Cooperative Autonomous Driving for Vehicular Networks

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Abstract: In this paper, we introduce cooperative autonomous driving algorithms for vehicular networks in urban environments that take human safety into account and are capable of performing vehicle-to-vehicle (V2V) and vehicle-to-pedestrian (V2P) collision avoidance. We argue that "flocks" are multi-agent models of vehicular traffic on roads and propose novel autonomous driving architectures for cyber-physical vehicles capable of performing autonomous driving tasks such as lane-driving, lane-changing, braking, passing, and making turns. These autonomous driving algorithms are inspired by the flocking theory of Olfati-Saber (Olfati-Saber, 2006), though, there are notable differences between autonomous driving on urban roads and flocking behavior—flocks have a single desired destination whereas most drivers on road do not share the same destination. We demonstrate that lane-driving for a vehicular network with $n > 3$ vehicles cannot necessarily be performed using pairwise vehicular interactions and might require triangular interactions among triplets of vehicles. The self-driving vehicles in our framework turn out to be nonlinear switching systems with discrete states that are related to the driving modes of the vehicles. Complex driving maneuvers can be performed using a sequence of mode switchings. We present several examples of driving tasks that can be effectively performed using our proposed driving algorithms.

1 INTRODUCTION

The existing transportation systems are mostly controlled by humans who are prone to making errors causing collisions and fatalities. According to a report by World Health Organization (WHO, 2004), more than 1.3 million road accident fatalities occur worldwide every year. Accident fatalities have been the 9th leading cause of death by 2004 and will be among the top 3 by 2020. Thirty-two percent of fatal crashes in the US are due to DUI[1] (FARS, 2011).

Some of the main causes of traffic accidents are 1) weather and road conditions; 2) drivers and pedestrians engaging in risky behaviors such as erratic driving, driving while being drowsy or intoxicated, and jay-walking while texting or being distracted by music; and 3) the inability of human drivers to predict and quickly react to imminent collision threats.

The main focus of past research on autonomous driving has been on detection and tracking of lanes, pedestrians, and obstacles for an individual vehicle with embedded laser radars and video cameras (Schneiderman and Nashman, 1994; Soelo et al., 2004; Kolski et al., 2006). The most notable of these autonomous vehicles is *Boss*, the winner of the DARPA Grand Challenge for autonomous driving in urban roads. There has been limited research on *cooperative autonomous driving for vehicular networks*. Some notable examples with a primary idea of forming rigid platoons of vehicles are in (Hedrick et al., 1994; Swaroop and Hedrick, 1996; Kato et al., 2002; Baber et al., 2005). The existing *fluidic models* of traffic flow (Helbing, 1995)—going back to the 1955 LWR model (Lighthill and Whitham, 1955)—are ideal for modeling and analysis of congestion in transportation networks. Fluidic models are incapable of capturing V2V and V2P collisions as they ignore the discrete nature of multi-agent interactions.

We propose *flocking*[2] as a multi-agent model of vehicular traffic on roads. Unlike rigid platoons, flocks are capable of performing split/rejoin maneuvers and passing disabled vehicles stuck on lanes by squeezing into a narrower road from a wider road. The two fundamental differences between driving on roads and flocking are as follows: i) all drivers do not share the same destination and ii) the objective of the drivers

[1] Driving Under the Influence of drugs or alcohol.

[2] Forming collision-free and flexible time-varying conformations that move towards the same direction (See Section 2).

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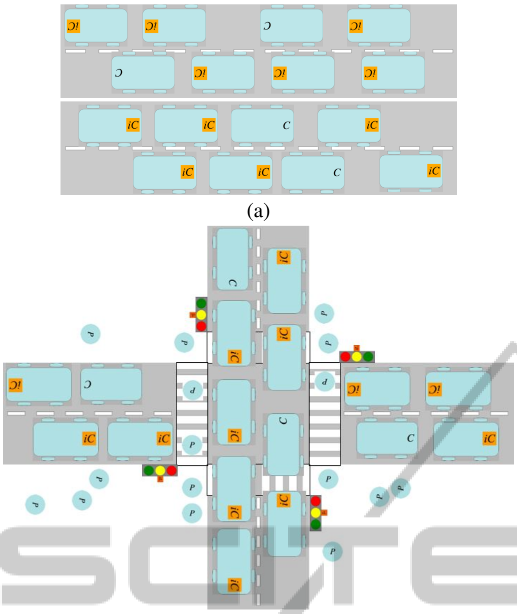

(a) (b)

Figure 1: An intelligent transportation system with a mix of manual vehicles (C), autonomous vehicles (iC), and pedestrians (P): a) a highway and b) an intersection.

on roads is not to form connected 1D-chains on lanes (i.e. platoons). The main similarity between driving and flocking is that the majority of drivers would like to avoid collisions with other vehicles and pedestrians[3] and they temporarily move towards the same direction as they share the road.

