Block Triangular Decoupling of General Neutral Multi Delay Systems

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Abstract: The problem of block triangular decoupling is studied for the case of general neutral multi delay systems. The system is not restricted to be square and invertible. The controller is of the general neutral dynamic type involving a dynamic feedback and dynamic precompensator. Two different cases of feedback are studied. The first is the case of measurable output feedback and second is the case of performance output feedback. The controller is restricted to be realizable. The necessary and sufficient conditions for the problem to be solvable are established. The general class of the realizable controllers solving the problem is derived. The closed loop transfer function is proven to have arbitrary characteristic polynomial thus facilitating command tracking and stability.

1 INTRODUCTION

The block triangular decoupling problem, has attracted considerable attention (see Commault and Dion, 1983; Lohmann, 1991; Morse and Wonham, 1970; Otsuka and Inaba, 1992; Otsuka, 1992; Park 2008; Park and Choi, 2011; Sourlas, 2001 and the references therein). The problem appears to be of great importance particularly for large scale and interconnected MIMO plants. For the case of retarded time delay systems (or more generally for systems over a ring or a principal ideal domain) the problem has been studied in Caturiyati (2003), Ito and Inaba (1997a) and Ito and Inaba (1997b).

The category of general neutral multi delay systems is more general than the aforementioned system cases and covers a wide range of applications (see Koumboulis and Panagiotakis 2008; Koumboulis, Kouvakas and Paraskevopoulos, 2009a-c and the references therein). In the present paper the block triangular decoupling problem is studied for the first time for the category of general neutral multi delay systems. The controller is of the measurement output feedback type with a dynamic feedback matrix and a dynamic precompensator. The controller is required to be realizable. The controller type covers the state feedback and the performance output feedback cases as special cases. The contribution of the present paper consists in establishing the necessary and sufficient conditions for the problem to be solvable and deriving the general class of the realizable controllers solving the problem. The closed loop transfer function is proven not to be restricted by the design requirement except of its realization index, thus achieving tracking and BIBO stability. It is important to mention that the special case of row by row triangular decoupling for the category of general neutral multi delay systems has been solved in Koumboulis and Panagiotakis (2008) using static controllers and in Koumboulis and Kouvakas (2010) using dynamic controllers. Also, the problem of diagonal block decoupling for the same system category has been solved in Koumboulis and Kouvakas (2011).

2 PRELIMINARIES

Consider the general class of linear neutral multi-delay differential systems

$$\sum_{j=1}^{q_0} \tilde{E}_j \dot{x}\left(t-\sum_{i=1}^{q} q_{j,i}\tau_i\right) = \sum_{j=1}^{q_0} \tilde{A}_j x\left(t-\sum_{i=1}^{q} q_{j,i}\tau_i\right) + \sum_{j=1}^{q_0} \tilde{B}_j u\left(t-\sum_{i=1}^{q} q_{j,i}\tau_i\right) \quad (1a)$$

$$\sum_{j=1}^{q_0} \bar{C}_j y\left(t-\sum_{i=1}^{q} q_{j,i}\tau_i\right) = \sum_{j=1}^{q_0} \tilde{C}_j x\left(t-\sum_{i=1}^{q} q_{j,i}\tau_i\right) \quad (1b)$$

where $x(t) \in \mathbb{R}^n$ denotes the vector of state

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variables, $u(t)\in\mathbb{R}^m$ the vector of control inputs, $y(t)\in\mathbb{R}^p$ the vector of performance outputs, τ_i ($i=1,\ldots,q$) are positive real numbers denoting point delays, and $q_{j,i}$ ($j=1,\ldots,q_0$; $i=1,\ldots,q$) is a finite sequence of integers with regard to i and j. The quantities q and q_0 are positive integers. Clearly, if the quantity $\sum_{i=1}^{q} q_{j,i}\tau_i$ is negative then it denotes prediction. The real matrices $\tilde{E}_j$, $\tilde{A}_j$, $\tilde{B}_j$ have n rows while the real matrices $\bar{C}_j$, $\tilde{C}_j$ have p rows. In general, $m\neq p$.

