Self–Scheduled H_∞ Control of a Wind Turbine

A Real Time Implementation

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Abstract: This paper is concerned with the design of robust gain-scheduled controllers with guaranteed H_∞ performance for a horizontal axis wind turbine (HAWT) with variable-speed and fixed-pitch. The control problem in terms of Linear Parameter-Varying (LPV) plants is stated and the theoretical background of the design method is given. Due to some interesting properties outlined in this paper, the synthesis problem is reduced to solving off-line a finite-dimensional set of Linear Matrix Inequalities (LMIs), making the controller suited for real-time applications. The computed LPV controller focuses on multiple objectives such as mechanical fatigue reduction, speed regulation and mode stabilization with simultaneously maximizing energy capture. The performances obtained through this control method are discussed and presented by means of a set of simulations. A real-time control algorithm for the large-scale wind turbines is also proposed.

1 INTRODUCTION

Nowadays, the development of electrical power generation from wind currents is a big concern for the society energy issue as well as for the management of electrical power systems. As wind turbines prove to be one of the cheapest, cleanest and most efficient sources of energy, it has become of great necessity to focus on complex algorithms to meet with multiple objectives.

The wind is the main energy source and, thus, it is of great importance to determine the characteristics of the wind currents passing through the turbine rotor. The stochastic nature of the wind determines the necessity of a wind turbine to be able to work under different wind velocities.

In order to keep the performance within these conditions, controllers have to be designed and implemented. Various control synthesis options have been applied in response to wind turbine control problem such as PID controllers, LQG controllers or fuzzy logic. The classical control structures proved to be simple and robust but most of the times they require the implementation of multiple control loops, in order to accomplish multiple control objectives. An interesting approach is the formulation of gain scheduling control. These techniques are largely used since they tackle the control of nonlinear systems with the tools of the well-known linear control theory. In the context of *Linear Parameter-Varying* (LPV) systems, the design follows a procedure similar to H_∞ synthesis.

Due to the development of the power converters and microcontrollers, a wind turbine can operate in a variable-speed mode, making it suitable for optimization. Thus, in this research a variable-speed fixed pitch wind turbine has been analyzed. The main goal was to design an LPV controller with guaranteed H_∞ - like performance, ensuring closed-loop stability.

This paper is structured as follows. Section 2 presents some theoretical aspects of the gain-scheduling problem and the synthesis procedure. In Section 3, an LPV model is determined in order to design the self-scheduled H_∞ controller. A set of simulations confirms the robustness of the system. Finally, the possibility of a digital implementation is discussed.

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2 THEORETICAL FRAMEWORK

2.1 LPV Models for Nonlinear Systems

It is a known fact that in control engineering, most of the dynamical systems are nonlinear. Nevertheless, they can be approximated as LTI systems around the equilibrium or some operating points. Then, by seeing the nonlinear dynamical system as a collection of LTI behaviors corresponding to different operating points, and using some well chosen variables to perform *switching* between them, one can have an approximation of the global behavior. Such a modeling approach, detailed in (Tóth, 2010), defines an LPV system. In the context of gain scheduling techniques, LPV models form a well known class of models, with practical applications in many fields of control engineering, *e.g.* modeling, system identification, and control.

An LPV system can be described by a state-space realization:

$$\mathbf{T}_{zw}:\begin{cases}\dot{\mathbf{x}}(t)=\boldsymbol{A}\left(\boldsymbol{\theta}(t)\right)\cdot\mathbf{x}(t)+\boldsymbol{B}\left(\boldsymbol{\theta}(t)\right)\cdot\mathbf{w}(t)\\ \mathbf{z}(t)=\boldsymbol{C}\left(\boldsymbol{\theta}(t)\right)\cdot\mathbf{x}(t)+\boldsymbol{D}\left(\boldsymbol{\theta}(t)\right)\cdot\mathbf{w}(t)\end{cases}, \quad (1)$$

where $\mathbf{x}\in\mathrm{R}^{nx}$ is the state vector, $\mathbf{z}\in\mathrm{R}^{nz}$ is the output or the error signal, $\mathbf{w}\in\mathrm{R}^{nw}$ is the input (disturbance), $\boldsymbol{\theta}\in\mathrm{R}^{n\theta}$ is the time-varying parameter vector, and $\boldsymbol{A}(\cdot),\boldsymbol{B}(\cdot),\boldsymbol{C}(\cdot),\boldsymbol{D}(\cdot)$ are continuous functions, evaluated at the operating points $\boldsymbol{\theta}$. When freezing $\boldsymbol{\theta}(t)$ to some given value $\boldsymbol{\theta}_0$, the LPV system (1) becomes an LTI system of transfer function:

$$\mathbf{T}_{zw}(s)=\boldsymbol{C}\left(\boldsymbol{\theta}_0\right)\left(s\mathbf{I}-\boldsymbol{A}\left(\boldsymbol{\theta}_0\right)\right)^{-1}\boldsymbol{B}\left(\boldsymbol{\theta}_0\right)+\boldsymbol{D}\left(\boldsymbol{\theta}_0\right).$$

Note that $\boldsymbol{\theta}(t)$ as well as its rate of variation $\dot{\boldsymbol{\theta}}(t)$ are assumed bounded, that is

$$\left(\boldsymbol{\theta}(t),\dot{\boldsymbol{\theta}}(t)\right)\in\Theta\times\tilde{\Theta},\quad\forall t\geq 0, \quad (2)$$

$$\begin{aligned}\Theta&:=\left\{\boldsymbol{\theta}(t):\ \left|\boldsymbol{\theta}(t)\right|\leq\boldsymbol{\theta}_{vi},\ i=1,\ldots,r,\ \forall t\geq 0\right\}\\ \tilde{\Theta}&:=\left\{\dot{\boldsymbol{\theta}}(t):\ \left|\dot{\boldsymbol{\theta}}(t)\right|\leq\tilde{\boldsymbol{\theta}}_{vi},\ i=1,\ldots,r,\ \forall t\geq 0\right\}\end{aligned},$$

which means that $\boldsymbol{\theta}(t)$ is valued in the polytope Θ, a bounded and connected set, with vertices in $\boldsymbol{\theta}_{vi}$, $i=1,\ldots,r$; similarly, $\dot{\boldsymbol{\theta}}(t)$ is valued in $\tilde{\Theta}$, having the same properties as Θ.