In this paper, we introduce a multi-agent framework for safety-aware cooperative autonomous driving for vehicular networks that involves making major modifications to the existing theory and algorithms of flocking in (Olfati-Saber, 2006). We propose novel algorithms that enable *cyber-physical vehicles*[4] (shown in Fig. 1) to autonomously perform driving tasks such as lane-driving, lane-changing, braking, passing, turning left and right, and avoiding tailgating and collision with other vehicles and pedestrians. Our **main contribution** is to *introduce multi-objective flocking (or autonomous driving) algorithms as a significant modification of (Olfati-Saber, 2006)* that enable cyber-physical vehicles to perform all the aforementioned tasks in a single distributed control framework. Our driving algorithm creates networks of autonomous vehicles with nonlinear switching dynamics that is extremely challenging to analyze. *Stability analysis of the novel autonomous driving algorithms is a major open problem* that simply cannot be addressed using the existing analytical tools in nonlinear control and switching systems.

Readers who expect the authors to provide stability analysis of the nonlinear switching systems arising in autonomous driving are greatly underestimating the complexities involved in such an analysis. One of our objectives is to present *autonomous driving* as a major open problem in nonlinear stability to the control community. Our algorithms result in natural forms of autonomous driving in urban environments for fleets of vehicles with linear and nonlinear dynamics based on multi-objective flocking that simply do not exist in the literature and that is our main contribution.

Extension of our autonomous driving algorithms to networked vehicles with nonlinear and nonholonomic dynamics is presented in a separate paper and relies on the use of *near-identity transformations* (Olfati-Saber and Iftekhar, 2012).

Here is an outline of the paper: Some background and notations on flocking are provided in Section 2. Our proposed autonomous driving algorithms for separate tasks are presented in Section 3. Our main result is a unified autonomous driving algorithm that is given near the end of that section. Simulation results are provided in Section 4. Finally, concluding remarks are made in Section 5.

2 FLOCKING ALGORITHMS AND THEORY: BACKGROUND AND NOTATIONS

In this section, we provide some background and notations on Olfati-Saber's flocking algorithms and theory (Olfati-Saber, 2006). Consider a group of n α-agents moving in an m-dimensional space with the dynamics

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i \end{cases} \tag{1}$$

where $q_i, p_i, u_i \in \mathcal{R}^m$ denote the position, velocity, and control of agent i, respectively. Let us denote the conformation of all α agents by $q = \mathrm{col}(q_1, \dots, q_n) \in \mathcal{R}^{mn}$. The *proximity network* of the agents is a dynamic graph $G(q) = (V, E(q))$ with the set of nodes $V = \{1, 2, \dots, n\}$ and set of edges

$$E(q) = \{(i, j) \in V \times V : \|q_j - q_i\| \leq r\}.$$

where r is the *interaction range* between two agents. The adjacency matrix of $G(q)$ is a non-negative matrix $A(q) = [a_{ij}(q)]$ with smooth elements $0 \leq a_{ij}(q) \leq 1$ defined in (Olfati-Saber, 2006). The set of

[3]The drivers who commit DUI or criminals who intentionally collide with other drivers and pedestrians are excluded.

[4]Vehicles with embedded sensing, control, computing, and communication devices.

neighbors of agent i are denoted by $N_i(q)\{j : (i, j) \in E(q)\}$. A conformation q is called a *quasi alpha-lattice* iff every agent is approximately equally distanced from all of its neighbors, i.e.

$$\exists \varepsilon, d > 0 : -\varepsilon \leq \|q_j - q_i\| - d \leq \varepsilon, \forall j \in N_i(q)$$

where d is the *desired distance* between two neighbors. An α-lattice is a quasi α-lattice with $\varepsilon = 0$. Based on (Olfati-Saber, 2006) (but slightly modified for autonomous driving), a group of α-agents perform *flocking behavior* if they asymptotically achieve the following three objectives:

1. quasi self-alignment, $\exists \varepsilon > 0 : \|p_j - p_i\| < \varepsilon, \forall j \neq i$;
2. form a quasi α-lattice; and
3. form a connected proximity network $G(q)$.