The interest is focused on the forced behaviour of the system, i.e. for zero initial and past conditions ($x(t)=0$, $u(t)=0$ for $t<0$). Defining

$$\mathbf{T}=\begin{bmatrix}\tau_1 & \cdots & \tau_q\end{bmatrix},$$
$$\mathbf{e}^{\text{-s}\mathbf{T}}=\begin{bmatrix}\exp(-s\tau_1) & \cdots & \exp(-s\tau_q)\end{bmatrix}$$

the system (1) can be described in the frequency domain by the following set of equations

$$s\tilde{E}(\mathbf{e}^{\text{-s}\mathbf{T}})X(s)=\tilde{A}(\mathbf{e}^{\text{-s}\mathbf{T}})X(s)+\tilde{B}(\mathbf{e}^{\text{-s}\mathbf{T}})U(s) \quad (2a)$$
$$\bar{C}(\mathbf{e}^{\text{-s}\mathbf{T}})Y(s)=\tilde{C}(\mathbf{e}^{\text{-s}\mathbf{T}})X(s) \quad (2b)$$

where $X(s)=\mathbf{L}_-\{x(t)\}$, $U(s)=\mathbf{L}_-\{u(t)\}$, $Y(s)=\mathbf{L}_-\{y(t)\}$ with $\mathbf{L}_-\{\bullet\}$ be the Laplace transform of the argument signal, while

$$\begin{bmatrix}\tilde{E}(\mathbf{e}^{\text{-s}\mathbf{T}})\\ \tilde{A}(\mathbf{e}^{\text{-s}\mathbf{T}})\\ \tilde{C}(\mathbf{e}^{\text{-s}\mathbf{T}})\end{bmatrix}=\sum_{j=1}^{q_0}\begin{bmatrix}\tilde{E}_j\\ \tilde{A}_j\\ \tilde{C}_j\end{bmatrix}\exp\left[-s\left(\sum_{i=1}^{q}q_{j,i}\tau_i\right)\right]$$
$$\tilde{B}(\mathbf{e}^{\text{-s}\mathbf{T}})=\sum_{j=1}^{q_0}\tilde{B}_j\exp\left[-s\left(\sum_{i=1}^{q}q_{j,i}\tau_i\right)\right]$$
$$\bar{C}(\mathbf{e}^{\text{-s}\mathbf{T}})=\sum_{j=1}^{q_0}\bar{C}_j\exp\left[-s\left(\sum_{i=1}^{q}q_{j,i}\tau_i\right)\right]$$

where $\exp[\cdot]=e^{[\cdot]}$ is the exponential of the argument quantity. The matrices $\tilde{E}(\mathbf{e}^{\text{-s}\mathbf{T}})$ and $\bar{C}(\mathbf{e}^{\text{-s}\mathbf{T}})$ are assumed to be invertible. Hence, the system of equations in (2) can be expressed in normal system form as follows

$$sX(s)=A(\mathbf{e}^{\text{-s}\mathbf{T}})X(s)+B(\mathbf{e}^{\text{-s}\mathbf{T}})U(s) \quad (3a)$$
$$Y(s)=C(\mathbf{e}^{\text{-s}\mathbf{T}})X(s) \quad (3b)$$

where

$$A(\mathbf{e}^{\text{-s}\mathbf{T}})=\left[\tilde{E}(\mathbf{e}^{\text{-s}\mathbf{T}})\right]^{-1}\tilde{A}(\mathbf{e}^{\text{-s}\mathbf{T}})$$
$$B(\mathbf{e}^{\text{-s}\mathbf{T}})=\left[\tilde{E}(\mathbf{e}^{\text{-s}\mathbf{T}})\right]^{-1}\tilde{B}(\mathbf{e}^{\text{-s}\mathbf{T}})$$
$$C(\mathbf{e}^{\text{-s}\mathbf{T}})=\left[\bar{C}(\mathbf{e}^{\text{-s}\mathbf{T}})\right]^{-1}\tilde{C}(\mathbf{e}^{\text{-s}\mathbf{T}})$$

