

SysML Parametric Models for Complex System Performance Analysis A Case Study

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Abstract: Parametric analysis is an essential tool in optimizing the performance of any system; it is, in particular, used to fine-tune key parameters in a system design process. In this paper, using a vehicle cruise control system as a non-trivial case study, we introduce a new approach for the performance parametric analysis of complex systems using SysML models and a parametric constraint solver. System requirements are taken into account to verify automatically whether the design solutions satisfy these requirements. This suggests that in order to reduce time and resources, it is possible to perform initial performance analysis in a modeling tool, just after the system functional and architectural analyses. Of course, once an approximate operating point has been determined using this approach, experiments in specialized simulation tools can be used to confirm and further refine the parameters of a system.

1 INTRODUCTION

SysML (SysML, 2010) is a visual modeling language used to support the specification, analysis, design, verification and validation of any engineered system. Taking advantage of SysML concepts such as requirements, blocks, flow ports, parametric diagrams and allocations, it is easy to model architectural and operational aspects of complex systems at various levels of abstraction.

In this paper, we perform a sensitivity analysis, which explores a parameter space, to find ideal operational parameters allowing the validation of alternative operational scenarios and system configurations. The impact of constraints on system properties and behaviours is analyzed in order to optimize global system performance. We use conjointly SysML parametric diagrams and a solver in support of analyzing system alternatives performances with respect to stakeholder requirements, derived system requirements and measures of effectiveness.

To illustrate our approach, we use a particular case study: a Cruise Control Engine system. Since many design parameters influence the operation of such a system, it is difficult to quantify their impact on the interactions within the system, and thus its performance. The purpose of this study is thus to investigate the consequences of varying some of these operating parameters on the performance of

the system and to report the results using more quantitative measures. The outcomes will be used to improve the understanding of the system operation and to optimize its performance by changing some operating parameters or improving components. Due to the large parameter space, and the complex, highly coupled hybrid nature of the different internal components of automatic systems, analysis is complicated and sometime more specialized simulation tools are necessary. The limits of our approach are also discussed.

The structure of the paper is the following. In Section 2, we describe briefly the functionalities of a cruise control engine, the related SysML requirement diagram and the dynamic model used in our case study. Section 3 presents the parametric analysis using the IBM Rhapsody SysML IDE, SysML parametric models and IBM add-on parametric constraint evaluator. Some preliminary parametric analysis of dynamic constraints in trade-off design activities and results of the case study are discussed in Section 4. Conclusions are outlined in Section 5.

2 CRUISE CONTROL SYSTEM

2.1 Functionalities

Cruise control is a system that automatically controls

the speed of a vehicle by maintaining a constant speed set by the driver. The implementation of a cruise control system may vary but respects in general the following principles. First, the cruise control system may need to be turned on before use: it passes from the disengaged state to the engaged state. While engaged, cruise control becomes activated when the driver sets the desired speed. A driver instruction, such as braking or throttle pedal depression, will put the cruise control on suspended mode. Of course, we can easily go back to the configuration before the suspension by using the resume function supported by almost all systems. Beside these operations, one can always increment or decrement the desired speed when the system is activated.

2.2 Requirements Model

Requirements analysis is the first step in the system design process, where stakeholder requirements are translated into system requirements that define what the system must do and how well it must perform. The result is a requirement diagram in which the requirements are classified hierarchically. Complex specifications are decomposed and categorized into simpler ones, leading to a better interpretation that will help with system verification and validation.

Figure 1 shows the requirement diagram for our cruise control system. In the performance requirement category, you can see some design constraints for cruise control systems taken from

Control Tutorials for Matlab (Michigan, 1997), with some of our own modifications. For example, when the motor yields a 1500 Newton force, the car must reach a maximum velocity of 30 m/s and be able to accelerate up to that speed in less than 5 seconds. Beside this, a 10% overshoot on the car speed and a 2% steady-state error are acceptable for the cruise control system. The above criteria can be used later to verify if the design solutions respect these requirements. To complete the requirement analysis phase, a model for system use cases must be built to establish traceability links between requirements and use cases provided, in order to ensure the coverage of functional and performance requirements by the use cases. These issues fall outside the scope of this paper.

2.3 Dynamic Model

Almost all cruise control systems follow the closed-loop control system principle. A sensor monitors the car speed and feeds data to a controller that adjusts the control as needed to maintain the reference speed. When the car goes uphill or downhill, the difference in speed is measured, and the throttle position changed to increase or decrease engine power, speeding or slowing respectively the vehicle. Feedback from measuring the car velocity allows the controller to dynamically compensate for changes to the car speed.