Algorithm 2, $u_i = f_i^\alpha + f_i^\gamma$, in (Olfati-Saber, 2006) leads to emergence of flocking behavior and can be described as follows:

$$u_i = \underbrace{\sum_{j \in N_i} \phi_\alpha(\|q_j - q_i\|_\sigma)\mathbf{n}_{ij}}_{\text{gradient-based term}} + \underbrace{\sum_{j \in N_i} a_{ij}(q)(p_j - p_i)}_{\text{consensus term}} + f_i^\gamma \quad (2)$$

where $\mathbf{n}_{ij} = (q_j - q_i)/\sqrt{1 + \varepsilon\|q_j - q_i\|^2}$ is a bounded vector along the line connecting q_i to q_j and $\varepsilon \in (0, 1)$. Moreover $\|s\|_\sigma = \frac{1}{\varepsilon}(\sqrt{1 + \varepsilon\|s\|^2} - 1)$ denotes the σ-*norm* of vector s. The potential function of the group is defined as $V(q) = \sum_{j \neq i} \psi_\alpha(\|q_j - q_i\|_\sigma)$ where $\psi_\alpha(s) = \int_{\|d\|_\sigma}^{s} \phi_\alpha(h)dh$. The scalar function $\phi_\alpha(s)$ is defined in eq. (15) of (Olfati-Saber, 2006). f_i^γ is a linear tracking controller

$$f_i^\gamma = -c_1^\gamma(q_i - q_r) - c_2^\gamma(p_i - p_r); \quad c_1^\gamma, c_2^\gamma > 0 \quad (3)$$

where (q_r, p_r) is the state of the γ-*agent*, i.e. a virtual moving rendezvous point (with double integrator dynamics). The objective of all α-agents is to asymptotically track a single γ-agent (or common goal) as depicted in Fig. 2.

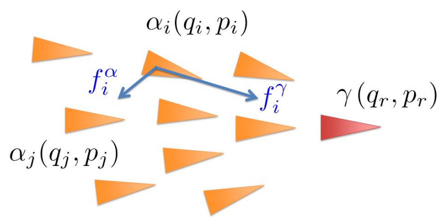

Figure 2: The schematic diagram of the flocking algorithm in (2).

The assumption of "tracking one γ-agent" needs to change entirely in the design of autonomous driving algorithms as different drivers do not necessarily share the same destination.

3 COOPERATIVE AUTONOMOUS DRIVING

In this section, we introduce a theoretical framework for design and analysis of novel autonomous driving algorithms for vehicular networks in urban environments that enables cyber-physical vehicles to autonomously perform a variety of driving tasks such as lane-driving, lane-changing, braking, passing other vehicles, making turns, and avoiding collision with pedestrians and nearby vehicles. The autonomous driving framework relies on the basic elements of the flocking theory described in Section 2, but requires making significant number of changes and new agent types that are necessary for urban driving.

3.1 Lane-driving

Lane-driving means moving along the mid-lane curve $\mu(t)$ of a lane without collision with other neighboring vehicles. This is one of the most basic driving tasks on roads, yet performing this task in a cooperative manner with other vehicles is rather challenging contrary to common belief.

We propose a lane-driving algorithm illustrated in Fig. 3 (a) where each α_i-agent has its own dedicated γ_i-agent with position $\hat{q}_i$ that is the projection of q_i on its desired mid-lane curve $\mu(t)$ with a unit tangent ξ_i and normal π_i. We refer to this model as *multi-objective flocking on curves*. The velocity $\hat{p}_i$ of γ_i is defined based on the *desired velocity* v_i^d of vehicle i and the curvature κ_i of the midlane curve at $\hat{q}_i$:

$$\hat{p}_i = \frac{1}{1 + \kappa_i} v_i^d \xi_i \quad (4)$$

For a straight midlane curve passing through a point ζ along ξ, the projection $\hat{q}_i$ of point q_i can be readily calculated as

$$\hat{q}_i = Pq_i + (I - P)\zeta$$

where $P = I - \pi_i\pi_i^T$ is the projection matrix. The tracking term of lane-driving algorithm takes the form

$$f_i^\gamma = -c_1^\gamma(q_i - \hat{q}_i) - c_2^\gamma(p_i - \hat{p}_i) \quad (5)$$

Unfortunately, a group of at least $n \geq 4$ applying a flocking-based lane-driving algorithm $u_i = f_i^\alpha + f_i^\gamma$ cannot always asymptotically achieve lane-driving even on a straight lane (see Fig. 3 (b)) due to the existence of structurally stable *entangled formations* such as the one shown in Fig. 3 (c) that are undesirable local minima of a potential function for a 1D α-lattice. Structural stability of flocks and formations is defined and analyzed in (Olfati-Saber and Murray, 2002a; Olfati-Saber and Murray, 2002b; Olfati-Saber, 2006).

