Flocking for Networks of Mobile Robots with Nonlinear Dynamics

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Keywords: Flocking, Nonholonomic Robots, Multi-robot Networks, Near-Identity Transformation, Nonlinear Systems.

Abstract: In this paper, we address the problem of flocking for networks of nonholonomic mobile robots with nonlinear dynamics given that a flocking algorithm for particles is known. Our approach relies on the use of near-identity change of coordinates that transform the nonlinear dynamics of the robot to a partially-linear normal form with a double-integrator linear subsystem. The flocking algorithm is then applied to the linear part. The inverse of the near-identity transformation provides the flocking algorithm for the networked nonholonomic robots. We prove the emergence of flocking behavior for robotic networks with nonlinear dynamics according to the formal definition of flocking in Olfati-Saber's flocking paper (TAC '06). Simulation results are provided for large-scale networks of two-wheeled robots with nonlinear dynamics as models of Khepera-III robots that demonstrate the effectiveness of our proposed transformation and algorithm.

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1 INTRODUCTION

Flocking is a form of collective behavior of a large number of interacting mobile agents with a unified objective (e.g. moving towards the same direction) with an elegant lattice shaped spatial order that reflects cohesive group motion (Olfati-Saber, 2006). Group behaviors such as flocking, schooling, and herding are one of the few research subjects that are of great interest to physicists (Vicsek et al., 1995; Shimoyama et al., 1996; Toner and Tu, 1998; Levine et al., 2001), computational social scientists (Helbing et al., 2000), animal behavior experts (Shaw, 1975; Potss, 1984; Partridge, 1982; Parrish et al., 2002), applied mathematicians (Mogilner and Edelstein-Keshet, 1999; Topaz and Bertozzi, 2004; D'Orsogna et al., 2006; Cucker and Smale, 2007), computer scientists (Reynolds, 1987), and robotics and control scientists (Olfati-Saber, 2006) alike.

Some of the existing results on flocking and formation control for groups of nonholonomic agents can be found in (Regmi et al., 2005; Dimarogonas and Kyriakopoulos, 2006a; Dimarogonas and Kyriakopoulos, 2006b; Dimarogonas and Kyriakopoulos, 2007; Moshtagh and Jadbabaie, 2007; Dong and Farrell, 2008; Chopra et al., 2008; Tanner et al., 2005). In this paper, we extend the flocking algorithms from (Olfati-Saber, 2006) for groups of particles with double-integrator dynamics $\ddot{q}_i = u_i$ to mobile robots with nonlinear dynamics. We address the problem of flocking for networked nonholonomic robots with nonlinear dynamics shown in Fig. 1.

Our approach to flocking for dynamic nonholonomic robots relies on application of near-identity diffeomorphisms (Olfati-Saber, 2002) that transform the nonlinear dynamics of the robot into a partially-linear normal form with a double-integrator linear part and applying the flocking algorithm to the reduced normal form. The flocking algorithm for the original network of mobile robots can be obtained using the inverse of the near-identity diffeomorphism.

The aforementioned reduction method is more general and can be applied to the design of flocking algorithms for other types of mobile robots with nonlinear dynamics if the corresponding near-identity transformation of the robot can be found. The main challenge is to prove that the resulting flocking algorithm for robotic networks with nonlinear dynamics leads to emergence of flocking behavior according to the formal definition of flocking in (Olfati-Saber, 2006). Establishing the emergence of flocking is one of our main results.

Here is an outline of the paper: Some background and notation on flocking are provided in Section 2. The nonholonomic robot and its dynamics are presented in Section 3, which also details a coordinate change using near identity diffeomorphism. Our main theoretical result is presented in Section 5. Simulation results are provided in Section 6. Finally, concluding remarks are made in Section 7.