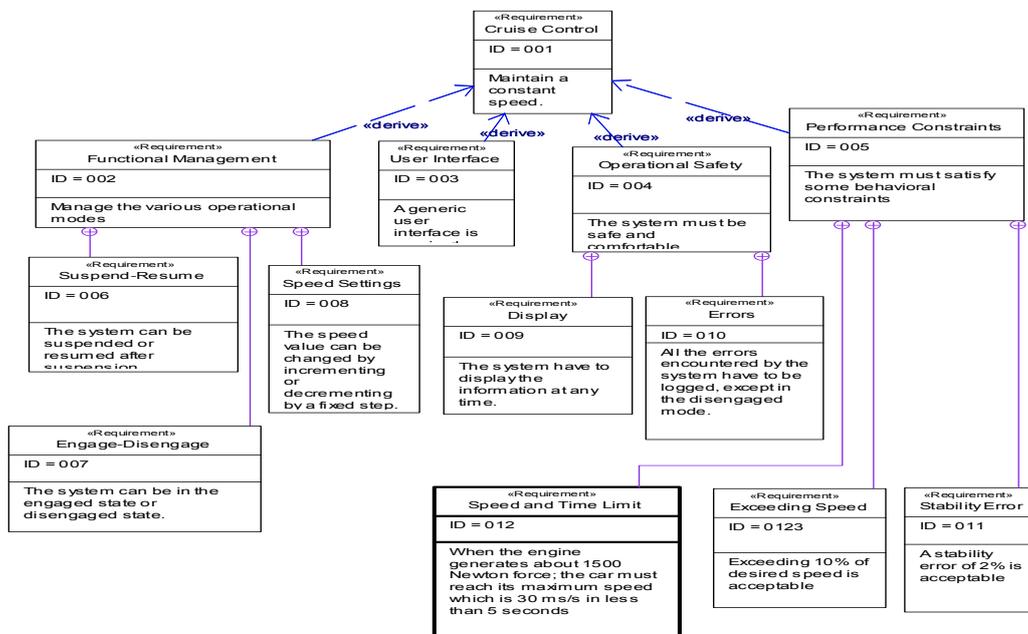


Figure 1: Requirement Diagram of a Cruise Control.

PID (proportional - integral - derivative) controller is widely used in industrial control system theory. The typical form of the PID algorithm is the following:

$$u(t) = k_p e(t) + k_i \int e(t) dt + k_d \frac{d}{dt} e(t)$$

where k_p , k_i and k_d are tuning parameters that have to be adjusted to optimum values to achieve the desired response while maintaining the stability of the system. The control signal u computed by the controller is used to rectify the throttle position and thus the torque delivered by the engine, generating a force that accelerates the vehicle.

To illustrate our case study, we used the dynamic model of a cruise control system found in Astrom and Murray's book (Astrom and Murray, 2010). Their proportional - integral (PI) controller has the form:

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau$$

$$e = v_r - v$$

where v_r is the desired speed, v the current speed and u the signal control. The torque T , controlled by the throttle position, delivered by the engine and transmitted through the gears and the wheels, depends also on engine speed ω :

$$T(\omega) = u T_m \left(1 - \beta \left(\frac{\omega}{\omega_m} - 1 \right)^2 \right)$$

where the maximum torque T_m is obtained at engine speed ω_m , and typical values are given for T_m , ω_m and β . The angular velocity is related to the speed through the expression:

$$\omega = \alpha_n v$$

And the driving force generated by the torque T is written as:

$$F = \alpha_n T(\omega)$$

Typical values of α_n (n is the gear ratio) for gears 1 through 5 are 40, 23, 16, 12 and 10. The car's motion is given by the following equation:

$$m \frac{dv}{dt} = F - F_d$$

where m is the mass, F_d the disturbance force which has three major components : gravity force (F_g), rolling friction force (F_r) and aerodynamic drag force (F_a). Different parameters such as the slope of the road, total mass of the car, gravitational constant, density of air, frontal area of the car as well as coefficients of various forces are taken into account in the model.

3 PARAMETRIC ANALYSIS

In this section, we provide information regarding the main components of our tools environment. Good tool integration is paramount here, since round-trip interoperability between SysML parametric models and an integrated solver is a key requirement of our approach.