Consider the open loop transfer matrix

$$P(s,\mathbf{e}^{\text{-s}\mathbf{T}})=C(\mathbf{e}^{\text{-s}\mathbf{T}})\left[sI_n-A(\mathbf{e}^{\text{-s}\mathbf{T}})\right]^{-1}B(\mathbf{e}^{\text{-s}\mathbf{T}}) \quad (4)$$

3 SOLUTION OF THE BLOCK TRIANGULAR DECOUPLING PROBLEM

Here, the design goal is that of block triangular decoupling via dynamic state feedback and dynamic precompensator, namely to derive a closed loop system in a block lower triangular form. The outputs of the system are grouped into blocks and each block of outputs is controlled by the corresponding group of external inputs and the previous groups of external inputs. To present the formal definition of the problem we will first present the form of the controller.

Let $\Psi(s)$ be the Laplace transform of the vector $\psi(t)\in\mathbb{R}^r$, denoting the measurement output of the system. It holds that $\Psi(s)=L(\mathbf{e}^{\text{-s}\mathbf{T}})X(s)$, where $L(\mathbf{e}^{\text{-s}\mathbf{T}})$ is a $r\times n$ matrix with elements being rational functions of $e^{-s\tau_1},\ldots,e^{-s\tau_q}$. The feedback is proposed to be of the form

$$U(s)=K(s,\mathbf{e}^{\text{-s}\mathbf{T}})\Psi(s)+G(s,\mathbf{e}^{\text{-s}\mathbf{T}})\Omega(s) \quad (5)$$

where $\Omega(s)$ is the $p\times 1$ vector of external inputs. The elements of the matrices $K(s,\mathbf{e}^{\text{-s}\mathbf{T}})$ and $G(s,\mathbf{e}^{\text{-s}\mathbf{T}})$ are rational functions of s. The respective numerator and denominator polynomial coefficients are multivariable rational functions of $e^{-s\tau_1},\ldots,e^{-s\tau_q}$. The controller is restricted to be realizable. This means that the elements of $K\left(s,\mathbf{e}^{\text{-sT}}\right)$ and $G\left(s,\mathbf{e}^{\text{-sT}}\right)$ are restricted to be realizable, i.e. their realizability index should be greater than or equal to zero. Substituting controller (5) to the open loop system (3) the problem of block decoupling is formulated as follows

$$C\left(\mathbf{e}^{\text{-sT}}\right)\left[sI_n - A\left(\mathbf{e}^{\text{-sT}}\right) - \right.$$
$$\left. -B\left(\mathbf{e}^{\text{-sT}}\right)K\left(s,\mathbf{e}^{\text{-sT}}\right)L\left(\mathbf{e}^{\text{-sT}}\right)\right]^{-1}B\left(\mathbf{e}^{\text{-sT}}\right)G\left(s,\mathbf{e}^{\text{-sT}}\right) =$$
$$= \underset{i=1,\ldots,\nu\,;\,j=1,\ldots,i}{\text{block.triang}}\left\{H_{i,j}\left(s,\mathbf{e}^{\text{-sT}}\right)\right\} \quad (6)$$

where

$$\underset{i=1,\ldots,\nu\,;\,j=1,\ldots,i}{\text{block.triang}}\left\{H_{i,j}\left(s,\mathbf{e}^{\text{-sT}}\right)\right\} =$$
$$= \begin{bmatrix} H_{1,1}\left(s,\mathbf{e}^{\text{-sT}}\right) & 0 & \cdots & 0 \\ H_{2,1}\left(s,\mathbf{e}^{\text{-sT}}\right) & H_{2,2}\left(s,\mathbf{e}^{\text{-sT}}\right) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_{\nu,1}\left(s,\mathbf{e}^{\text{-sT}}\right) & H_{\nu,2}\left(s,\mathbf{e}^{\text{-sT}}\right) & \cdots & H_{\nu,\nu}\left(s,\mathbf{e}^{\text{-sT}}\right) \end{bmatrix}$$