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2 FLOCKING ALGORITHMS FOR PARTICLES

In this section, we provide some background and notations on Olfati-Saber's flocking algorithms and theory (Olfati-Saber, 2006). Consider a group of n particles called α-*agents* moving in $\mathbb{R}^m$ with the following dynamics

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i \end{cases} \tag{1}$$

where $q_i, p_i, u_i \in \mathbb{R}m$ denote the position, velocity, and control of agent i, respectively. Let us denote the conformation of all α-agents by $q = \text{col}(q_1, \ldots, q_n) \in \mathbb{R}^{mn}$. The *proximity network* of the agents is a dynamic graph $G(q) = (V, E(q))$ with the set of nodes $V = \{1, 2, \ldots, n\}$ and the set of edges

$$E(q) = \{(i, j) \in V \times V : \|q_j - q_i\| \leq r\}.$$

The adjacency matrix of $G(q)$ is a non-negative matrix $A(q) = [a_{ij}(q)]$ with smooth elements $0 \leq a_{ij}(q) \leq 1$ defined in (Olfati-Saber, 2006). The set of neighbors of agent i is defined as $N_i(q) = \{j : (i, j) \in E(q)\}$. A conformation q is called a *quasi* α-*lattice* if every agent is approximately equally distanced from all of its neighbors, i.e.

$$\exists \varepsilon, d > 0 : -\varepsilon \leq \|q_j - q_i\| - d \leq \varepsilon, \forall j \in N_i(q).$$

An α-*lattice* is a quasi α-lattice with $\varepsilon = 0$. The formal definition of flocking is given in (Olfati-Saber, 2006) as follows:

Definition 1. (flocking) A group of α-agents with the trajectory $(q(\cdot), p(\cdot))$ perform *flocking behavior* over a time interval $[t_0, t_f)$, if every agent i only communicates with its neighbors N_i and the trajectories of the agents satisfy the following properties over the period $[t_0, t_f)$:

1. Cohesion: $\exists \rho > 0 : \|q_i(t) - q_c(t)\| \leq \rho$ and $q_c(t) = \frac{1}{n}\sum_{i=1}^{n} q_i(t)$.
2. Self-assembly of a quasi α-lattice after some finite time;
3. Self-assembly of a connected proximity network $G(q(t))$ after some finite time;
4. Self-alignment: a relatively small velocity mismatch

$$\exists 0 < \varepsilon \ll 1 : K(p) = \frac{1}{2} \sum_{(i,j) \in E(q)} \|p_j(t) - p_i(t)\|^2 < \varepsilon$$

It is shown in (Olfati-Saber, 2006) that Algorithm 2 leads to emergence of flocking behavior. This flocking algorithm can be described as follows:

$$u_i = f_i^\alpha + f_i^\gamma \tag{2}$$

or in a more explicit form, it can be expressed as

$$u_i = \underbrace{\sum_{j \in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma)\mathbf{n}_{ij}}_{\text{gradient-based term}} + \underbrace{\sum_{j \in N_i} a_{ij}(q)(p_j - p_i)}_{\text{consensus term}} + f_i^\gamma \tag{3}$$

where $\mathbf{n}_{ij} = (q_j - q_i)/\sqrt{1 + \varepsilon\|q_j - q_i\|^2}$ is a bounded vector along the line connecting q_i to q_j and $\varepsilon \in (0, 1)$. Moreover $\|s\|_\sigma = \frac{1}{\varepsilon}(\sqrt{1 + \varepsilon\|s\|^2} - 1)$ denotes the σ-*norm* of vector s.The potential function of the group is defined as $V(q) = \sum_{j \neq i} \psi_\alpha(\|q_j - q_i\|_\sigma)$ where $\psi_\alpha(s) = \int_{\|d\|_\sigma}^{s} \phi_\alpha(h)dh$. The scalar function $\phi_\alpha(s)$ is defined in eq. (15) of (Olfati-Saber, 2006). f_i^γ is a linear tracking controller

$$f_i^\gamma = -c_1^\gamma(q_i - q_r) - c_2^\gamma(p_i - p_r); \quad c_1^\gamma, c_2^\gamma > 0 \tag{4}$$

where (q_r, p_r) is the state of the γ-*agent*, i.e. a virtual moving rendezvous point (with double integrator dynamics). The objective of all α-agents is to asymptotically track a single γ-agent (or common goal).