2.2 Stability and Performance

Stability of the LPV system defined in (1) can be established by finding a parameter-dependent Lyapunov function. This approach leads to the concept of *parameter-dependent quadratic* (PDQ) stability introduced in (Wu *et al.*, 1996). It was also shown that the PDQ stability condition implies the autonomous LPV system $\dot{\mathbf{x}}(t)=\boldsymbol{A}\left(\boldsymbol{\theta}(t)\right)\cdot\mathbf{x}(t)$ is uniformly exponentially stable.

The performance of a closed-loop system can be characterized in several ways. In LTI theory, the performance is commonly measured by the induced L_2-norm, using the well known Bounded Real Lemma (BRL). This famous lemma can be extended for an LPV system as a Linear Matrix Inequality (LMI) problem, as stated in (Becker and Packard, 1994), (Wu et al., 1996), (Apkarian and Adams, 1998), with quadratic parameter-dependent Lyapunov functions:

$$V\left(\mathbf{x},\boldsymbol{\theta}\right)=\mathbf{x}^T\mathbf{P}\left(\boldsymbol{\theta}\right)\mathbf{x}, \quad (3)$$

where $\mathbf{P}(\boldsymbol{\theta}):\Theta\to\mathrm{R}^{nx\times nx}$. In order for the problem to have a solution to the extended problem, the time-varying parameter $\boldsymbol{\theta}(t)$ has to be bounded, as in (2).

The induced L_2-norm for the LPV system (1) is defined as:

$$\left\|\mathbf{T}_{zw}\right\|_2:=\sup_{\left(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}\right)\in\Theta\times\tilde{\Theta}}\ \sup_{\mathbf{w}\in L_2}\frac{\left\|\mathbf{z}\right\|_2}{\left\|\mathbf{w}\right\|_2}, \quad (4)$$

where L_2 denotes the space of the Lebesgue square integrable vector functions with the corresponding norm. If the input-output operator $\mathbf{T}_{zw}:\mathbf{w}\to\mathbf{z}$ has an induced L_2-norm bounded by $\gamma>0$, *i.e.*:

$$\left\|\mathbf{T}_{zw}\right\|_2<\gamma,$$

then, according to (4), we can write:

$$\int_0^\infty\mathbf{z}^T(\tau)\mathbf{z}(\tau)\,\mathrm{d}\tau<\gamma^2\int_0^\infty\mathbf{w}^T(\tau)\mathbf{w}(\tau)\,\mathrm{d}\tau. \quad (5)$$

The bounded real lemma states that the LPV system (1) is PDQ stable over Θ and has $\left\|\mathbf{T}_{zw}\right\|_2<\gamma$ if there exists a differentiable matrix function $\mathbf{P}(\boldsymbol{\theta}):\Theta\to\mathrm{R}^{nx\times nx}$ such that $\mathbf{P}(\boldsymbol{\theta})>0$ and the symmetric matrix:

$$\begin{bmatrix} \mathcal{A}^{T}(\boldsymbol{\theta})\mathbf{P}(\boldsymbol{\theta})+\mathbf{P}(\boldsymbol{\theta})\mathcal{A}(\boldsymbol{\theta})+\dot{\mathbf{P}}(\boldsymbol{\theta}) & * & * \\ \mathcal{B}^{T}(\boldsymbol{\theta})\mathbf{P}(\boldsymbol{\theta}) & -\gamma\mathbf{I} & * \\ \mathcal{C}(\boldsymbol{\theta}) & \mathcal{D}(\boldsymbol{\theta}) & -\gamma\mathbf{I} \end{bmatrix} < 0 \tag{6}$$

for all $(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \in \Theta\times\tilde{\Theta}$. In this formulation, $*$ denotes the transpose of the corresponding block matrix, and $\dot{\mathbf{P}}(\boldsymbol{\theta})$ can be expressed as:

$$\mathbf{P}(\boldsymbol{\theta}) = \sum_{k=1}^{n\theta} \dot{\boldsymbol{\theta}} \frac{\partial \mathbf{P}(\boldsymbol{\theta})}{\partial \theta_k}. \tag{7}$$

Note that (6) is an infinite-dimensional LMI problem. Also note that this problem represents a generalization of the standard sub-optimal H_∞ control problem (Zhou *et.al*, 1996) and conceptually expands the applicability and usefulness of the H_∞ methodology.

2.3 Problem Statement

Roughly speaking, there are two main approaches to design LPV gain-scheduled controllers. One of them is based on a version of the *Small Gain Theorem*, applicable to LPV systems with fractional parameter dependence, namely *the LFT gain scheduling technique*, devised in (Packard, 1994), (Scorletti and El Gahoui, 1998). A drawback of the LFT formulation is that the variations of $\boldsymbol{\theta}$ are allowed to be *complex*, thus introducing some conservatism when parameters are known to be real. The other approach, namely *the quadratic gain scheduling*, based on Lyapunov theory and the notion of *Quadratic H_∞ performance* (Apkarian and Adams, 1998), (Apkarian *et al.*, 1995), (Wu, 2001), is used in this paper.