and where $H_{i,j}\left(s,\mathbf{e}^{\text{-sT}}\right) \in \mathbb{R}\left(s,\mathbf{e}^{\text{-sT}}\right)^{p_i \times p_j}$ ($i,j \in \{1,\ldots,\nu\}$) are matrices whose elements are rational functions of s while the respective numerator and denominator polynomial coefficients are multivariable rational functions of $e^{-s\tau_1},\ldots,e^{-s\tau_q}$. Obviously it holds that $\sum_{i=1}^{\nu} p_i = p$. The matrices $H_{i,i}\left(s,\mathbf{e}^{\text{-sT}}\right)$ ($i=1,\ldots,\nu$) are square and invertible. From (6) we also observe that $G\left(s,\mathbf{e}^{\text{-sT}}\right)$ is constrained to be of full column rank while $K\left(s,\mathbf{e}^{\text{-sT}}\right)$ is constrained to preserve the solvability of the closed loop system, i.e.

$$\det\left[sI_n - A\left(\mathbf{e}^{\text{-sT}}\right) - \right.$$
$$\left. -B\left(\mathbf{e}^{\text{-sT}}\right)K\left(s,\mathbf{e}^{\text{-sT}}\right)L\left(\mathbf{e}^{\text{-sT}}\right)\right] \not\equiv 0 \quad (7)$$

Defining

$$Q\left(s,\mathbf{e}^{\text{-sT}}\right) = L\left(\mathbf{e}^{\text{-sT}}\right)\left[sI_n - A\left(\mathbf{e}^{\text{-sT}}\right)\right]^{-1}B\left(\mathbf{e}^{\text{-sT}}\right)$$

and applying elementary computations, relation (6) can be rewritten as

$$P\left(s,\mathbf{e}^{\text{-sT}}\right)\left\{sI_n - K\left(s,\mathbf{e}^{\text{-sT}}\right)Q\left(s,\mathbf{e}^{\text{-sT}}\right)\right\}^{-1}G\left(s,\mathbf{e}^{\text{-sT}}\right) =$$
$$= \underset{i=1,\ldots,\nu\,;\,j=1,\ldots,i}{\text{block.triang}}\left\{H_{i,j}\left(s,\mathbf{e}^{\text{-sT}}\right)\right\} \quad (8)$$

Define

$$\tilde{G}\left(s,\mathbf{e}^{\text{-sT}}\right) = \left\{sI_n - K\left(s,\mathbf{e}^{\text{-sT}}\right)Q\left(s,\mathbf{e}^{\text{-sT}}\right)\right\}^{-1}$$
$$\times G\left(s,\mathbf{e}^{\text{-sT}}\right) \quad (9)$$

After dividing $P\left(s,\mathbf{e}^{\text{-sT}}\right)$ into row blocks as follows

$$P\left(s,\mathbf{e}^{\text{-sT}}\right) = \begin{bmatrix} P_1\left(s,\mathbf{e}^{\text{-sT}}\right) \\ \vdots \\ P_\nu\left(s,\mathbf{e}^{\text{-sT}}\right) \end{bmatrix}$$
$$P_i\left(s,\mathbf{e}^{\text{-sT}}\right) \in \left[\mathbb{R}\left(s,\mathbf{e}^{\text{-sT}}\right)\right]^{p_i \times m}$$

the equation (8) takes on the form

$$\begin{bmatrix} P_1\left(s,\mathbf{e}^{\text{-sT}}\right) \\ \vdots \\ P_\nu\left(s,\mathbf{e}^{\text{-sT}}\right) \end{bmatrix}\tilde{G}\left(s,\mathbf{e}^{\text{-sT}}\right) = \underset{i=1,\ldots,\nu\,;\,j=1,\ldots,i}{\text{block.triang}}\left\{H_{i,j}\left(s,\mathbf{e}^{\text{-sT}}\right)\right\} \quad (10)$$