In the presence of obstacles, Algorithm 3 in (Olfati-Saber, 2006) is applicable. Let us refer to the projection of an α-agent on the boundary of a convex obstacle (e.g. a wall, or an ellipse) as a β-*agent*. Obstacle avoidance can be achieved by establishing a repulsive potential between an α-agent and its β-agent projection on a neighboring obstacle. The flocking algorithm with obstacle avoidance capabilities is in the form

$$u_i = f_i^\alpha + f_i^\beta + f_i^\gamma \tag{5}$$

where f_i^β is the obstacle avoidance force and is given by

$$\begin{aligned} f_i^\beta = {} & c_1^\beta \sum_{k \in N_i^\beta} \phi_\beta(\|\hat{q}_{i,k} - q_i\|_\sigma)\hat{\mathbf{n}}_{i,k} \\ & + c_2^\beta \sum_{j \in N_i^\beta} b_{i,k}(q)(\hat{p}_{i,k} - p_i); \quad c_1^\beta, c_2^\beta > 0 \end{aligned} \tag{6}$$

Here, $\hat{q}_{i,k}$ and $\hat{p}_{i,k}$ with $(i, k) \in V_\alpha \times V_\beta$ denote the position and velocity of a β-agent generated by an α-agent with state (q_i, p_i) on an obstacle. The functions $b_{i,k}(q)$ and $\phi_\beta(s)$ are defined in equations (55) and (56) respectively of (Olfati-Saber, 2006) and $\hat{\mathbf{n}}_{i,k} = (\hat{q}_{i,k} - q_i)/\sqrt{1 + \varepsilon\|\hat{q}_{i,k} - q_i\|^2}$ Consider a wall obstacle (a hyperplane) with a unit normal $\mathbf{a}_k$ passing through a point y_k. The projection matrix for this obstacle is $P = I - \mathbf{a}_k\mathbf{a}_k^T$. The position and velocity of the β-agent are given by

$$\hat{q}_{i,k} = Pq_i + (I - P)y_k, \hat{p}_{i,k} = Pp_i. \tag{7}$$

3 NONHOLONOMIC ROBOT AND TRANSFORMATION

In this section, we study a two-wheeled mobile robot with nonlinear dynamics shown in Fig. 1 that can be used as the model of commercial robots such as Khepera-III. After presenting the nonlinear dynamics of the mobile robot, we will demonstrate how the robot dynamics can be transformed into a partially-linear system using *near-identity diffeomorphisms* (Olfati-Saber, 2002) where the linear part is a particle model (double integrator) and therefore flocking algorithms are applicable to the transformed system. Inversion of the transformation provides flocking algorithms for networked mobile robots with nonlinear dynamics.

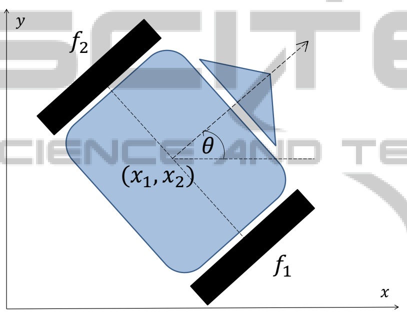

Figure 1: Configuration variables and controls of a two-wheeled mobile robot.

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3.1 Dynamics of the Mobile Robot

Fig. 1 shows a two-wheeled mobile robot with the following nonlinear dynamics

$$\begin{cases} \dot{x}_1 = v\cos\theta \\ \dot{x}_2 = v\sin\theta \\ \dot{\theta} = \omega \\ m\dot{v} = f_1 + f_2 \\ J\dot{\omega} = \bar{r}(f_1 - f_2) \end{cases} \tag{8}$$

where $(x_1, x_2)^T \in \mathbb{R}$ is the position of the center of mass, θ is the orientation angle of the robot, f_1 and f_2 are control forces and m, J, and $\bar{r}$, are mass, moment of inertia, and half of the length of the wheel-base. Defining the following change of control

$$\begin{cases} u_1 = (f_1 + f_2)/m \\ u_2 = \bar{r}(f_1 - f_2)/J \end{cases} \tag{9}$$

the dynamics of the robot in (8) can be rewritten as

$$(\dot{x}_1, \dot{x}_2, \dot{\theta}, \dot{v}, \dot{\omega}) = [v\cos\theta, v\sin\theta, \omega, u_1, u_2]^T \tag{10}$$

Notice that for all $t \in \mathbb{R}$, the velocities $\dot{x}_1$ and $\dot{x}_2$ satisfy the following first-order nonholonomic constraint

$$\dot{x}_2\cos\theta - \dot{x}_1\sin\theta = 0 \tag{11}$$

Denoting $x = (x_1, x_2)^T$, one can write the dynamics of the robot in a coordinate-independent manner as

$$\begin{cases} \dot{x} = (Re_1)v \\ \dot{v} = u_1 \\ \dot{R} = R\hat{\omega} \\ \dot{w} = u_2 \end{cases} \tag{12}$$

where $e_1 = (1,0)^T$, $e_2 = (0,1)^T$, Re_k is the kth column of $R \in SO(2)$ for $k = 1, 2$ and