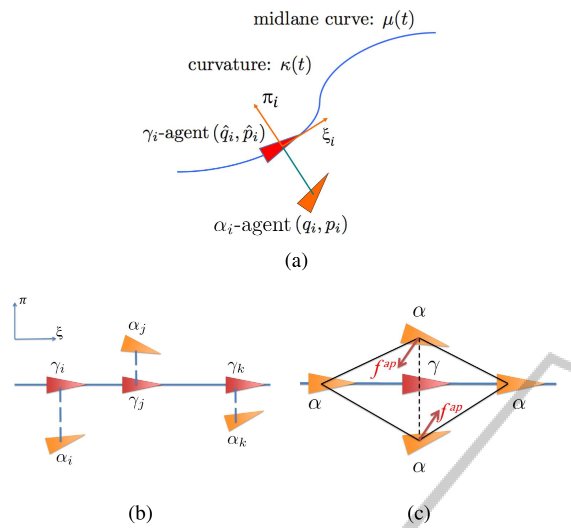

Figure 3: Multi-objective flocking-based model of lane-driving: (a) curved lanes, (b) straight lanes, and (c) entangled conformations with $n > 3$ vehicles that require applying area-potential based forces f_i^{ap}.

To resolve the triangular entanglement shown in Fig. 3 (c), we propose to add *signed-area potential function* terms to the structural potential $V(q)$ of the agents. Signed-area potentials were first applied to formation control in (Olfati-Saber and Murray, 2002c). Let q_i, q_j, q_k be the position of three agents that form a triangle. Then, the signed-area error term of the triangle, or face (i, j, k) is in the form

$$\eta_{ijk} = (q_j - q_i) \otimes (q_k - q_i) - \bar{a}_{ijk} \tag{6}$$

where $\bar{a}_{ijk}$ is the desired signed-area (zero for lane-driving on a straight lane) and $\otimes$ is a tensor product defined by $u \otimes v = u^T \cdot v^\perp = v^T \cdot u^\perp$ and $v^\perp = (v_2, -v_1)^T$ is right orthogonal to the vector $v = (v_1, v_2)^T$. The signed-area potential is $\psi(\eta_{ijk})$ with a scalar potential function $\psi(s) = \sqrt{1+s^2} - 1$.

Let F denote the set of all faces in the Delaunay triangulation of $q_1, \ldots, q_n$. The triplets (i, j, k) are ordered such that $i < j < k$. Then, the set of neighboring faces F_i of node i is the set of faces that include node i.

Define the subunit vector $\mathbf{n}_i = (\hat{q}_i - q_i)/\sqrt{1+\varepsilon\|\hat{q}_i - q_i\|^2}$ that is orthogonal to ξ_i and let $\rho_i = \mathbf{n}_i + \mathbf{n}_i^\perp$ be a *perturbation vector* associated with the α_i-agent. We define the *area potential* force as a perturbation force given by

$$f_i^{ap} = \sum_{(i,j,k) \in F_i} \psi((q_j - q_i) \otimes (q_k - q_i))\rho_i \tag{7}$$

Suppose that an agent i is very close to its associated γ_i-agent, or $\|\hat{q}_i - q_i\| \leq \varepsilon$. If agent i belongs to a face with an area of order $O(1)$, $\|\rho_i\| < 2\varepsilon$ and thus agent i feels a relatively small perturbation of order $O(\varepsilon)$ (e.g. the front and end agents in the diamond formation of Fig. 3 (c) that lie on the midlane curve). However, if agent i is relatively far from its associated γ_i agent, or $\|\hat{q}_i - q_i\| = O(1)$, and it belongs to a face with an area of order $O(1)$, then agent i feels a significant perturbation of order $O(1)$ along π_i as shown in Fig. 3 (c) for the two agents far from the midlane curve. Here is our *cooperative lane-driving algorithm*:

$$u_i = f_i^\alpha + f_i^\gamma + f_i^{ap}. \tag{8}$$

The area potential term f_i^{ap} has a major role in avoiding vehicle-to-vehicle collisions while two lanes merge (e.g. the highway up-ramp and the highway), or when multi-lane roads get narrower.

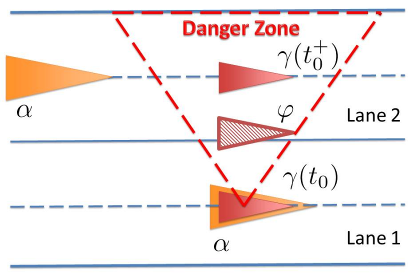

Figure 4: The schematic diagram of the lane-changing maneuver.