or equivalently the form

$$P_i\left(s,\mathbf{e}^{\text{-sT}}\right)\tilde{G}\left(s,\mathbf{e}^{\text{-sT}}\right) =$$
$$\left[H_{i,1}\left(s,\mathbf{e}^{\text{-sT}}\right) \quad \cdots \quad H_{i,i-1}\left(s,\mathbf{e}^{\text{-sT}}\right)\right.$$
$$\left. H_{i,i}\left(s,\mathbf{e}^{\text{-sT}}\right) \quad 0 \quad \cdots \quad 0\right];\; i=1,\ldots,\nu \quad (11)$$

From (11) the following necessary condition is derived

$$\text{Rank}\left\{\begin{bmatrix} P_1\left(s,\mathbf{e}^{\text{-sT}}\right) \\ \vdots \\ P_\nu\left(s,\mathbf{e}^{\text{-sT}}\right) \end{bmatrix}\right\} = p_1 + p_2 + \cdots + p_\nu \quad (12)$$

Condition (12) implies that $m \geq p$. Before presenting the necessary and sufficient conditions and the general solutions of the controller matrices, three definitions will be presented. If $m = p$ define $\hat{P}\left(s,\mathbf{e}^{\text{-sT}}\right) = P\left(s,\mathbf{e}^{\text{-sT}}\right)$. If $m > p$ define

$$\hat{P}\left(s,\mathbf{e}^{\text{-sT}}\right) = \begin{bmatrix} P\left(s,\mathbf{e}^{\text{-sT}}\right) \\ \tilde{P}\left(s,\mathbf{e}^{\text{-sT}}\right) \end{bmatrix};\; \det\hat{P}\left(s,\mathbf{e}^{\text{-sT}}\right) \not\equiv 0 \quad (13)$$

Clearly, the choice of $\tilde{P}\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ is not unique. Here, the following choice is proposed

$$\tilde{P}\left(s,\mathbf{e}^{\textbf{-sT}}\right)=\left[k_1 e_{\upsilon_1}^T \quad \cdots \quad k_{m-p} e_{\upsilon_{m-p}}^T\right]^T$$

where e_j is an $1\times m$ unity row vector having the unity in its $j-$th position and k_i are appropriate different than zero reals. The integers $\upsilon_1,\ldots,\upsilon_{m-p}$ are chosen in a way that the set of integers $\left\{1,\ldots,m\right\}-\left\{\upsilon_1,\ldots,\upsilon_{m-p}\right\}$ corresponds to the indices of the linear independent columns of $P\left(s,\mathbf{e}^{\textbf{-sT}}\right)$. The parameters k_i may be used to adjust the characteristics of $\left[\hat{P}\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]^{-1}$ (e.g. to adjust its norm, to achieve stability). Divide $\left[\hat{P}\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]^{-1}$ in column blocks as follows

$$\left[P_1^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right) \quad \cdots \quad P_\nu^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right) \quad P^\perp\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]= =\left[\hat{P}\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]^{-1}$$

where

$$P_i^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right)\in\left[\mathbb{R}\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]^{m\times p_i}$$

$$P^\perp\left(s,\mathbf{e}^{\textbf{-sT}}\right)\in\left[\mathbb{R}\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]^{m\times(m-p)}$$

In the following theorem, the solvability conditions and the general class of all realizable controllers solving the problem are presented.