3.1 SysML Parametric Models

SysML provides mechanisms and constructs necessary to successfully describe all the structural and behavioural specifications and constraints of a model of a system. In the design phase, it is essential to annotate these models with qualitative and quantitative requirements, known as non-functional properties, aiming at verifying and validating the temporal behaviour, power estimation and other various constraints.

Block diagrams are the natural approach used by SysML for expressing system-level models, providing a standardized form of representation for both the structure of a system and the equations that characterize its dynamic and its functional and behavioural constraints. Blocks are extended into constraint blocks that can be used in parametric diagrams, which enable users to model equations in terms of constraints in SysML, establishing a network of relations among the properties of a system (Peak, et al., 2007). These mathematical expressions can represent the physical properties of a system (e.g., relevant physics laws) or non-functional properties (e.g., cost, risk, performance, reliability, etc).

Simulation and system parametric analysis then can be realized to check that a system definition meets a certain system requirement, which can be modelled explicitly using the SysML "verify" dependency stereotype. Furthermore, some non-functional requirements can be written as constraints so they can be automatically verified by an integrated solver. Instead of using specific simulation tools such as Simulink or Modelica, we decided to exploit a lightweight solver already integrated in a SysML supported toolset to carry out parametric analyses.

3.2 IBM Rhapsody Toolset

IBM Rational Rhapsody(Hoffmann, 2010) is a collaborative, model-based systems engineering development platform providing simulation for early requirement, architecture and behavioural validation.

We decided to use it in our research because it provides tools to dynamically analyze and execute SysML parametric diagrams to assist in trade study analysis. The integrated Parametric Constraint Evaluator is a Rhapsody add-on that allows the evaluation of parametric diagrams via a Computer Algebra System (CAS) that solves the corresponding mathematical expressions. Matlab Toolbox and Maxima are two CAS supported by IBM Rhapsody; given its ready availability, we chose the open-source Maxima for this work. Maxima manipulates symbolic and numerical expressions, and includes many operations such as differentiation, integration, Taylor series expansion, Laplace transform, ordinary differential equation solving, systems of linear equations manipulation, etc. It yields high precision numeric results, using exact fractions and variable precision floating point numbers. Thanks to Maxima, PCE is able to compute the values that satisfy mathematical constraints or to solve constraints to minimize or maximize a value of an attribute for linear algebraic equations. Besides this, it can also produce graphs showing how values behave over time or over a range of values of other parameters. These possibilities allow a system engineer to analyze the behaviours of the system, to validate the constraints and to perform trade studies. However, some solver limitations that are discussed later led us to an interesting debate about the choice of lightweight simulation tools such as parametric diagrams with respect to dedicated tools such as Simulink or Modelica.

4 SENSIBILITY ANALYSIS

Instead of using specialized simulation tools, we have tried to use the SysML parametric diagram coupled with a solver to carry out 2 different simulations: an open loop cruise control as in (Michigan, 1997) and a closed loop with the dynamic model in Section 2.3. Throughout our experiments, the limits of the solver integrated in Rhapsody were encountered and are discussed here.

4.1 Open Loop Cruise Control

For an open loop experiment with linear differential equations, our solver with Maxima succeeded at producing desired results as with Matlab from the parametric diagram in Figure 2.

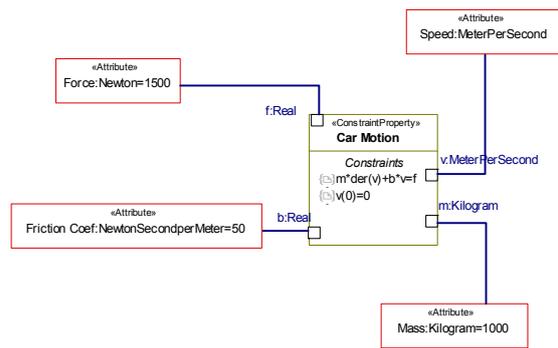


Figure 2: Parametric Diagram of an Open Loop Cruise Control.

We used here the same physical setup and system equations as in (Michigan, 1997): friction opposing the motion of the car is proportional to the car speed. With an initial condition, a graph is generated from the constraint view referencing the corresponding parametric diagram, which is shown in Figure 3. From the plot, we see that the car needs more than 100 seconds to reach the steady-state speed of 30m/s, which does not satisfy the performance requirement about rise time (less than 5 seconds).