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \hat{\omega} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \tag{13}$$

3.2 Near-identity Transformation

The dynamics of the nonholonomic mobile robot in (12) can be transformed into a cascade of a double-integrator and attitude dynamics. This can be performed using a nonlinear change of coordinates called a *near-identity transformation* and introduced in (Olfati-Saber, 2002). This transformation is given by

$$z = x + \lambda(Re_1) \tag{14}$$

where λ is relatively small constant.

By direct differentiation, we get

$$\begin{aligned} \dot{z} &= \dot{x} + \lambda(\dot{R}e_1) \\ &= (Re_1)v + \lambda(R\hat{\omega}e_1) \\ &= (Re_1)v + \lambda\omega(Re_2) \\ &= R_\lambda p \end{aligned} \tag{15}$$

with $p = (v, \omega)^T \in \mathbb{R}^2$ and $R_\lambda = [(Re_1)|\lambda(Re_2)]$. The matrix R_λ can be explicitly expressed as

$$R_\lambda = \begin{bmatrix} \cos(\theta) & -\lambda\sin(\theta) \\ \sin(\theta) & \lambda\cos(\theta) \end{bmatrix} \tag{16}$$

Clearly , $\det(R_\lambda) = \lambda$ and thus R_λ is an invertible matrix if and only if $\lambda \neq 0$. For $\lambda \neq 1$, R_λ is not a $SO(2)$ matrix. Notice that $\dot{p} = u$, thus

$$\ddot{z} = R_\lambda u + \dot{R}_\lambda p = \tau \tag{17}$$

where τ is the *new control* input for the linear system

$$\ddot{z} = \tau. \tag{18}$$

To find an explicit relation between u and τ , we need to calculate $\dot{R}_\lambda p$ as follows

$$\begin{aligned}\dot{R}_\lambda p &= [R\hat{\omega}e_1|\lambda R\hat{\omega}e_2].(v,\omega)^T \\ &= [\omega Re_2|-\lambda\omega Re_1](v,\omega)^T \\ &= v\omega Re_2 - \lambda\omega^2 Re_1 \\ &= R(-\lambda\omega^2, v\omega)^T\end{aligned} \tag{19}$$

Hence, $u = (u_1, u_2)^T$ can be obtained as

$$u = R_\lambda^{-1}\tau - R_\lambda^{-1}R(\lambda\omega^2, v\omega)^T \tag{20}$$

On the other hand, we have

$$R_\lambda^{-1}R = \begin{bmatrix}1 & 0 \\ 0 & \dfrac{1}{\lambda}\end{bmatrix} \tag{21}$$

thus

$$u = R_\lambda^{-1}\tau + \begin{bmatrix}\lambda\omega^2 \\ \dfrac{-v\omega}{\lambda}\end{bmatrix} \tag{22}$$

The complete dynamics of the mobile robot in new coordinates in given by

$$\begin{cases}\dot{z} = w \\ \dot{w} = \tau \\ \dot{R} = R\hat{\omega} \\ \dot{\omega} = e_2^T u = e_2^T R_\lambda^{-1}\tau - \dfrac{v\omega}{\lambda}\end{cases} \tag{23}$$

with a partially-linear (z, w)-subsystem. We apply our particle-based flocking algorithms to this partially-linear subsystem of the transformed dynamics of the mobile robot.

4 FLOCKING FOR NONLINEAR ROBOTIC NETWORKS

Consider a network of robots with the dynamics in (8). To implement the flocking algorithm (2) on this network, we perform a change of coordinates as in the previous section obtaining the double integrator for the ith robot

$$\begin{cases}\dot{z}_i = w_i \\ \dot{w}_i = \tau_i\end{cases} \tag{24}$$

which is the same as the particle dynamics in (1) after relabeling of the variables. Applying the following algorithm from (2)

$$\tau_i = f_i^\alpha + f_i^\gamma \tag{25}$$

leads to flocking behavior for networked mobile robots with nonlinear dynamics. For group motion in an environment with obstacles, collision-free flocking behavior is achieved using the application of the algorithm in (5), i.e.