Theorem 1: The necessary and sufficient condition for the solvability of the Block Triangular Decoupling problem via a dynamic measurement output feedback controller of the form (5) is condition (12). The general class of the realizable controller matrices solving the problem is

$$\begin{aligned}G\left(s,\mathbf{e}^{\textbf{-sT}}\right)=&\left\{I_m-K\left(s,\mathbf{e}^{\textbf{-sT}}\right)Q\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right\}\times\\ &\times\left\{\left[\sum_{\kappa=1}^{\nu}P_\kappa^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right)H_{\kappa,1}\left(s,\mathbf{e}^{\textbf{-sT}}\right) \quad \cdots\right.\right.\\ &\cdots \quad \sum_{\kappa=i}^{\nu}P_\kappa^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right)H_{\kappa,i}\left(s,\mathbf{e}^{\textbf{-sT}}\right) \quad \cdots\\ &\left.P_\nu^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right)H_{\nu,\nu}\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]+\\ &\left.+P^\perp\left(s,\mathbf{e}^{\textbf{-sT}}\right)\left[\Lambda_1\left(s,\mathbf{e}^{\textbf{-sT}}\right) \quad \cdots \quad \Lambda_\nu\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]\right\}\end{aligned} \quad (14a)$$

$K\left(s,\mathbf{e}^{\textbf{-sT}}\right)$: arbitrary, realizable and preserve the closed loop solvability (14b)

where $\Lambda_i\left(s,\mathbf{e}^{\textbf{-sT}}\right)\in\mathbb{R}\left(s,\mathbf{e}^{\textbf{-sT}}\right)^{(m-p)\times p_i}$ $(i=1,\ldots,\nu)$ $H_{i,j}\left(s,\mathbf{e}^{\textbf{-sT}}\right)\in\mathbb{R}\left(s,\mathbf{e}^{\textbf{-sT}}\right)^{p_i\times p_j}$ ($i\in\left\{1,\ldots,\nu\right\}$, $j\in\left\{1,\ldots,i\right\}$) are arbitrary matrices being enough realizable to satisfy the realizability of (14a). The matrices $H_{i,i}\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ ($i=1,\ldots,\nu$) are square and invertible.

Proof: Using the definitions before Theorem 1, we observe that the general solution of (10) is

$$\begin{aligned}\tilde{G}\left(s,\mathbf{e}^{\textbf{-sT}}\right)=&\left[\sum_{\kappa=1}^{\nu}P_\kappa^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right)H_{\kappa,1}\left(s,\mathbf{e}^{\textbf{-sT}}\right) \quad \cdots\right.\\ &\cdots \quad \sum_{\kappa=i}^{\nu}P_\kappa^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right)H_{\kappa,i}\left(s,\mathbf{e}^{\textbf{-sT}}\right) \quad \cdots\\ &\left.P_\nu^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right)H_{\nu,\nu}\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]+\\ &+P^\perp\left(s,\mathbf{e}^{\textbf{-sT}}\right)\left[\Lambda_1\left(s,\mathbf{e}^{\textbf{-sT}}\right) \quad \cdots \quad \Lambda_\nu\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right]\end{aligned} \quad (15)$$

where $\Lambda_i\left(s,\mathbf{e}^{\textbf{-sT}}\right)\in\mathbb{R}\left(s,\mathbf{e}^{\textbf{-sT}}\right)^{(m-p)\times p_i}$ ($i=1,\ldots,\nu$) are arbitrary matrices. Substituting (15) to (9), the relation (14a) is derived. Clearly, $K\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ is arbitrary but it should also be chosen to be realizable and to satisfy the constraint (7). For $G\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ to be left invertible (of full column rank) and realizable, it is necessary for $H_{i,j}\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ and $\Lambda_i\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ ($i\in\left\{1,\ldots,\nu\right\}$, $j\in\left\{1,\ldots,i\right\}$) to be sufficiently realizable and $H_{i,i}\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ to be invertible. For example the index of realizability of $\Lambda_i\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ should be greater than or equal to the minus of the index of realizability of $\left\{I_m-K\left(s,\mathbf{e}^{\textbf{-sT}}\right)Q\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right\}P^\perp\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ and the index of realizability of $H_{i,j}\left(s,\mathbf{e}^{\textbf{-sT}}\right)$ should be greater than or equal to the minus of the realizability index of $\left\{I_m-K\left(s,\mathbf{e}^{\textbf{-sT}}\right)Q\left(s,\mathbf{e}^{\textbf{-sT}}\right)\right\}P_i^\dagger\left(s,\mathbf{e}^{\textbf{-sT}}\right)$. ■

The expressions of the general class of the controller matrices proposed in Theorem 1 are implicit. In the following corollary explicit and analytic expressions of a class of controllers solving the problem at hand are proposed. The derivation of this class is based on the observation that $Q\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ and $P\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ are strictly proper with respect to s.