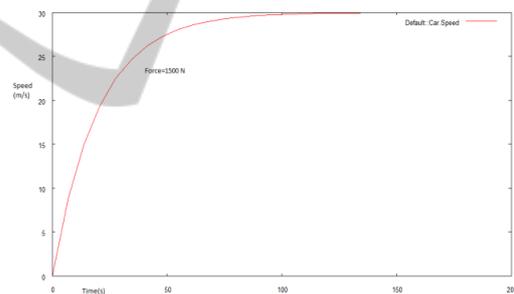


Figure 3: Speed vs. Time in Open Loop Cruise Control (generated by PCE/Maxima).

4.2 Closed Loop with Dynamic Model

Our second implementation is the dynamic model of the cruise control system described in Section 2.3. The corresponding SysML parametric diagram is provided in Figure 4. Each parametric constraint represents an equation dealing with the comparator, the controller, the torque related to the throttle position, the generated force, the car motion and the different disturbance forces. Constants are given in the value properties. Almost all physical units are available in the Systems Engineering (SE) profile of Rhapsody.

Some simplifications have been made due to software limitations: not all the possibilities of Maxima are fully implemented. As PCE does not

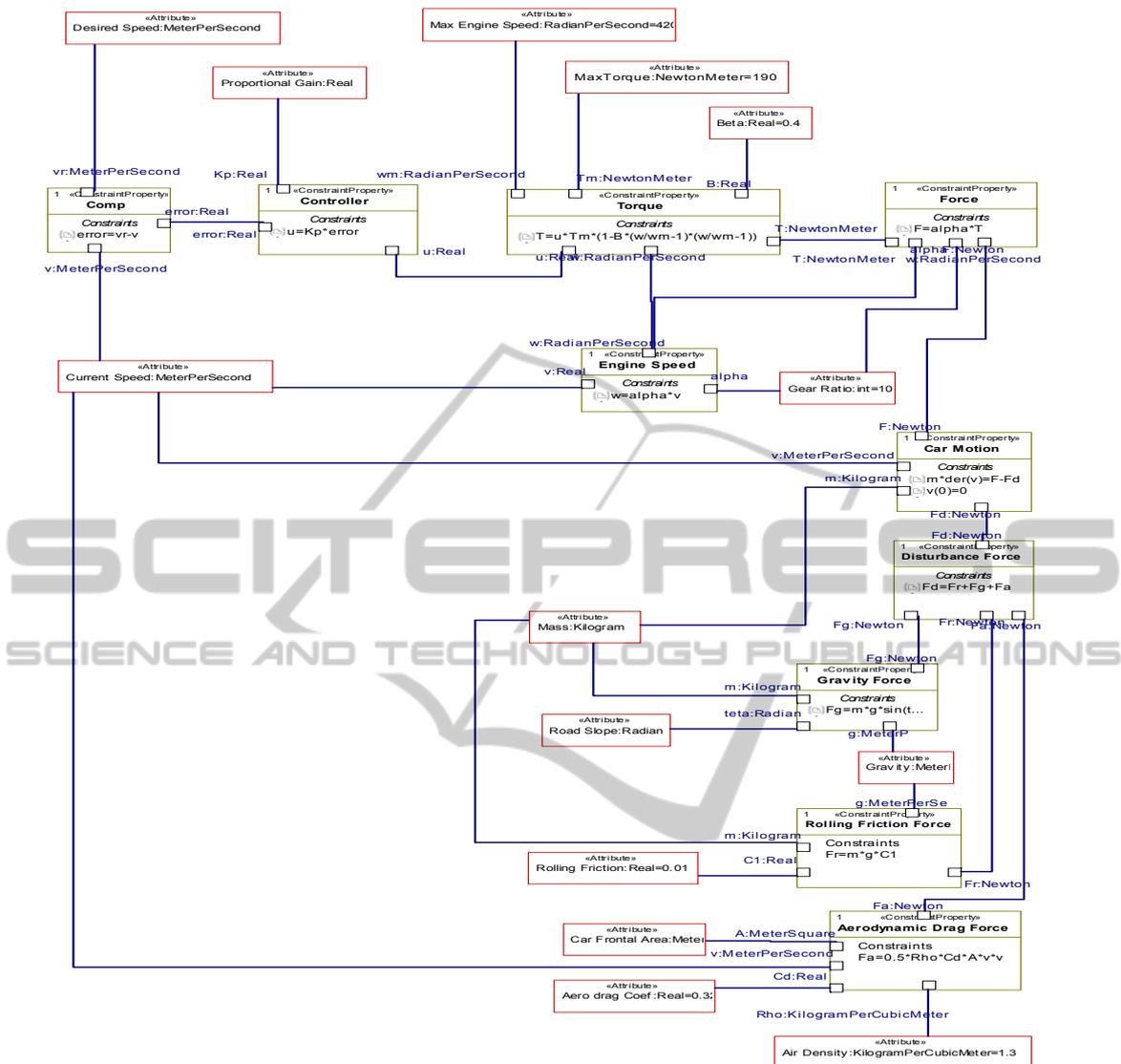


Figure 4: Parametric Diagram of the Dynamic Model.