Consider an open-loop LPV system $\mathbf{G}(\boldsymbol{\theta})$ with state-space realization:

$$\mathbf{G}(\boldsymbol{\theta}):\begin{bmatrix}\dot{\mathbf{x}}\\ \mathbf{z}\\ \mathbf{y}\end{bmatrix}=\begin{bmatrix}\mathbf{A}(\boldsymbol{\theta}) & \mathbf{B}_1(\boldsymbol{\theta}) & \mathbf{B}_2(\boldsymbol{\theta})\\ \mathbf{C}_1(\boldsymbol{\theta}) & \mathbf{D}_{11}(\boldsymbol{\theta}) & \mathbf{D}_{12}(\boldsymbol{\theta})\\ \mathbf{C}_2(\boldsymbol{\theta}) & \mathbf{D}_{21}(\boldsymbol{\theta}) & \mathbf{0}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{x}\\ \mathbf{w}\\ \mathbf{u}\end{bmatrix}, \tag{8}$$

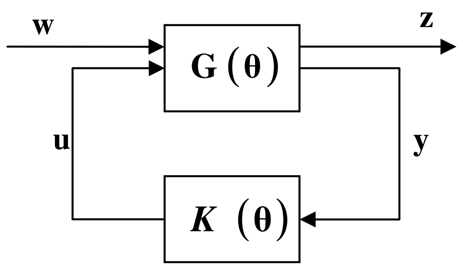

Figure 1: The closed-loop system.

where $\mathbf{x}(t)\in \mathrm{R}^{nx}$ is the state vector, $\mathbf{w}(t)\in \mathrm{R}^{nw}$ is the disturbance, $\mathbf{u}(t)\in \mathrm{R}^{nu}$ is the control input, $\mathbf{z}(t)\in \mathrm{R}^{nz}$ is the error signal, and $\mathbf{y}(t)\in \mathrm{R}^{ny}$ is the output measured vector. The time variation of each of the parameters $\boldsymbol{\theta}(t)$ is not known in advance, but is assumed to be measurable in real-time.

The gain-scheduled output-feedback control problem consists of finding a dynamic LPV controller $\mathcal{K}(\boldsymbol{\theta})$ with state space equations:

$$\mathcal{K}(\boldsymbol{\theta}):\begin{bmatrix}\dot{\mathbf{x}}_K\\ \mathbf{u}\end{bmatrix}=\begin{bmatrix}\mathbf{A}_K(\boldsymbol{\theta}) & \mathbf{B}_K(\boldsymbol{\theta})\\ \mathbf{C}_K(\boldsymbol{\theta}) & \mathbf{D}_K(\boldsymbol{\theta})\end{bmatrix}\cdot\begin{bmatrix}\mathbf{x}_K\\ \mathbf{y}\end{bmatrix}, \tag{9}$$

which ensures PDQ stability and a guaranteed L_2 − gain bound $\gamma > 0$ for the closed-loop system interconnected as shown in Figure 1.

Note that the closed-loop system has an input-output operator $\mathbf{T}_{zw}$ described by (1) & (2), bounded as in (5):

$$\mathbf{T}_{zw}:\begin{bmatrix}\dot{\mathbf{x}}_{CL}(t)\\ \mathbf{z}(t)\end{bmatrix}=\begin{bmatrix}\mathcal{A}(\boldsymbol{\theta}) & \mathcal{B}(\boldsymbol{\theta})\\ \mathcal{C}(\boldsymbol{\theta}) & \mathcal{D}(\boldsymbol{\theta})\end{bmatrix}\begin{bmatrix}\mathbf{x}_{CL}(t)\\ \mathbf{w}(t)\end{bmatrix}, \tag{10}$$

where $\mathbf{x}_{CL}^{T}=\left[\mathbf{x}^{T},\mathbf{x}_{K}^{T}\right]$ denotes the state space vector of the closed-loop system.

The basic characterization of LPV controller (9) with guaranteed stability and performance is given by the *Basic Characterization Theorem*, stated and proved in (Scherer, 1995). Basically, the theorem states that the controller can be easily obtained if there exists some parameter-dependent matrices such that an infinite-dimensional set of LMI problems holds (one for each $(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\in\Theta\times\tilde{\Theta}$). The unknown parameter-dependent matrices can be found by solving an infinite-dimensional convex optimization problem with LMIs and an infinite set of decision variables, where the objective function is γ. The set of LMIs can be obtained after replacing the closed-loop system matrices $\mathcal{A}(\cdot),\mathcal{B}(\cdot),\mathcal{C}(\cdot)$, $\mathcal{D}(\cdot)$, derived from (8) & (9), in (6).

Techniques to reduce the infinite-dimensional problem to finite-constraint and finite-dimensional, practical validity of gain-sheduled controllers, and some computational aspects have also been treated in (Apkarian *et al.*, 1995), (Wu *et al.*, 1996), (Apkarian and Adams, 1998). In this paper, we will focus on the case when the matrices of the plant are *affine* in the parameter. Some interesting properties for this particular case (important for this research) will be revealed and analyzed in the following.

2.4 Self-scheduled H_∞ Control

Consider the LPV plant (8) with matrices affine in paramenter $\boldsymbol{\theta}$, *i.e.*:

$$\begin{bmatrix}\dot{\mathbf{x}}\\ \mathbf{z}\\ \mathbf{y}\end{bmatrix}=\left(\begin{bmatrix}\mathbf{A}_0 & \mathbf{B}_{1,0} & \mathbf{B}_2\\ \mathbf{C}_1 & \mathbf{D}_{11} & \mathbf{0}\\ \mathbf{C}_2 & \mathbf{D}_{21} & \mathbf{0}\end{bmatrix}+\sum_{k=1}^{n_\theta}\boldsymbol{\theta}_k\cdot\begin{bmatrix}\mathbf{A}_k & \mathbf{B}_{1,k} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\right)\cdot\begin{bmatrix}\mathbf{x}\\ \mathbf{w}\\ \mathbf{u}\end{bmatrix} \quad (11)$$

In addition, assume that the parameter $\boldsymbol{\theta}$ varies in a convex polytope Θ with *r* vertices, *i.e.*:

$$\boldsymbol{\theta}(t)\in\Theta:=\mathrm{Co}\{\boldsymbol{\theta}_{v1},\ldots,\boldsymbol{\theta}_{vr}\}, \quad (12)$$

where the convex hull of a finite number of matrices $\mathbf{M}_i$ (with the same dimensions) is defined as:

$$\mathrm{Co}\{\mathbf{M}_i : i=1,\ldots,r\}:=\left\{\sum_{k=1}^{r}\alpha_i\mathbf{M}_i : \alpha_i\ge 0,\sum_{k=1}^{r}\alpha_i=1\right\} \quad (13)$$

Note that matrices $\mathbf{B}_2,\mathbf{C}_1,\mathbf{C}_2,\mathbf{D}_{11}$ and $\mathbf{D}_{21}$ are parameter independent. If this assumption is not satisfied, the computation of solutions is not easily tractable. Though not fully general, this description encompasses many practical situations, including our case study.