$$\tau_i = f_i^\alpha + f_i^\gamma + f_i^\beta \tag{26}$$

5 ANALYSIS OF FLOCKING FOR ROBOTIC NETWORKS

Our main result is that if the near-identity transformation of mobile robots (oriented particles) perform flocking, then the group of mobile robots perform flocking as well—given some assumptions. Here is our main analytical result:

Theorem 1. *Consider n mobile robots with nonlinear dynamics given in* (12) *applying the flocking algorithm* (25). *After applying a near-identity transformation*

$$\begin{cases}z_i = x_i + \lambda(R_i e_1) \\ w_i = v_i(R_i e_1) + \lambda\omega_i(R_i e_2)\end{cases} \tag{27}$$

suppose that the n particles with linear dynamics

$$\begin{cases}\dot{z}_i = w_i \\ \dot{w}_i = \tau_i\end{cases} \tag{28}$$

perform flocking behavior (as defined in Section 2) so that the angular velocity of all robots remain uniformly bounded, i.e. $|\omega_i| \le \bar{\omega}$. *Then, the n mobile robots with nonlinear dynamics perform flocking behavior as well.*

Proof. To establish that the mobile robots with nonlinear dynamics perform flocking behavior, we need to prove that the four conditions of flocking in Definition 1 are satisfied, particularly, achieving a relatively small velocity mismatch

$$K(\nu) = \frac{1}{2}\sum_{(i,j)\in E(x)} \|\nu_j - \nu_i\|^2 \ll 1$$

where $\dot{x}_i = \nu_i$.

Suppose that the particle-based model of the robots form a cohesive trajectory over the time interval $[t_0, t_f])$ with a cohesion radius of $\rho > 0$. Then, for any given $\lambda \ge 0$, a slight perturbation of every particle $x = z - \lambda(Re1)$ can be geometrically contained in a slightly larger ball of radius $\rho_\varepsilon = \rho + \varepsilon(\lambda)$ and thus the trajectory of the group of mobile robots is cohesive. Formally, this can be shown by defining a compact set containing all the balls around n

Note that the topology of the proximity network of the mobile robots is the same as the proximity network of their induced particle-based models after near-identity transformation. Therefore, connectivity of the latter implies connectivity of the former. This proves property (3).

We prove that the mobile robots form a quasi α-lattice if their perturbed particle-based models (near-identity transformations) form a quasi α-lattice. Define the unit vectors $\mathbf{r}_i = R_i e_1$ and $\mathbf{s}_i = R_i e_2$. For a pair of neighboring robots i and j and a small parameter

$\lambda > 0$, we have

$$z_i = x_i + \lambda \mathbf{r}_i,\ z_j = x_j + \lambda \mathbf{r}_j$$

and

$$\begin{aligned} \|x_j - x_i\| &= \|(z_j - z_i) + \lambda(\mathbf{r}_j - \mathbf{r}_i)\| \\ &\leq \|z_j - z_i\| + 2\lambda \leq d + (\varepsilon + 2\lambda) \end{aligned}$$

Setting $\varepsilon_1 = \varepsilon + 2\lambda \ll 1$ guarantees that $\|x_j - x_i\| \leq d + \varepsilon_1$. Similarly, one can show $\|z_j - z_i\| \leq \|x_j - x_i\| + 2\lambda$. Hence

$$\begin{aligned} \|x_j - x_i\| &= \|z_j - z_i\| - 2\lambda \\ &\geq d - (\varepsilon + 2\lambda) \end{aligned}$$

In other words, we get $d - \varepsilon_1 \leq \|x_j - x_i\| \leq d + \varepsilon_1$, or the mobile robots form a quasi α-lattice.

Finally, we need to show that if the velocity mismatch is relatively small ($K(w) < \varepsilon \ll 1$) for the particle-based models of robots, then the velocity mismatch for the robots $K(\nu)$ will also be relatively small. Note that the velocities of the particles and robots satisfy $w_i = \nu_i + \lambda \omega_i \mathbf{s}_i$ where $\mathbf{s}_i$ is a unit vector. Let $n_e = |E|$ denote the number of edges of the proximity network $G(q)$—this is a finite number and equal to $\sum_i k_i$ where k_i is the degree of node i. We have

$$\begin{aligned} K(\nu) &= \frac{1}{2} \sum_{(i,j)\in E} \|\nu_j - \nu_i\|^2 \\ &= \frac{1}{2} \sum_{(i,j)\in E} \|(w_j - w_i) + \lambda(\omega_j \mathbf{s}_j - \omega_i \mathbf{s}_i)\|^2 \\ &\leq \frac{1}{2} \sum_{(i,j)\in E} \|w_j - w_i)\|^2 + 2n_e \lambda^2 \bar{\omega}^2 \\ &\quad + 2\lambda\bar{\omega} \sum_{(i,j)\in E} \|w_j - w_i\| \end{aligned}$$