support the integral operation or the Laplace transform, we had to give up the integral part of the controller; only proportional gain is taken into account in the controller. That gives us the following nonlinear differential equation for the system:

$$m \frac{dv}{dt} = \alpha_n k_p (v_r - v) T_m \left(1 - \beta \left(\frac{\alpha_n v}{\omega_m} - 1 \right)^2 \right) - m g C_r - 0.5 \rho C_d A v^2 - m g \sin \theta \quad (1)$$

where the last three terms represent respectively the three disturbance forces : rolling friction, aerodynamic drag and gravity force. Every variable of the equation, except speed (v) and time (t), is either constant or of known values.

A constraint view referencing the above parametric diagram can be generated, as shown in

Figure 5. Normally, through the fixing of some values in the spreadsheet, such as the value for the proportional gain, the road slope, the car mass, the desired speed..., we can ask Rhapsody to evaluate the system in order to detect inconsistent constraints or to display graphs that show interesting relationships, for instance between time and speed.

Unfortunately, since Maxima only gives us the exact analytical solution for the nonlinear differential equation (1), which is not exploitable, PCE cannot be used here. So, we decided to fall back to using Scilab, an open-source solver using numerical solutions, which allowed us to plot graphs that represent the relationship between speed and time from Equation (1), when varying some system

parameters. Some experimental results are given in Figure 6, 7 and 8.

In Figure 6, we see that with $k_p = 1$, the car almost reaches the desired speed which is 30 m/s in less than 5 seconds, so the performance requirements about steady-state error and rise time are verified.

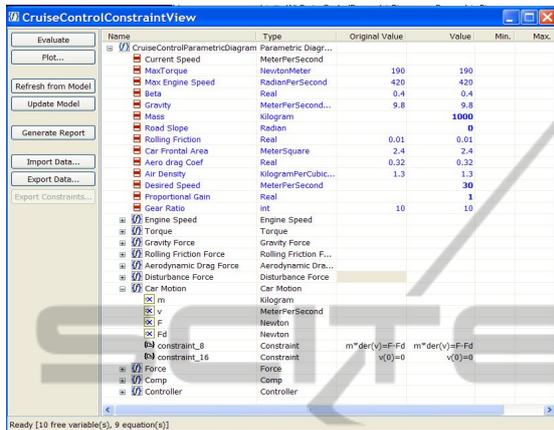


Figure 5: Constraint View of the Dynamic Model.

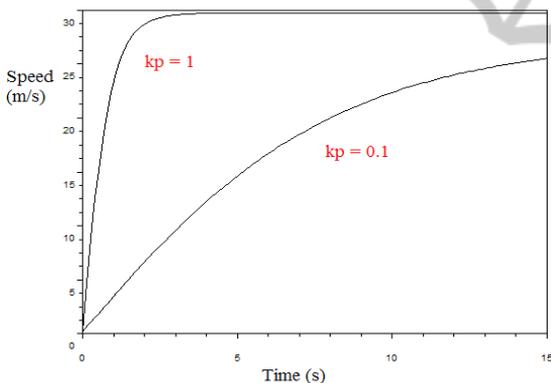


Figure 6: Speed vs. Time with Different Proportional Gains (generated by Scilab).

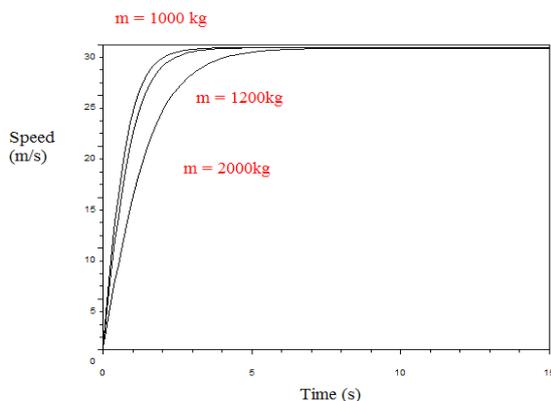


Figure 7: Speed vs. Time with Different Masses (generated by Scilab).