In this case, because of the LMIs properties (multi-convexity and vertex property), the infinite number of constraints is reduced to a finite set of LMIs. To allow quadratic stabilization of the LPV system (11) by an output feedback controller, one assumes that the pairs $(\mathbf{A}(\cdot),\mathbf{B}_2)$ and $(\mathbf{A}(\cdot),\mathbf{C}_2)$ are quadratically stabilizable and quadratically detectable over Θ. Furthermore, assuming that the Lyapunov matrix $\mathbf{P}(\boldsymbol{\theta})$ is constant, *i.e.* $\mathbf{P}(\boldsymbol{\theta})=\boldsymbol{P}$, $\forall\boldsymbol{\theta}\in\Theta$, a H_∞-like control problem arises. The synthesis procedure for a self-scheduled H_∞ controller (Apkarian *et al.*, 1995), reformulated for our case study, is formalized in the next theorem.

Theorem. *Consider the LPV system* (11). *Given some* $\gamma>0$*, the following statements are equivalent*
(i) there exists an LPV controller (9) *such that the closed-loop system is stable and* $\|\mathbf{T}_{zw}\|_2<\gamma$*;*
(ii) there exists $\boldsymbol{P}>0$ *and LTI controllers* $\boldsymbol{K}(\boldsymbol{\theta}_{vi})$ *such that:*

$$\begin{bmatrix}\boldsymbol{A}^T(\boldsymbol{\theta}_{vi})\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}(\boldsymbol{\theta}_{vi}) & * & *\\ \boldsymbol{B}^T(\boldsymbol{\theta}_{vi})\boldsymbol{P} & -\gamma\mathbf{I} & *\\ \boldsymbol{C}(\boldsymbol{\theta}_{vi}) & \boldsymbol{D}(\boldsymbol{\theta}_{vi}) & -\gamma\mathbf{I}\end{bmatrix}<0 \quad (14)$$

where $\boldsymbol{\theta}_{vi}$, $i=1,\ldots,r$ *are the vertices of* Θ*, defined in* (12)*. If (i) or (ii) is satisfied, the LPV controller matrices can be computed as follows:*

$$\begin{bmatrix}\mathbf{A}_K(\boldsymbol{\theta}) & \mathbf{B}_K(\boldsymbol{\theta})\\ \mathbf{C}_K(\boldsymbol{\theta}) & \mathbf{D}_K(\boldsymbol{\theta})\end{bmatrix}=\sum_{i=1}^{r}\alpha_i\begin{bmatrix}\mathbf{A}_K(\boldsymbol{\theta}_{vi}) & \mathbf{B}_K(\boldsymbol{\theta}_{vi})\\ \mathbf{C}_K(\boldsymbol{\theta}_{vi}) & \mathbf{D}_K(\boldsymbol{\theta}_{vi})\end{bmatrix} \quad (15)$$

where $\alpha_1,\ldots,\alpha_r$ *is any solution of the convex decomposition problem:*

$$\boldsymbol{\theta}(t)=\sum_{i=1}^{r}\alpha_i\boldsymbol{\theta}_{vi}. \quad (16)$$

Note that a single Lyapunov function $V(\mathbf{x})=\mathbf{x}^T\boldsymbol{P}\,\mathbf{x}$, ensuring stability and performance over Θ, is used over the entire operating range. The controller implementation requires the on-line solution of the factorization problem (16), while the vertex controllers $\boldsymbol{K}(\boldsymbol{\theta}_{vi})$ can be computed off-line. Thus, the controller $\boldsymbol{K}(\boldsymbol{\theta})$ is updated in *real time* based on the parameter measurement $\boldsymbol{\theta}(t)$.

From (11) and (12), it is clear that the system state space matrices range in a polytope of matrices whose vertices are the images of the vertices $\boldsymbol{\theta}_{v1},\ldots,\boldsymbol{\theta}_{vr}$. Thus, if we restrict ourselves to LPV controllers, there is no loss of generality in assuming that the controller ranges in a polytope of matrices.

3 SELF-SCHEDULED H_∞ CONTROL OF A HAWT

3.1 The HAWT as a System

The research concerning modeling and control of renewable energy production systems based on wind activity has known an impressive development in the last years (Jain, 2011), (Pao and Johnson, 2011). One of the most targeted such systems is the *Horizontal Axis Wind Turbine* (HAWT), with a three blades propeller.

As Figure 2 is displaying, the structure of a HAWT is modular. The main blocks and signals that define the HAWT as a system to be modeled and controlled are also illustrated, where V is the wind speed (the main input of HAWT), ω_R and ω_G are the angular speed of the rotor and generator, respectively, T_R and T_G are the aerodynamic torque and the electromagnetic torque, β_{ref} and β are the desired/actual pitch angle of the blades, while

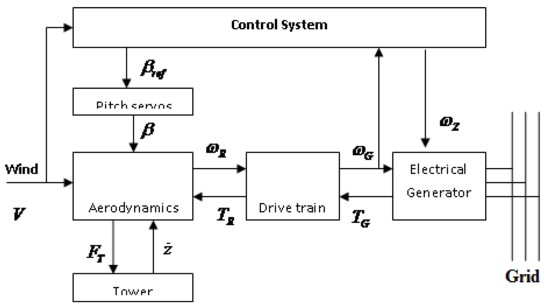

Figure 2: Wind energy conversion structure.

ω_Z is the control input of the electrical generator. The structure comprised by the tower and the foundation supports the thrust force F_T, producing an axial displacement z of the tower, nacelle and blades.