3.2 Lane-changing and the φ-Agent

Lane-changing maneuver is schematically depicted in Fig. 4. A vehicle can initiate a lane-changing maneuver if its danger zone (i.e. an obtuse isosceles triangle) does not contain other vehicles—it is safe to change lanes. The obtuse angle of the zone is $\theta = 2\arctan(\varepsilon\|p_i\|/l_w)$ where l_w is the lane-width and $\varepsilon > 0$ is an appropriate small constant.

If it is safe to change lanes at time $t = t_0$, an autonomous vehicle switches its desired lane variable $l(t_0) = 1$ to $l(t_0^+) = 2$ and as a result its γ-agent switches from midlane curve 1 to midlane curve 2. Note that l, the desired lane, is a *discrete state* of the γ-agent that takes a finite set of values in multi-lane roads.

If the α-agent tries to directly track its γ-agent, the lane-changing maneuver will be rather abrupt and "jerky." Most human drivers engage in a gradual lane-changing maneuver to avoid accidents and possible getting out of the passing lane. To create a gradual lane-changing maneuver, we introduce a "filtering agent" called the φ-*agent* with the state $(\bar{q}_i, \bar{p}_i)$ that acts as a low-pass filter on the motion of the γ-agent. In other words, for lane-driving or lane-changing, we propose that an α-agent i should track its φ-agent and the φ-agent should track the γ-agent associate with α-agent i. The new *multi-lane lane-driving and lane-changing algorithm* for a vehicular network has the following form

$$\alpha:\begin{cases}\dot{q}_i = p_i\\ \dot{p}_i = f_i^{\alpha}+f_i^{ap}+f_i^{\varphi}\\ f_i^{\varphi} = -c_1^{\varphi}(q_i-\bar{q}_i)-c_2^{\varphi}(p_i-\bar{p}_i)\end{cases} \quad (9)$$

$$\varphi:\begin{cases}\dot{\bar{q}}_i = \bar{p}_i\\ \dot{\bar{p}}_i = -c_1^{\gamma}(\bar{q}_i-\hat{q}_i)-c_2^{\gamma}(\bar{p}_i-\hat{p}_i)\end{cases}$$

3.3 Braking and Stopping

Braking is a mode (or action) that occurs during lane-driving when a vehicle i gets too close to the vehicle j in front of it. Let $\delta_b \in \{0,1\}$ denote the braking state and d_1, d_2 satisfying $d_1 < d < d_2$ be the minimum and maximum distances between vehicles i and j that trigger and end braking, respectively (Recall, d is the desired distance between two neighbouring α-agents, i.e, two consecutive vehicles here). Then, δ_b switches from 1 to 0 and 0 to 1 according to the following conditions

$$\delta_b = \begin{cases}1 & \text{if } \|q_j-q_i\| \le d_1\\ 0 & \text{if } \|q_j-q_i\| \ge d_2 \quad \text{once it brakes}\end{cases}$$

The braking force is defined as $f_i^b = -c_b(p_i - p_j)$ with $c_b > 0$. In cases of low-speed braking with the possibility of the vehicle coming very close to a stop, the desired velocity is set to 0. The cooperative lane-driving algorithm with braking ability is given by

$$u_i = f_i^{\alpha}+f_i^{ap}+(1-\delta_b)f_i^{\varphi}+\delta_b f_i^b \quad (10)$$

If there is no vehicle in front of vehicle i, then the following braking force is applied for stopping with a braking state $\delta_b = 1$

$$f_i^b = -\bar{c}_b(\rho)p_i \quad (11)$$

where $\rho \ge 0$ is the target stopping range (the distance to the target stopping point), as shown in Fig. 5 (b), and $\bar{c}_b(\rho)$ is defined by

$$\bar{c}_b(\rho) = \frac{1}{2}(c_1+c_2)+(c_1-c_2)\sigma(\rho-\rho_0);\ c_2 > c_1 > 0 \quad (12)$$

with $\sigma(s) = s/\sqrt{1+s^2}$. The constants c_1 and c_2 correspond to the minimum and maximum strength of braking, respectively, and $\rho_0 > 0$ is a critical range of switching from soft braking to hard braking. In addition, the desired velocity of the vehicle is set to 0.