Corollary 1: If $Q\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ is realizable, a class of the realizable controller matrices solving the problem is given by (14a) and (14b) with

i) $K\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ proper with respect to s and realizable while the minimum index of realizability of its elements is restricted to be greater than or equal to zero

ii) the minimum index of realizability of the elements of $H_{i,j}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ is restricted to be greater than or equal to the minus of the minimum realizability index of the elements of $P_i^{\dagger}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$

iii) the minimum index of realizability of the elements of $\Lambda_i\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ is restricted to be greater than or equal to the minus of the minimum realizability index of the elements of $P^{\perp}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$. ■

Corollary2: If $Q\left(s,\mathbf{e}^{-s\mathbf{T}}\right)=P\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ (the case of performance output feedback) and $P\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ is realizable, a class of the realizable controller matrices solving the problem is given by

$$G\left(s,\mathbf{e}^{-s\mathbf{T}}\right)=\left\{\left[\sum_{\kappa=1}^{\nu}P_{\kappa}^{\dagger}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)H_{\kappa,1}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)\ \cdots\ \sum_{\kappa=i}^{\nu}P_{\kappa}^{\dagger}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)H_{\kappa,i}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)\ \cdots\ P_{\nu}^{\dagger}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)H_{\nu,\nu}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)\right]+ +P^{\perp}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)\left[\Lambda_1\left(s,\mathbf{e}^{-s\mathbf{T}}\right)\ \cdots\ \Lambda_{\nu}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)\right]\right\}+ -K\left(s,\mathbf{e}^{-s\mathbf{T}}\right)\left\{\underset{i=1,\ldots,\nu;\,j=1,\ldots,i}{\text{block.triang}}\left\{H_{i,j}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)\right\}\right\} \quad (16)$$

where $K\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ is restricted only to be proper with respect to s and realizable while the minimum index of realizability of its elements is restricted to be greater than or equal to zero and the minimum index of realizability of the elements of $H_{i,j}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ is restricted to be greater than or equal to the minus of the minimum realizability index of the elements of $P_i^{\dagger}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$. The minimum index of realizability of the elements of $\Lambda_i\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$ is restricted to be greater than or equal to the minus of the minimum realizability index of the elements of $P^{\perp}\left(s,\mathbf{e}^{-s\mathbf{T}}\right)$. ■

Remark 1: In Theorem 1 and Corollaries 1 and 2 the blocks of the closed loop transfer matrix are restricted only to have enough large realizability index. This way regional BIBO stability of the closed loop system can be achieved, while tracking and command following are satisfied as fast as is allowed by the realizability indices of the elements of the closed loop transfer matrix.

Remark 2: The condition (12) is the same with the necessary and sufficient condition for the solvability of the diagonal block decoupling (Koumboulis and Kouvakas, 2011). As was expected, the class of controller, solving the problem at hand, is much wider.

4 CONCLUSIONS

In the present paper the problem of block triangular decoupling has been studied for the first time for the category of general neutral multi delay systems. The controller has been selected of the general neutral dynamic type involving a dynamic feedback and dynamic precompensator. The controller has been restricted to be realizable. The necessary and sufficient condition for the problem to be solvable has been established and the general class of the realizable controllers solving the problem has been derived. The realizability indices of the elements of the closed loop transfer matrix are restricted to be large enough. Except this restriction, the closed loop transfer matrix has been proven to have arbitrary characteristic polynomial thus offering itself for command tracking and regional BIBO stability.

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