Various analytical models of wind turbines were introduced in the literature so far. A complete description of wind energy conversion systems can be found in (Burton *et al.*, 2001), (Manwell *et al.*, 2009). Nowadays, there is a trend to take into account even the smallest constructive details. Quite complex models based on finite element theory are also adopted, in order to describe the blades variable geometry. But the most important and complex subsystem of a wind turbine is the electrical generator. Many wind turbines installed in grid connected application use squirrel cage induction generators (SQIG), operating within a range of speeds slightly higher than the synchronous speed. Driven by the desire of operating the wind turbine at maximum efficiency, an increasingly popular option today is the doubly fed IG (DFIG), being used in variable-speed applications.

In this paper, the main goal is to shape and design the control strategy for a variable speed wind turbine. Thus, by using suitable power electronic converters in our variable-speed machine, a robust controller could be implemented. The pitch angle will be fixed at its optimum value, that is $\beta_{opt} = 1^\circ$.

3.2 Control Objectives and Strategies

One promising way to reduce the electricity cost produced by a wind energy conversion system is to improve its control system. This involves a series of partial objectives and the judicious balancing of their requirements. First of all, maximizing the energy production is a main requirement. This involves optimum conversion of wind energy, guaranteeing both maximum yield and a good power quality. Another objective is to maximize the faultless life of the rotor drive train and other structural components (actuators, mechanical structure) in the presence of changes in the wind (direction, speed, turbulence), as well as start-stop cycles. These two objectives are actually conflicting (the tighter the closed loop tracks the control strategy, the larger the transient loads will be), and therefore well balanced compromise must be formulated.

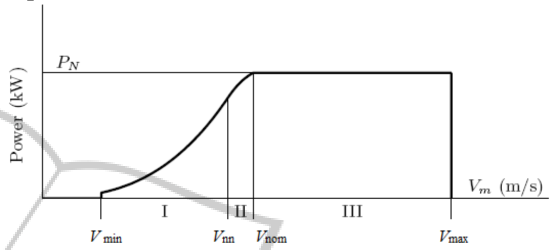

Figure 3: Power characteristic of a HAWT.

As already known, the ideal power characteristic of a HAWT looks like in Figure 3. The turbine analyzed here is generating the nominal power $P_{\text{nom}} = 400\,\text{kW}$ it was designed to provide, only in case the wind speed is large enough (at least 12 m/s) and varies in range III, between V_{nom} and $V_{\max} = 25$ m/s, the cut-off speed. When V varies in range I, that is between the cut-in speed $V_{\min} = 5$ m/s and $V_{\text{nn}} = 10$ m/s, the generated power is smaller. Thus, the generation objective is to extract all the available power. Finally, there is region II, which is a transition between region I and III.

A well chosen control strategy can provide a trade-off between the ideal power characteristics and the maximum faultless life of the structural components. The *basic control strategy*, adopted in this research, and detailed in (Lescher, 2006), is plotted in Figure 4, in the parameter space $(\bar{V}, \bar{\omega})$, formed by the generator rotational speed and the wind speed.

This strategy is selected to make the best use of the HAWT. The function that describes the dependence $\bar{\omega}(\bar{V})$ is defined as:

$$\bar{\omega}(\bar{V}) = \begin{cases} \dfrac{\lambda_{opt}\bar{V}}{R} & \bar{V} \le V_{\text{nn}} \\ \omega_{\text{nn}} & V_{\text{nn}} \le \bar{V} \le V_{\text{nom}} \\ \omega_G \ \text{ s.t. } kC_P\left(\dfrac{R\omega_G}{\bar{V}}\right)\bar{V}^3 = P_{\text{nom}} & \bar{V} \ge V_{\text{nom}} \end{cases} \quad (17)$$

Note that this curve represents the desired trajectory (thus, the locus) of all the operating points

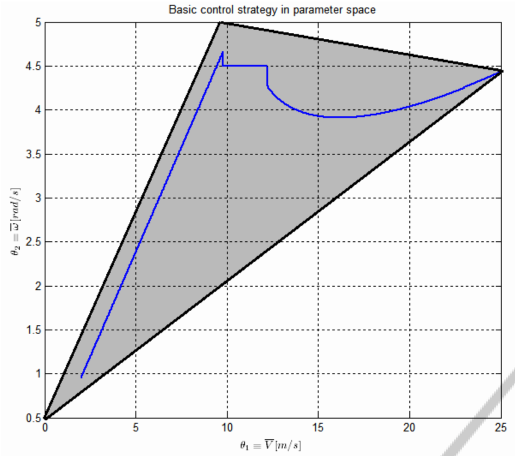

Figure 4: Parameter trajectory and polytope Θ.

in the parameter space $\boldsymbol{\theta}$. Hence, the controller setup and design involve the optimization of the control strategy tracking. Also note that the operating points have been covered with a convex polytope of three vertices

$$\Theta = \mathrm{Co}\{\boldsymbol{\theta}_{v1}, \boldsymbol{\theta}_{v2}, \boldsymbol{\theta}_{v3}\}, \tag{18}$$

with

$\boldsymbol{\theta}_{v1} = (0; 0.5)$, $\boldsymbol{\theta}_{v2} = (9; 5)$, $\boldsymbol{\theta}_{v3} = (25; 4.43)$.

In this favorable situation, as discussed in Section 2, the synthesis of the LPV controller is very simple. In fact, due to the LMIs multi-convexity properties, we only have to check the set of LMIs (14) at the vertices of Θ; the controller is then obtained as a linear combination of three LTI controllers.

3.3 LPV Model of Hawts

In case of VS-FP wind turbines, there is only one control action, applied to the electrical machine. Hence, a reduced model of HAWT will be used, which neglects the high-frequency dynamics, treated as model uncertainty. Despite the simplicity, the reduced model describes well the dominant system dynamics at low frequencies, thus is suitable for designing a robust self-scheduled H_∞ controller.

As stated in section 2, an LPV model of the nonlinear system can be obtained by linearization around a set of equilibrium points. Although the LPV model is known only for a finite set of scheduled variables $\boldsymbol{\theta}$, it is well defined for all $(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \in \Theta \times \tilde{\Theta}$.