Let us define $\delta_s \in \{0,1\}$ as a *stopping flag*. Every time the stopping flag is triggered ($\delta_s = 1$), the vehicle

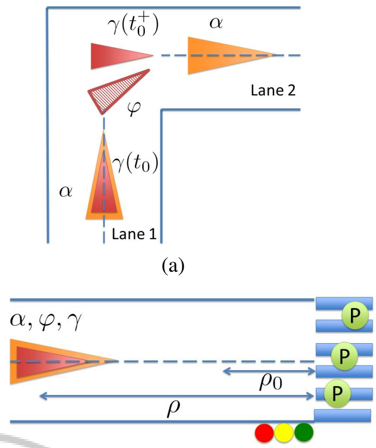

Figure 5: (a) Turning left or right amounts to a lane-changing maneuver and (b) stopping is a form of braking that avoids collision with pedestrians at a crosswalk, or jay-walkers.

enters the braking mode and $\delta_b = 1$. The following equation determines the *braking force*:

$$f_i^b = \begin{cases}c_b(p_j-p_i), & \delta_s = 0\\ -\bar{c}_b(\rho)p_i, & \delta_s = 1\end{cases} \quad (13)$$

3.4 Turning and Collision Avoidance with Pedestrians

Making left or right turns amounts to performing lane-changing maneuvers with lanes that are orthogonal to each other as illustrated in Fig. 5 (a). Stopping is one possible way to avoid V2P collisions without changing lanes. and by entering the braking mode with flags $\delta_b = \delta_s = 1$ as shown in Fig. 5 (b). Similarly, the intention state for stopping is the stopping flag $\delta_s \in \{0,1\}$ that appears in (13).

3.5 Intention State for Lane-changing

Before an autonomous vehicle physically engages in lane-changing or stopping, it needs to “anticipate” performing such actions in advance by triggering its *intention states* associated with those actions since its “danger zone” might not be empty for a safe lane-changing maneuver.

Let $\delta_l \in \{-1,0,1\}$ denote the *lane-changing intention state* where $\delta_l = 0$ means no lane-changing, and $\delta_l = 1$ and $\delta_l = -1$ mean changing lanes to the left and right lanes, respectively, whenever such lanes (or shoulders) exist.

3.6 Tailgating behavior and Asymmetric Weights

Consider the braking mode. If the speed of vehicle i is significantly greater than vehicle j, or $\|p_i\| \gg \|p_j\|$, after some finite time their distance becomes smaller than d_1 and braking is triggered which temporarily increases the distance between the two vehicles. The mode resulting from frequent switches in the braking flag δ_b of agent i is commonly known as *tailgating behavior* that is illegal and often could result in accidental collisions.

To avoid the tailgating behavior, we propose the use of asymmetric interaction weights w_{ij} as multiplicative factors of $a_{ij}(q)$ (recall eq. (2)) in vehicular networks. Consider a pair (i, j) of α-agents in which one has a significantly higher desired speed than the other. Let us assume the slower vehicle i is driving on a two-lane road and consider two cases: 1) the slow vehicle is driving in the slower lane $l_i = 1$ and 2) the slow vehicle is driving in the faster lane $l_i = 2$. Define

$$w_{ij} = \begin{cases} 1 - \tanh(k_w\|p_j\|/\|p_i\|) & \text{Case 1} \\ 1 & \text{Case 2} \end{cases} \qquad (14)$$

with $0 < k_w < 1$. In Case 1, $w_{ij} \approx 0$ and vehicle i ignores a relatively fast vehicle j behind it. In this case, vehicle j has to engage in lane-changing to avoid tailgating vehicle i. In Case 2, vehicle i sets its weight to $w_{ij} = 1$ forcing it to go as fast as vehicle j and eventually change lanes to a slower lane whenever possible. Note that there is nothing about the tailgating behavior that limits it to two-lane roads, though we chose this scenario for clarity of presentation.

3.7 Algorithm and Modes of Driving

Self-driving vehicles applying our cooperative autonomous driving algorithms have three basic modes of driving m_1 = lane-driving, m_2 = lane-changing, and m_3 = braking as shown in Fig. 6. For example, *passing another vehicle* amounts to a sequence of lane-changing and lane-driving modes (and if necessary) followed by another lane-changing. Therefore, passing can be achieved using consecutive mode switchings and is not an independent mode of driving. The changes in the *discrete state* vector $Q = (\delta_b, \delta_s, \delta_l, l)^T$ can directly trigger mode switchings. Here is our main autonomous driving algorithm:

Algorithm 1: Every vehicle applies the following cooperative autonomous driving algorithm:

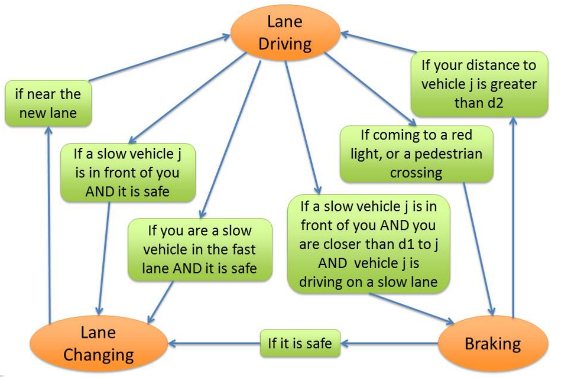

Figure 6: The modes of driving for self-driving vehicles and the logical conditions for mode switchings.