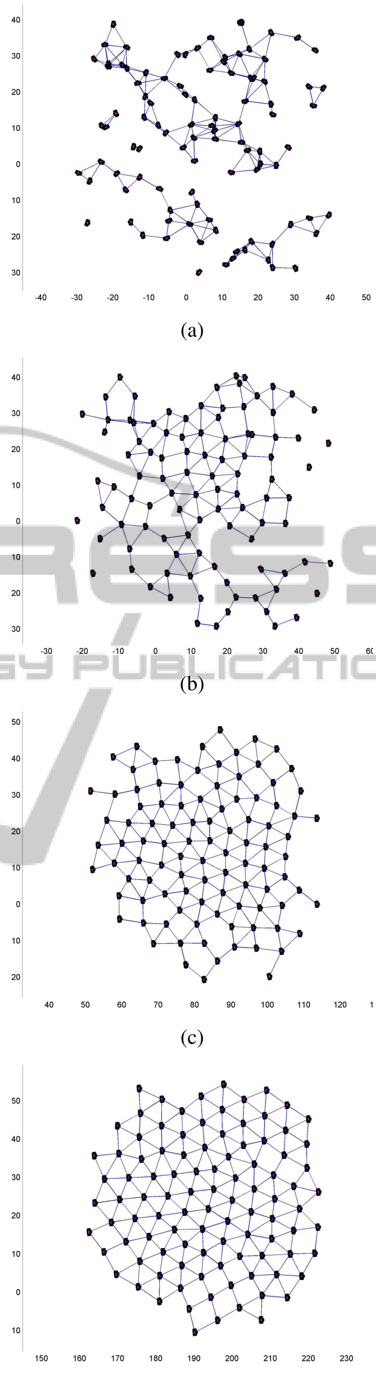
From the property of p-norms, for any vector $\xi \in \mathbb{R}^m$, $\|\xi\|_1 \leq \sqrt{m}\|\xi\|_2$ which implies

$$\sum_{(i,j)\in E} \|w_j - w_i\| \leq \sqrt{n_e}\Big(\sum_{(i,j)\in E} \|w_j - w_i)\|^2\Big)^{\frac{1}{2}} \leq \sqrt{n_e}\varepsilon$$

Therefore, we obtain a positive upper bound for $K(\nu)$

$$\begin{aligned} K(\nu) &\leq \varepsilon + 2\sqrt{n_e \varepsilon}\lambda\bar{\omega} + 2n_e\lambda^2\bar{\omega}^2 \\ &\leq (\sqrt{\varepsilon} + \sqrt{n_e}\lambda\bar{\omega})^2 + n_e\lambda^2\bar{\omega}^2 \ll 1 \end{aligned}$$

because $\varepsilon, \lambda \ll 1$. The trajectory of the group of mobile robots satisfy all four conditions of flocking behavior in Definition 1 and perform flocking if their near-identity transformations do the same. □



(a) (b) (c) (d)

Figure 2: Flocking for networked nonholonomic robots with nonlinear dynamics: Consecutive snapshots of the proximity structure for $n = 100$ mobile robots in free-space. Flocking behavior emerges in (c) and is maintained thereafter.

6 SIMULATIONS

In this section, we present simulation results for flocking behavior by networks of mobile robots with nonlinear dynamics both in free-space in Fig. 2 and in presence of wall-shaped obstacles in Fig. 3. Parame-