After the order reduction and the linearization around a set of equilibrium points, the dynamic model of the HAWT is described by (Bianchi *et al.*, 2007):

$$\mathbf{M}(\boldsymbol{\theta}): \begin{cases} \dot{\mathbf{x}} = \mathbf{A}(\boldsymbol{\theta})\mathbf{x} + \mathbf{B}_V(\boldsymbol{\theta})\hat{V} + \mathbf{B}_Z\hat{\omega}_Z \\ \begin{bmatrix} \hat{T}_S \\ \hat{\omega}_G \\ \hat{T}_G \end{bmatrix} = \mathbf{Cx} + \mathbf{D}\begin{bmatrix} \hat{V} \\ \hat{\omega}_Z \end{bmatrix} \end{cases}, \tag{19}$$

where the state and parameter vectors are:

$$\begin{aligned} \boldsymbol{\theta} &= [\bar{V} \quad \bar{\omega}]^T \\ \mathbf{x} &= [\hat{\theta}_S \quad \hat{\omega}_R \quad \hat{\omega}_G]^T \end{aligned}. \tag{20}$$

In this formulation, the bars and the hats over the variables means operating point (steady-state) value, and small variations with respect to the operating point, *e.g.* $V(t) = \bar{V}(t) + \hat{V}(t)$, respectively. The model's inputs are the turbulence $\hat{V}$, regarded as a disturbance, and the control action ω_Z. The outputs are the shaft torque T_S, the generator speed and torque. The matrices of the model are:

$$\mathbf{A}(\boldsymbol{\theta}) = \begin{bmatrix} 0 & 1 & -1 \\ -\dfrac{k_S}{J_R} & -\dfrac{b_R(\boldsymbol{\theta}) + b_S}{J_R} & \dfrac{b_S}{J_R} \\ \dfrac{k_S}{J_G} & \dfrac{b_S}{J_G} & -\dfrac{b_S + b_G}{J_G} \end{bmatrix};$$

$$\mathbf{B}_V(\boldsymbol{\theta}) = \begin{bmatrix} 0 & \dfrac{k_R(\boldsymbol{\theta})}{J_R} & 0 \end{bmatrix}^T;$$

$$\mathbf{B}_Z = \begin{bmatrix} 0 & 0 & \dfrac{b_G}{J_G} \end{bmatrix}^T;$$

$$\mathbf{C} = \begin{bmatrix} k_S & b_S & -b_S \\ 0 & 0 & 1 \\ 0 & 0 & b_G \end{bmatrix}; \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -b_G \end{bmatrix}. \tag{21}$$

In order to have a full description of the model (19), a mathematical characterization of the coefficients $b_R(\boldsymbol{\theta})$ and $k_R(\boldsymbol{\theta})$ is required. These coefficients have been obtained by linearization of the power and the thrust coefficients, $C_P(\lambda)$ and $C_T(\lambda)$, which are usually available for a given HAWT. Thus, we have:

$$\begin{aligned} b_R(\boldsymbol{\theta}) &\equiv b_R\left(\bar{V}, \bar{\varpi}\right) = a_1\bar{V} + a_2\bar{\varpi} \\ k_R(\boldsymbol{\theta}) &\equiv k_R\left(\bar{V}, \bar{\varpi}\right) = a_3\bar{V} + a_4\bar{\varpi} \end{aligned} \quad (22)$$

The remaining parameters from equations (19) to (22) are defined in Appendix. Note that the LPV model (19) is affine in the parameter $\boldsymbol{\theta}$. This property will turn out to be very useful for an LPV controller design.

To accomplish the aforementioned control objectives, we need to develop a series of tasks such as the selection of the control scheme and controlled variables, and computation of the reference signals. A control scheme typically used to implement VS-FP control strategy is a common speed feedback loop. The speed reference is defined according to *the basic control strategy*, *i.e.* $\omega_{ref} = \omega_{ref}(V)$ is the function depicted in (17). Thus, the graph of $\omega_{ref}(V)$ has the same shape as the basic control strategy, plotted in Figure 4.

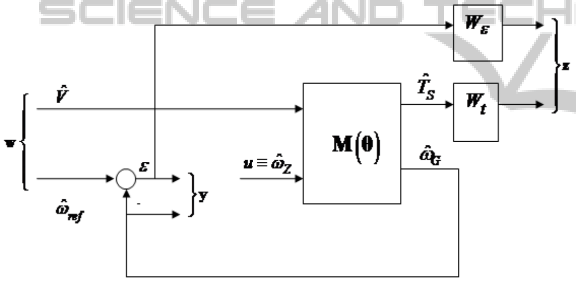

Figure 5: The open-loop LPV plant.

The first step of the LPV controller design is to state the control objectives in terms of the minimization of induced $L_2 - \text{norm}$ of certain input-output operator $\mathbf{T}_{zw} : \mathbf{w} \to \mathbf{z}$. This entails the selection of the input variable **w**, the disturbance, the virtual output variable **z** (called *performance output*) and some weighting functions. Recall that the first objective is to follow the control strategy, *i.e.* to minimize the error $\varepsilon = \omega_{ref} - \omega_G$, and the second objective is to the HAWT from excessive dynamic loads. Therefore, by tacking:

$$\begin{aligned} \mathbf{w} &:= \begin{bmatrix} \hat{V} & \hat{\omega}_{ref} \end{bmatrix}^T \\ \mathbf{z} &:= \begin{bmatrix} \tilde{\varepsilon} & \tilde{\hat{T}}_S \end{bmatrix}^T \end{aligned} \quad (23)$$

the objectives are introduced into the problem. Thus, the corresponding LPV plant of the HAWT is sketched in Figure 5, where W_ε and W_t are the weighting functions (to be determined), and $\mathbf{M}(\boldsymbol{\theta})$ is the input-output operator of HAWT, defined in (19). It is worth mentioning that the output $\mathbf{y} = \begin{bmatrix} \varepsilon & \hat{\omega}_G \end{bmatrix}^T$ and parameter vectors are measurable, which is an important aspect for the implementability of the controller.