$$\text{vehicle } i: \begin{cases} \alpha_i: \begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = f_i^\alpha + f_i^{ap} + (1-\delta_b) f_i^\varphi + \delta_b f_i^b \\ f_i^\varphi = -c_1^\varphi(q_i - \bar{q}_i) - c_2^\varphi(p_i - \bar{p}_i) \end{cases} \\ \varphi_i: \begin{cases} \dot{\bar{q}}_i = \bar{p}_i \\ \dot{\bar{p}}_i = -c_1^\gamma(\bar{q}_i - \hat{q}_i) - c_2^\gamma(\bar{p}_i - \hat{p}_i) \end{cases} \\ \gamma_i: \begin{cases} \hat{q}_i = Pq_i + (I-P)\zeta_l \\ \hat{p}_i = v_i^d \xi_i \end{cases} \end{cases} \qquad (15)$$

where ζ_l is a point on the midlane curve of lane l and $P = I - \pi_i\pi_i^T$ is the projection matrix (note: for curved lanes, the last two equations need to be modified based on the definition of the γ_i-agent). Note that the choice of desired lane state l directly influences the continuous-time dynamics of vehicle i via ζ_l.

The autonomous vehicle in (15) is a hybrid system with a nonlinear switching dynamics and the continuum state $\Sigma_i = \mathrm{col}(q_i, p_i, \bar{q}_i, \bar{p}_i)$, the discrete state $Q_i = (\delta_b, \delta s, \delta_l, l_i)$, and the input $\Upsilon_i = (\tau, \zeta_l, v_i^d, \rho_i)$ where ρ_i is the target stopping range whenever it exists. The γ_i-agent with the state $(\hat{q}_i, \hat{p}_i)$ acts as an input signal to the φ_i-agent and thus the *γ-agents in autonomous driving are not dynamic systems* unlike in flocking; yet, we take the liberty to refer to the pair $(\hat{q}_i, \hat{p}_i)$ as the state of the γ_i-agent.

3.8 Fundamental Challenges and Differences with Stability Analysis of Flocking

The dynamics of an α-agent for flocking behavior can be stated as follows:

$$\dot{q}_i = p_i, \; \dot{p}_i = f_i^\alpha + f_i^\gamma \qquad (16)$$

where the state of the γ-agent is independent of all α-agents that have fixed dynamics. In comparison, the dynamics of vehicle i in (15) is far more complex and the states of the γ-agents depend on both the inputs and continuum and discrete states of the autonomous vehicles. Moreover, unlike in flocking, the interaction weights between vehicles is no longer symmetric. The complexities of dynamics, asymmetric inter-agent interactions, and interactions with the environment makes the structural stability analysis of self-driving vehicular networks tremendously more challenging. Formal stability analysis of the autonomous vehicular networks will be presented in upcoming papers and fundamentally relies on flocking theory.

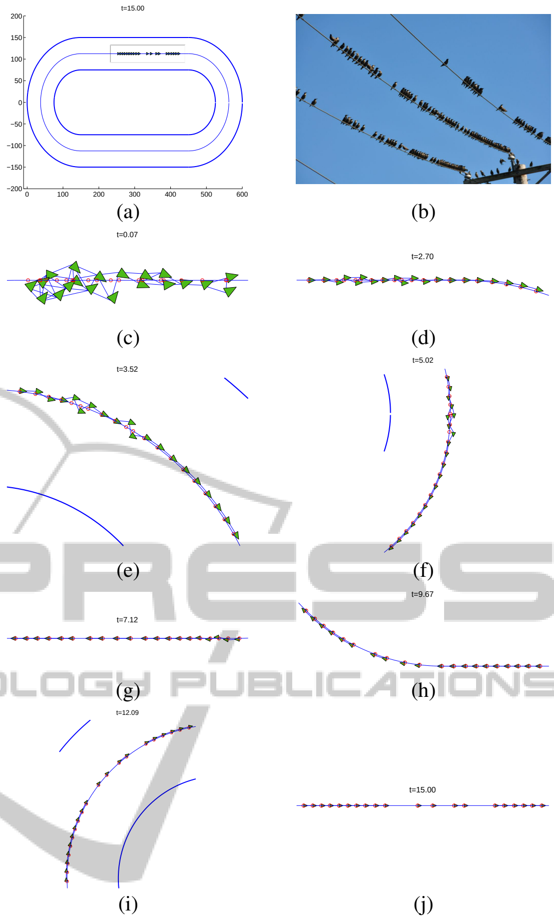

Figure 7: Autonomous driving for a vehicular network with $n = 20$ vehicles with random initial conditions (uniform) and desired velocities (Gaussian). Three 1D α-lattices (or platoons) asymptotically emerge and the vehicles within each platoon move with a consensus speed. (a) the track; (b) a 1D flock of birds forming α-lattices; (c) though (j) consecutive zoomed snapshots of the self-driving vehicular networks on the track in (a). Snapshot (a) is the global view of (j). The red circles show the γ-agents.

