white cases, the input variables and in gray, the output variables. Finally, this table describes the inference rules that define the reasoning of an addition of two operands. Then, we conclude in a generation of SAT equations, where from each line of the table corresponds two clauses (one clause to each output variable). For instance, let c_i, a_i, b_i, c_{i+1}, s_i be five boolean variables, the second line of the table in Fig.2 can be read as c_i = 0, a_i = 0, b_i = 1 implies c_{i+1} = 0 and s_i = 1. This can be formulated as follows:



c_i	a_i	b_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Figure 2: Model of an addition of two operands and its coresponding truth table of two 1-bit operands.

$$(\overline{c_i} \wedge \overline{a_i} \wedge b_i \Rightarrow \overline{c_{i+1}}) \wedge (\overline{c_i} \wedge \overline{a_i} \wedge b_i \Rightarrow s_i)$$

And then:

$$(c_i \vee a_i \vee \overline{b_i} \vee \overline{c_{i+1}}) \wedge (c_i \vee a_i \vee \overline{b_i} \vee\ s)$$

This type of algebraic modelisation leads to lost the notion of temporality during the solving process. We propose to illustrate this crucial point with an instance of a 4 bits addition with holes. We denote by x_i the variable $x \in \{c,a,b,s\}$ at the index i.

Index	3	2	1	0	
car c	1	-	-	0	
op a	-	1	-	-	+
op b	-	-	1	-	
sum s	1	1	-	1	=

Figure 3: Holed addition of two binary operands.

Mathematically, looking at the Fig.3 it is quite easy to see $a \geq 100_{(2)}$, $b \geq 10_{(2)}$, either a or b is odd and $s \in \{((1)1101)_{(2)}, ((1)1111)_{(2)}\}$. However, it is not so easy to export something from the row of the carries because c depends of a, b and c. Note also that work in binary field leads to have a very detailed notion of carries while it's not the case from a larger scale. Moreover, in SAT, as the reasoning is logical we deduce some other bits. Firstly, from c_0 and s_0, we can deduce $c_1 = 0$ and from c_3, a_2 and s_3, we can infer $c_2 = b_2 = 1$. Accordingly, only four solutions stay possible:

$$(a,b) \in \{(0110,0111)_{(2)}, (1110,1111)_{(2)},$$
$$(0111,0110)_{(2)}, (1111,1110)_{(2)}\}$$

3.3 Modeling the Four Operands Addition

To implement the four operands addition we need to consider that two levels of carries is outputed. Therefore, as for the addition of two operands, carries in output must be considered as input in the next row. Consequently, the addition of four operands becomes an addition of six operands (See Fig.4).

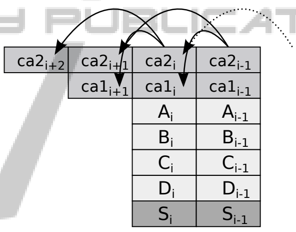


Figure 4: Model of an addition of four operands.

3.4 The Non-linear Functions

Anchored in each of the 64 steps, non-linear functions are another mean to give rise to chaos and strengthen to the MD5 cipher. Although the logical formalism of these functions is an additional constraint to the cryptanalysts, for us it's an advantage as we just need to add them to our modeling.

3.5 Modelisation of MD$\star$

Following the previously described method, thanks to all the bricks we define, we are able to model the whole MD5. These components are then cimented during the generation of clauses thanks to the variable encoding. Finally, the complete MD5 process consists in a SAT formula with 12,721 variables and 171,235 clauses. In the same way, we use this framework to generate SAT formula for the complete MD4 and some reduced instances for both MD5 and MD4.

4 RESULTS

In order to tackle the second preimage of MD⋆, we first generate a formula representing a reduced-step process of the hash function and secondly instance variables corresponding to a particular digest. Thanks to some of reasoning techniques described in the section 3 the formula is preprocessed to decrease its practical complexity. The resulting formula is then solved with a SAT engine. In our knowledge, the best practical result of a second preimage attack of reduced version MD⋆ is described in (De et al., 2007). They broke 1 round 12 steps of MD4 and 1 round 10 steps of MD5. With our approach, we break 1 round 15 steps for MD4 and 1 round 12 steps for MD5 in a few minutes. Our benchmarks have been achieved on a Westmere-EP 12 cores thanks to `plingeling` (Biere, 2010) for both MD4 and MD5. Hereafter, some results input/output values, in big endian, where the digests correspond to the ones obtained after the last addition of the MD⋆ algorithm.