Under these assumptions, the open-loop LPV model can be characterized as in (8), that is:

$$\mathbf{G}(\boldsymbol{\theta}) : \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{z} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{A}(\boldsymbol{\theta}) & \mathbf{B}_1(\boldsymbol{\theta}) & \mathbf{B}_2 \\ \mathbf{C}_1 & \mathbf{D}_{11} & \mathbf{0} \\ \mathbf{C}_2 & \mathbf{D}_{21} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{x} \\ \mathbf{w} \\ \mathbf{u} \end{bmatrix}, \quad (24)$$

where $\mathbf{A}(\boldsymbol{\theta})$ is the same as in expresion (21), and:

$$\mathbf{B}_1(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{B}_V(\boldsymbol{\theta}) & \mathbf{0}_{3\times 1} \end{bmatrix}; \ \mathbf{B}_2 = \mathbf{B}_Z;$$

$$\mathbf{C}_1 = \begin{bmatrix} 0 & 0 & -W_\varepsilon \\ W_t k_S & W_t b_S & -W_t b_S \end{bmatrix}; \ \mathbf{D}_{11} = \begin{bmatrix} 0 & W_\varepsilon \\ 0 & 0 \end{bmatrix}; \quad (25)$$
$$\mathbf{C}_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}; \qquad \mathbf{D}_{21} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

3.4 Controller Synthesis and Results

The model described in (24) and (25) is affine in the parameter $\boldsymbol{\theta}$, that is:

$$\begin{aligned} \mathbf{A}(\boldsymbol{\theta}) &= \mathbf{A}_0 + \theta_1\mathbf{A}_1 + \theta_2\mathbf{A}_2 \\ \mathbf{B}_1(\boldsymbol{\theta}) &= \mathbf{B}_{1,0} + \theta_1\mathbf{B}_{1,1} + \theta_2\mathbf{B}_{1,2} \end{aligned} \quad (26)$$

Moreover, the time-varying parameter $\boldsymbol{\theta}$ is valued in the convex polytope Θ (18). Hence, based on the theorem presented in Section 2, we devise the following constructive approach to the LPV self-scheduled H_∞ controller synthesis:

- compute a matrix $\boldsymbol{P} > 0$ and adequate LTI controllers $\boldsymbol{K}(\boldsymbol{\theta}_{vi})$ at the vertices $\boldsymbol{\theta}_{vi}$ of the parameter polytope, solving the LMIs (14);
- define LPV controller $\boldsymbol{K}(\boldsymbol{\theta})$ as an interpolant of the vertex controllers $\boldsymbol{K}(\boldsymbol{\theta}_{vi})$, as in (15). The interpolation is based on the position of $\boldsymbol{\theta}$ in the polytope Θ, given by the decomposition (16).

Note that, in this case, the decomposition problem has an unique solution:

$$\begin{bmatrix} \boldsymbol{\theta}_{v1} & \boldsymbol{\theta}_{v2} & \boldsymbol{\theta}_{v3} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1(t_k) \\ \alpha_2(t_k) \\ \alpha_3(t_k) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}(t_k) \\ 1 \end{bmatrix}. \quad (27)$$

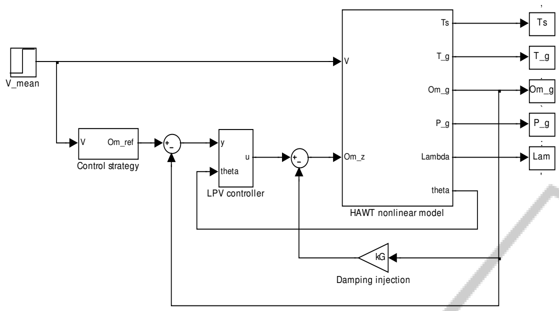

Figure 6: Speed control loop for VS operation.

The LMI problem has been solved using the Matlab's LMI solver. The function *feasp* (Robust Control Toolbox) solves the feasibility problem defined by the given LMIs constraints (14). The algorithm reaches convergence within 16 steps, for a $\gamma = 1.03$. For a real time implementation, this solution is computed off-line. With these specifications, we can formulate a real time synthesis algorithm.

Algorithm (Real Time controller)

Input. *Matrices* $\mathbf{A}_{K,i}, \mathbf{B}_{K,i}, \mathbf{C}_{K,i}, \mathbf{D}_{K,i}$, $i = 1,2,3$*;*

1. For $k \geq 0$

1.1. At time t_k*, the scheduling variable* $\boldsymbol{\theta}(t_k) = \boldsymbol{\theta}_k$ *is measured and the coefficients* $\alpha_i(t_k)$ *satisfying* (27) *are computed ;*

1.2. The LPV controller matrices are computed:

$$\begin{bmatrix} \mathbf{A}_K(\boldsymbol{\theta}) & \mathbf{B}_K(\boldsymbol{\theta}) \\ \mathbf{C}_K(\boldsymbol{\theta}) & \mathbf{D}_K(\boldsymbol{\theta}) \end{bmatrix} = \sum_{i=1}^{3} \alpha_i \begin{bmatrix} \mathbf{A}_{K,i} & \mathbf{B}_{K,i} \\ \mathbf{C}_{K,i} & \mathbf{D}_{K,i} \end{bmatrix}$$

Output. *The control signal* $u = \omega_Z$*, obtained by integration of* (10).

Simulation Results and Discussion

The nonlinear model of the HAWT, presented in (Tudor, 2011), is used in the following simulations. The wind speed signal is modeled as a non-stationary random process, split in two components, $V(t) = \bar{V}(t) + \hat{V}(t)$. The mean wind speed is the low frequency component, describing the behavior of the wind currents on a long term. The turbulent component $\hat{V}$ corresponds to fast variations (high frequency). In this paper, the turbulence is modeled as a unity intensity white noise process filtered by an adaptive stable filter von Karman.

The implemented speed feedback loop is sketched in Figure 6. Its external signal $\boldsymbol{\theta}$ is the measured scheduling variable.