4 SIMULATIONS

Fig. 7 shows the consecutive snapshots of a vehicular network applying the autonomous lane-driving algorithm in (8). Four 1D α-lattices of autonomous vehicles (or platoons) asymptotically emerge. These platoons closely resemble 1D flocks of birds on power lines as seen in Fig. 7 (b)). The role of the area potential forces f_i^{ap} is critical in untangling challenging initial conditions of the vehicles with several triangular faces in Figs. 7 (c) through (f).

Fig. 8 shows the snapshots of the *passing maneuver* for a fast vehicle behind a group of slower vehicles with random desired speeds in a collision-free manner for all the vehicles involved. The sequence of mode switching of the fast vehicles are as follows: lane-driving, braking, lane-changing, lane-driving (for a relatively long period until all three slower vehicles in front of it change lanes), lane-changing, and lane-driving. The slower vehicles in the fast lane engage in lane-driving, lane-changing, and lane-driving. All other vehicles perform lane-driving throughout the passing maneuver of the fast vehicle. Notice that the slower vehicles in the fast lane can easily merge into the slower lane without collisions.

5 CONCLUSIONS

We introduced a multi-objective flocking framework for cooperative autonomous driving for vehicular networks and design of safety-aware intelligent transportation systems. We demonstrated that *flocking* is a valid particle-based model of vehicular traffic on roads. Using significant modifications of Olfati-Saber's flocking algorithms, we proposed novel algorithms for three basic modes of autonomous driving, namely, lane-driving, lane-changing, and braking. The combination of these three modes enables autonomously performing the passing maneuver, making turns, stopping, avoid tailgating, and V2P and V2V collision avoidance. Our main contribution is to introduce a unified autonomous driving algorithm called "Algorithm 1" in (15) which encompasses all the individual driving algorithms for every mode as special cases. The γ and φ agents are novel types of agents that did not exist in the original flocking theory in (Olfati-Saber, 2006). Our simulation results demonstrate that the proposed cooperative autonomous driving algorithm is effective and capable of successfully performing complex maneuvers such as lane-driving from random initial conditions and passing without collisions.

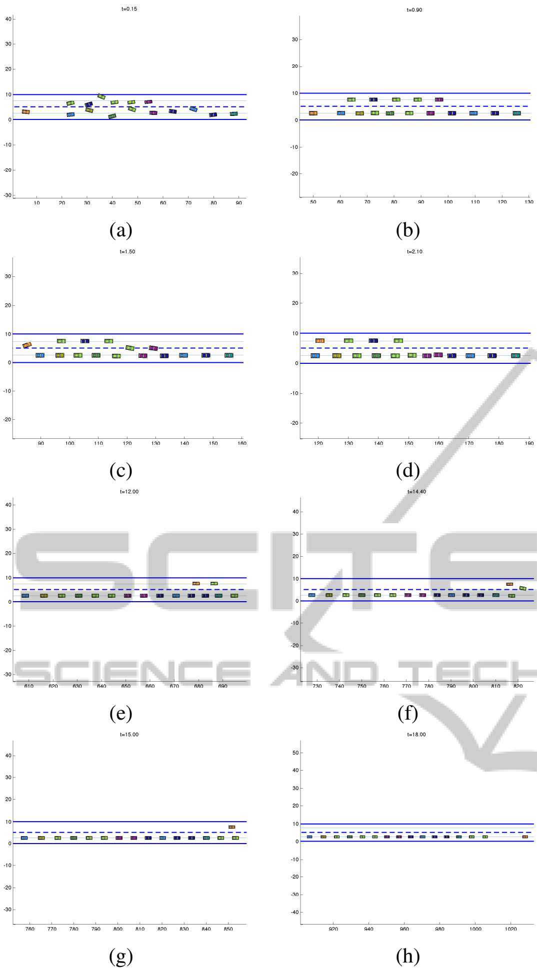

Figure 8: Consecutive snapshots of the *passing maneuver* for a fast vehicle behind a number of slower vehicles moving with a random set of uniformly distributed desired speeds on a two-lane highway starting from a random set of positions/velocities in (a). The slower vehicles in the fast lane perform lane-changing as soon as they detect a faster vehicle on the fast lane according to mode switching rules in Fig. 6.

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