1 round 15 steps on MD4

Fixed Hash:
`0x00000000 0x00000000 0x00000000 0x00000000`
Input found:
```
0x184937d5 0x6348828c 0x65e7547c 0x0201b903
0xba4f5298 0x12edc6df 0xbbe4a23e 0xa4c25972
0x5d9019f8 0x40bd880b 0x352f6960 0xbcb22ec4
0x43e0debc 0x0a4838d4 0xdf6a3b9f 0xcec88113
```

1 round 12 steps on MD5

Fixed Hash:
`0x01234567 0x89abcdef 0xfedcba98 0x76543210`
Input found:
```
0x0bd86c16 0x6dea158a 0x3fea904c 0x5930a4a1
0xf733709c 0x7e818951 0xdc6f481b 0x21f85c42
0x7a6b2051 0x09762af5 0xbf21286b 0xb70fe9bc
0xb6e76e81 0x0ba31a2c 0x71512697 0xbc2931af
```

We are also interested in the evolution of the runtime relative to the number of steps modeled. We draw these observations in Fig.5, achieved on an average of several instances.

For both MD4 and MD5 an exponential growth of the runtime according to the number of steps modeled is observed. In this manner, these hash functions are *a priori* relatively secure against preimage attacks by SAT SOLVER. However, the solving method presented in this paper is near from a brute-force attack in that the generated CNF just represents the hashing process with a fixed-hash. To improve our results, we should add some pertinent information to reduce the search space. In this way, we transpose the Dobbertin's algebraic attack (Dobbertin, 1996) to our reduced-step MD4 formulae.

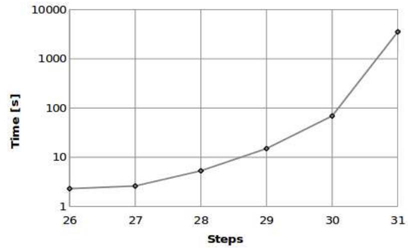


Figure 5: Runtime for solving reduced-step MD4.

Improving Attacks

A good mean to improve our preimage attack is to use some tricks of crytanalysts. As in (De et al., 2007), we used the Dobbertin's attack by instantiating some variables. More precisely, let be Q_i, the modified state at the step $i \in \{1, \ldots, 64\}$, and M_j the j^{th} input 32-bit sub-block, $j \in \{0, \ldots, 15\}$. We fixed:

- Q_{14}, Q_{15}, Q_{17} = `0x00000000`
- M_1, M_2, M_4, M_5, M_6, M_8, M_9, M_{10} = `0xa57d8667`

By propagation, we also have:

- Q_{18}, Q_{19}, Q_{21}, Q_{22}, Q_{23}, Q_{25}, Q_{26}, Q_{27} = `0x00000000`

The instance is preprocessed, reduced to its most compacted expression and `plingeling` is then applicated in order to find a solution. Our best result for MD4 was improved from 1 round 12 steps to 2 rounds 7 steps. Hereafter an example of a input/output value.

2 rounds 7 steps on MD4

Fixed Hash:
`0x00000000 0x00000000 0x00000000 0x00000000`
Input found:
```
0x321838cd 0x67867da5 0x67867da5 0x4bd844ff
0x67867da5 0x67867da5 0x67867da5 0x60babe30
0x67867da5 0x67867da5 0x67867da5 0x2e731890
0xb84655eb 0x1094c071 0xce0cfe36 0x0252233c
```

We focus on the evolution of the runtime of our solved instances and note the representative curve is very special. Indeed, there is a gap between steps 35 and 36, then the runtime is quasi-linear to 39 steps and a gap is again observed to up to 40 steps (unsolved after several hours of computation). This is due to the search space is correlated to the steps affected by the Dobbertin's attack. In fact, Dobbertin's attack fix some input sub-blocks that appear at steps 34, 35, 37, 38, 39, 41,... Note, steps

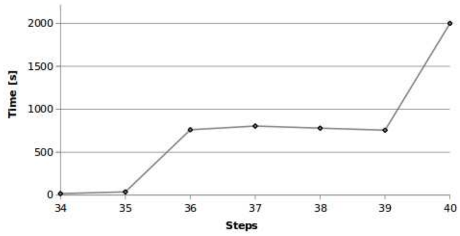


Figure 6: Runtime for reduced-step MD4 + Dobbertin's attack.

36 and 40 are not in this set that is why constraints are more numerous and the search space is decreased.

5 CONCLUSIONS

In this paper, we considered second preimage attack against MD⋆. Our work is based on logical cryptanalysis and described a two phases approach. As a result, we broke step-reduced instances for both MD4 and MD5 and improved results in existing practice (See Table.1). Since many other hash functions like RIPE-MD, TIGER, SHA-⋆ are built on the same schema as MD4, our angle of view is hopeful to be generalized. Indeed, we show an application of SAT as a great tool to cryptanalyse hash functions. Furthermore, our instance combined with added information led us to believe we could improve our attack. From our studies or from the literature about MD⋆, we can adapt and exploit weaknesses of hash functions to enrich our reverse engineering.

Table 1: Practical attacks on step-reduced MD4 and MD5 second preimage.

Type of CNF	In [*]	Our attack
MD4 Brute force	28 steps	31 steps
MD4 + info	up to 39 steps	up to 39 steps
MD5 Brute force	26 steps	28 steps
MD5 + info	X	X

[*] *(De et al., 2007)*

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