The step response of the closed-loop nonlinear system is assessed first. Figure 7 shows the response to a mean wind speed step in region I, from 9 m/s to 7 m/s, at t = 25 s. The control strategy is designed to maximize the power conversion efficency, which is equivalent to track the HAWT's tip speed ratio λ at its optimum value $\lambda_{opt} = 8$. In region II, large oscillations are expected. The control objective is therefore to limit the rotor speed at some well chosen value (in this case, $\omega_{\text{nom}} = 4.5$ rad/s). This situation is depicted in Figure 8; the mean wind speed step is from 10.2 m/s to 11.8 m/s, at t = 25 s.

4 CONCLUSIONS

The simulations show that, in regions I and II, the controller performances are quite good. More than that, by analyzing the response of the closed-loop system to a realistic wind profile, we can conclude that the control objectives are fulfilled.

The real time algorithm, presented in section 3, is efficient from the numerical point of view (the number of arithmetic operations is quite small – the convex decomposition problem consists in a 3x3 linear equation system; the computation of the controller requires a low number of multiplications). Thus, the LPV self-scheduled H_∞ controller is suitable for a real time implementation.

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APPENDIX

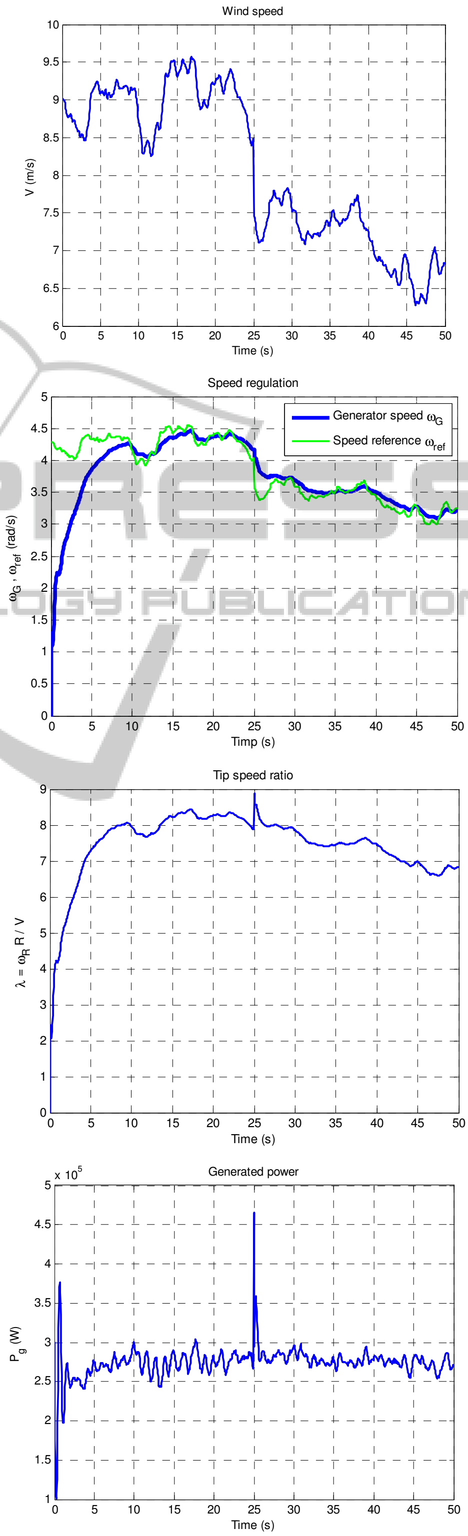

Figure 7: Closed-loop response in region I.

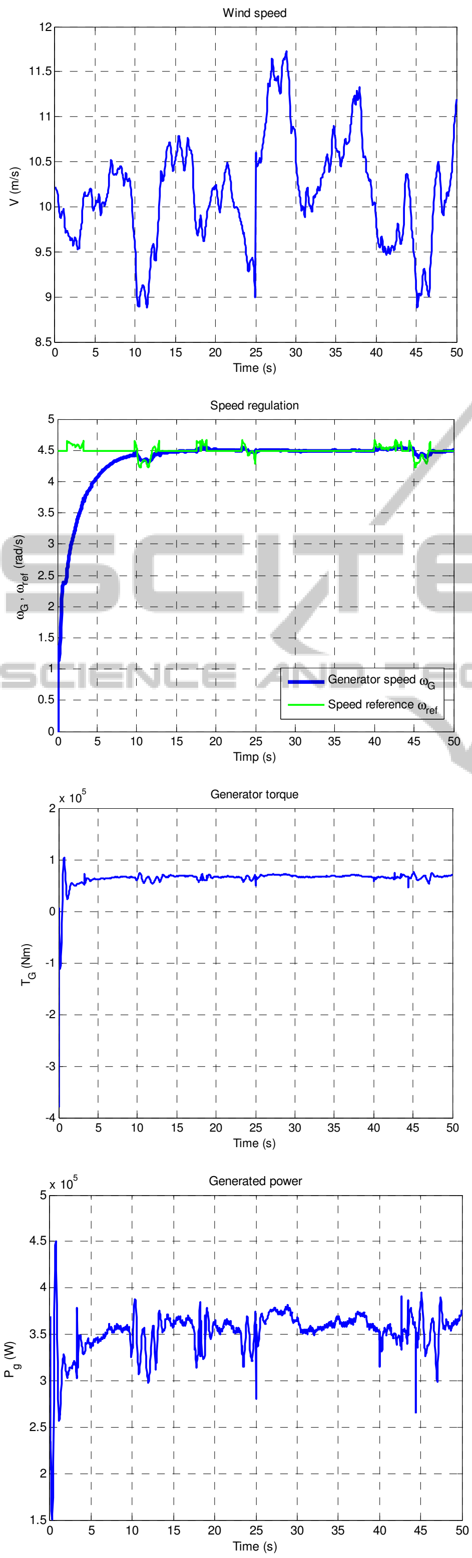

Figure 8: Closed-loop response in region II.