Modified Hybrid Evolutionary Strategies Method for Termination Control Problem with Relay Actuator

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Keywords: Termination Control Problem, Evolutionary Strategies, Relay Control, Two-point Boundary Problem, Dynamic Systems.

Abstract: The termination control problem, i.e., the problem with finite time and relay control function, is considered. The proposed approach fits different control problems, e.g., problems with the fixed or free time, problems with the set-up actuator characteristics and problems where the actuator can be tuned. The considered system is the nonlinear dynamic one and the number of relay switch points assumed to be tuned indirectly. To find out the solution of the given problem, the modified evolution strategies method is suggested. The proposed approach is useful also for the non-analytical system models and systems that can be evaluated numerically.

1 INTRODUCTION

The termination control task is the special case of the optimal control task with the goal of finding out a control function that would bring the system from the given initial point to the desired point within a finite time. We can also say, that the termination control task is a special case of two-point boundary values problem. Solutions of the two-point boundary problem and termination control task are described, for example, in (Cash and Mazzia, 2006) and (Tewari, 2011), respectively. Due to the practical needs, searching for the programmed control function is restricted by the actuator characteristics. The actuator can be an engine with uncontrolled pull, so it posses only the values from some finite set. As a special case, actuator can be a relay. Also, the system itself is often nonlinear or algorithmically represented. It is the main reason to investigate the new method of programmed relay control generation. For example, in the article (Aida-zade and Anar, 2010) an approach to relay switch points correction is given, when the control function is a relay, and the determination of the switch points number in the case of linear dynamic system is fulfilled. As we can see the relay program control is still an actual problem. But for the application of this approach one needs at least some control function which could be then tuned. In this study, we consider the termination relay control problem for nonlinear dynamic systems. Our approach does not require any initial approximation of control function. In the article (Kucherov et al., 2009) a method for the relay control synthesis is proposed also for a linear dynamic system. Generally speaking, the methods for solving the termination control problem with relay actuator for non-linear systems are not well known. However, such problem statements are very important in many significant areas, such as aircrafts and spacecrafts control.

Moreover, every optimal control problem with Hamiltonian linear over control is reduced to the termination control problem with known relay characteristics. All above is a reason for our study.

2 TERMINATION RELAY CONTROL PROBLEM DEFINITION

Let the system be described with nonlinear differential equation

$$\frac{dx}{dt} = f(x,u,t)\,, \tag{1}$$

Ryzhikov I. and Semenkin E..
Modified Hybrid Evolutionary Strategies Method for Termination Control Problem with Relay Actuator.
DOI: 10.5220/0004044403330337
In *Proceedings of the 9th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2012), pages 333-337
ISBN: 978-989-8565-21-1

where

$f(\cdot): R^n \times R \times R^+ \to R^n$ is a vector function of its arguments;

$x \in R^n$ is a vector of system state;

$u(t): R^+ \to U_u, U_u = \{u_i \in R, i = \overline{1, N_u}\}$ is a piecewise continuous function;

n is the system dimension.

We need to find a control function $u(t)$ that brings the system from the initial point $x(0) = x^0$ to the end point $x(T) = x^*$ within finite time T.

Though the piecewise continuous function fits only special systems, the relay approach is useful for every optimal control problem if the Hamiltonian is linear over control:

$H(x, p, u) = \varphi_1(x, p) + \varphi_2(x, p)u$,

and the control function itself is modulo limited.

Solving a problem with an ideal relay, we find the control function with the structure

$$u(t) = \begin{cases} -A, & t \in I_1, \\ A, & t \in I_2. \end{cases} \quad (2)$$

where I_1, I_2 are sets of intervals determined by the switch points and $I_1 \cup I_2 = [0, T]$; A is the relay amplitude.

For multilevel control problems we find a control function with the structure

$$u(t) = \begin{cases} u_1, & t \in I_1, \\ \vdots & \\ u_{N_u}, & t \in I_{N_u}. \end{cases}, \quad (3)$$

where

$I_i, i = \overline{1, N_u}$ are intervals, determined by switch points, $\bigcup_{i=1}^{N_u} I_i = [0, T]$, $\bigcap_{i=1}^{N_u} I_i = \varnothing$;

$U = \{u_i \in R, i = \overline{1, N_u}\}$ is a control function which posses values only from given set;

N_u is a number of relay positions (levels).

Let $P = \left\{ r_i : r_i < r_{i+1}, r_i \in R^+ \ \forall i = \overline{0, k}, r_0 = 0 \right\}$ be the set of switch points, k be the number of switch points chosen by user. Let $L = \left\{ l_i : l_i \in N \ \forall i = \overline{1, k} \right\}$ be the set of indexes. Then each interval $I_i, i = \overline{1, N_u}$ for control (3) can be determined with the equation

$$I_i = \{\bigcup_{j=0}^{N_u} (r_j, r_{j+1}] \cdot f_I(i, j), r_j \in P \ \forall j \},$$

where $f_I(i, j) = \begin{cases} 1, & i = L_j \\ 0, & i \neq L_j \end{cases}$ is an index function. For ideal relay (2) we can describe another scheme for the problem solving. We demand the change of sign at every switch point. It means that all we need to know for ideal relay are the sign of control at $t = 0$ and the switch points, because these characteristics are enough to describe the control structure.

In the practice, there are two different problem statements:

- the control problem, where the determination of the actuator and determination of control function are necessary. It means that we need to find the characteristics of the actuator and programmed control function that fit the termination control goal within given finite time T .