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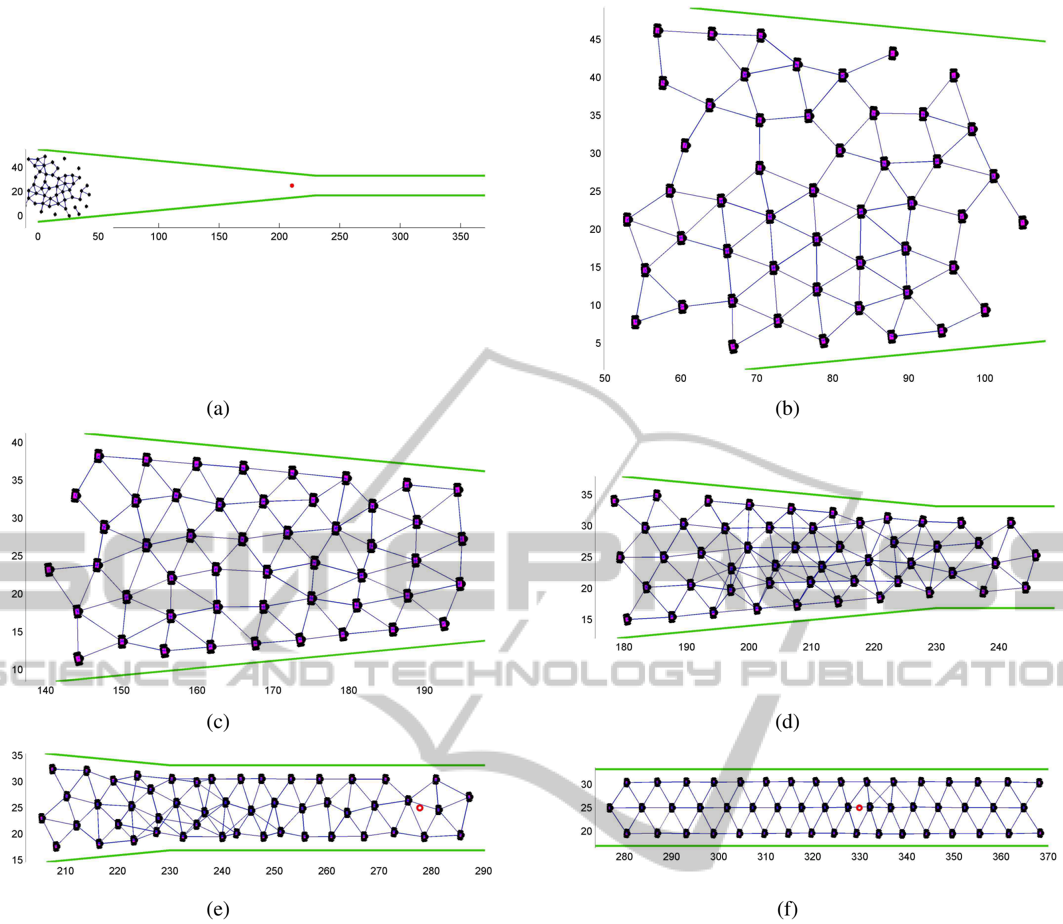

Figure 3: Flocking for networked nonholonomic robots with nonlinear dynamics in presence of wall-shaped obstacles: A group of $n = 50$ mobile robots start randomly within the boundaries of two wall-shaped obstacles. As the agents follows a γ-agent to the right, the space becomes increasingly narrower until the agents are forced to go through a tunnel about one-third in width compared to that of the initial space. The agents demonstrate squeezing maneuver where they move within a tight space without collisions: (a) a global view of the obstacles and (b) through (f) consecutive snapshots of the robots as they move within the boundaries. A quasi α-lattice can be observed in (b), (c), and (f).

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ters that are different from those in the flocking simulations of (Olfati-Saber, 2006) are as follows: $a = 5$ and $b = 10$ for $\phi(z)$ and $\lambda = 0.25$ for the near-identity transformation.

7 CONCLUSIONS

We addressed the problem of flocking for networks of nonholonomic robots with nonlinear dynamics given that a flocking algorithm for particles was known from (Olfati-Saber, 2006). Near-identity change of coordinates (Olfati-Saber, 2002) was applied to nonlinear dynamics of the robot to transform it into a partially-linear normal form with a double-integrator linear part. Then, the flocking algorithm was applied to the linear subsystem. The inverse of the near-identity transformation provides the flocking algorithm for the networked mobile robots with nonlinear dynamics. The approach is more general and applicable to any partially-linearizable robot dynamics. We proved the emergence of flocking behavior for robotic networks according to the formal definition of flocking in (Olfati-Saber, 2006). Simulations results were provided for large-scale networks of two-wheeled robots with nonlinear dynamics.

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