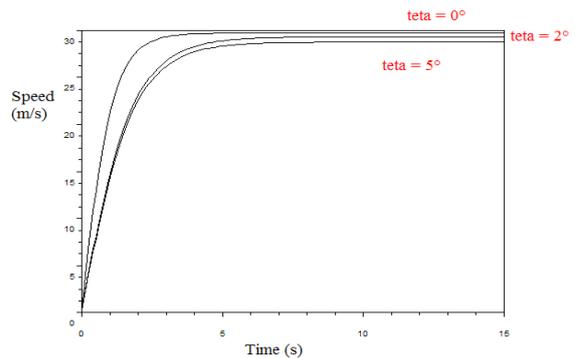


Figure 8: Speed vs. Time with Different Road Slopes (generated by Scilab).

With $k_p = 0.1$, the system also arrives at the desired target speed, but with a much longer rise time. In Figure 7, by varying the total mass of the car and keeping the value 1 for k_p , we get different traces in which only the trace for $m = 1000$ kg verifies the condition of rise time less than 5 seconds. For the other values of mass, further parameters must be adjusted to achieve satisfying results. In Figure 8, by varying the road slope while keeping the same value for k_p and m (1 and 1000kg respectively), we get different outcomes. Since the vehicle has to provide much more effort to go uphill ($\theta = 2^\circ$ and 5°), with the same value of k_p , it cannot reach the desired speed of 30 m/s.

The above results show that more experiments must be run in order to find out optimum values for the system parameters. All possible combinations of the different value ranges for mass, road slope, desired speed, etc must be taken into account. And, of course, the integral and derivative gains are necessary if we want to have more precision with our cruise control system.

5 CONCLUSIONS

Due to the large parameter space and the highly coupled hybrid nature of the different internal components of automatic systems, their analysis is usually complex and time consuming. In the case of performance analysis, rather than performing complete parametric analyses of complex systems in the field, our paper suggests that initial parametric performance analyses can be performed, in the lab, using SysML parametric diagrams as main tool, while at the same time leveraging conventional modeling and simulation tools including spreadsheets, math solvers, finite element analysis, discrete event solvers and optimization tools. This

approach allows for greater repeatability and requires less time and resources. Of course, once an approximate operating point has been determined using this approach, field experiments with more specialized tools (Simulink, OpenModelica, ...) can be used to confirm and further refine the parameters of a system.

Some works have been done by an OMG group for the integration of SysML and Modelica to profit the strength of two complementary modeling languages: the descriptive power from SysML and the analytic and computational power from Modelica (Johnson and Jobe and Paredis and Burkhart, 2007), (Paredis, et al., 2010). In fact, Modelica is well suited for representing differential algebraic equations to model the flow of energy, materials, signals ... in complex system. Transformation specification has been proposed to provide a bi-directional mapping between the two languages. However, the requirement models of SysML are not considered in this mapping. In our approach, we can integrate requirement information directly into parametric diagrams to validate the design. By rewriting requirement constraints in a formal language such as OCL or a temporal logic language, we can put them in the parametric diagrams and then formal methods can be used to verify if there are errors in system design.

The preliminary results presented in this paper are quite encouraging. With a lightweight system, we achieved results similar to those provided using specialized tools and the perspective to be able to combine directly in the same tool structural and behavioural specifications with requirement constraints to validate the design process is promising.

Nevertheless, this solution presents some limitations: although parametric diagrams are non-causal, they do not separate effort and flow variables, which is a fundamental issue when modeling physical systems. For example, Modelica (using flow) and VHDL-AMS (using across/through) contain such constructs. Beside this, although the Rhapsody tool is well suited for implementing the first steps of complex system development process, i.e., requirement analysis, system functional analysis and design synthesis, the architecture mismatch in its Parametric Constraint Evaluator integrated with Maxima should be corrected to represent more complicated mathematical relations. For instance, a solver providing numerical solutions for nonlinear

differential equations and supporting Laplace or Z transforms (Wescott, 2012) would be highly appreciated.

The next step of our work is to make a complete survey of different SysML parametric solving tools such as ParaSolver for Artisan Studio, ParaMagic for MagicDraw, etc. in order to compare how far these tools are able to support complex system models. Actually, the tutorial examples given by these tools are rather not very complicated.

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