- the control problem, where the control program determination only is necessary and when T is undefined.

The question about the number of switch points determination is left open though there is an indirect tuning of this number. First of all, if there is any switch point $r > T$ then it will not result on the control process. If researcher wants to fix the number of switch points, there is another way to for the set

$$\tilde{P} = \left\{ r_i : r_i < r_{i+1}, r_i \in R^+, r_i \le T \ \forall i = \overline{0, k}, r_0 = 0 \right\}$$

determination in the way that all the switch points will belong to the control interval. The second way is determined by the feature of numerical scheme that would be used to describe the process (1). Let h be the integration step, so $|r_{i+1} - r_{i-1}| < h$ means that control interval $(r_i, r_{i+1}]$ would not influent on the system behaviour, so these control switching points could be ignored.

Let S be the set of parameters that determine the control structure of the task. For multilevel relay systems $S = \{P, L, U\}$ and for the reduced scheme of the ideal relay $S = \{P, u(0)\}$. If we have the control problem with unfixed time and given relay characteristics then $S = \{P, L\}$ for multilevel relay and $S = \{P\}$ for ideal relay.

Now let us consider criteria for different control problems and ways of realization. As there are different sets determined control problem, then the problem itself can be reduced to the optimization task with real variables, or the optimization task with real and integer variables. If the time is fixed, the criterion can be defined with the function

$$F_1(\tilde{S}) = \left\| X^* - x(T)\big|_{S=\tilde{S}} \right\| \to \min_{\tilde{S}}, \qquad (4)$$

where $x(T)\big|_{S=\tilde{S}}$ is the system (1) state at the point T, and the control function is determined by $\tilde{S}$. If the time is free, then the criterion is

$$F_2(\tilde{S}, T) = \left\| X^* - x(T)\big|_{S=\tilde{S}} \right\| \to \min_{\tilde{S},T}. \qquad (5)$$

There are also inequality constraints for both criteria

$$r_i < r_{i+1}, r_i \in R^+ \ \forall i = \overline{1,k}, \qquad (6)$$

which ensure every switch point to be inside the $[0,T]$ interval. It means that we use $\tilde{P}$ instead of P that gives one more constraint: $r_k < T$.

Let introduce a special penalty function

$$\varphi(x) = \begin{cases} \|x\|, x > 0 \\ 0, x \le 0 \end{cases},$$

and weight coefficient α. Now the constrained minimization problem becomes the unconstrained optimization problem. After adding the penalty function into criteria (4) and (5), we have, respectively

$$F_3(\tilde{S}) = F_1(\tilde{S}) + \alpha \cdot \varphi(\tilde{r}_k - T) \to \min_{\tilde{S}}, \qquad (7)$$

$$F_4(\tilde{S}, T) = F_2(\tilde{S}, T) + \alpha \cdot \varphi(\tilde{r}_k - T) \to \min_{\tilde{S},T}. \qquad (8)$$

For the constraints $r_i < r_{i+1}$, we can add the sum of penalty functions $\beta \cdot \sum_{j=1}^{k-1} \varphi(r_{i+1} - r_i)$ with weight coefficient β to every criterion (4), (5), (7), (8). This sum gives a penalty for violation of the constraint $r_i < r_{i+1}, i = \overline{1,k-1}$. Adding it to criteria (4), (5), (7) or (8) gives us unconstrained optimization problem.

3 MODIFIED EVOLUTIONARY STRATEGIES ALGORITHM

Thus, the termination control problem was reduced to the optimization task with one of objective functions (4), (5), (7), (8) with real and integer numbers. The objective function, in general case, has no analytical form and has to be evaluated numerically. This is why evolution-based optimization algorithm has to be used. Genetic algorithm (GA) does not fit to the given task, because it needs the a priori known range for every variable. Also, the discretization of real numbers for GA adds extra troubles to the computation process.

The main principle of evolutionary strategies (ES) is described in (Schwefel, 1995). Additionally, we borrowed the operand definitions for integer numbers from the GA. Our ES-based optimization algorithm uses selection, recombination, mutation and local optimization operands. The selected pair of parents creates an offspring with given probability. Then the offspring is mutated. The population size is constant for all generations. The following GA selection types were used: fitness proportional, rank based and tournament based. Let every individual be represented with a tuple

$$Id_i = \left\langle op^i, sp^i, fitness(op^i) \right\rangle, i = \overline{1, N_p},$$

where

$$fitness(op) = \frac{1}{1 + F_q(op)}, q \in \{1, ..., 4\}$$

is the fitness function for problems (4), (5), (7), (8), respectively, $op_j^i \in R,\ j = \overline{1,k}$ is the set of objective parameters, $sp_j^i \in R^+,\ j = \overline{1,k}$ is the set of method strategic parameters and N_p is the size of population.

The solution of any task with criteria (4), (5), (7), (8) determines the set of objective parameters $op = \bigcup_{j=1}^{4} \rho_j$, where the every criterion defines sets $\rho_j,\ j = \overline{1,4}$. Here $\rho_1 = \{t_j \in R, j = \overline{1,k}\}$ is the set of switch points, $\rho_2 = \{l_j \in N, j = \overline{1,k}\}$ is the set of indexes, $\rho_3 = \{T \in R\}$ is the time and $\rho_4 = \{u_j \in R, j = \overline{1, N_u}\}$ is the U_u set. According to the nature of given sets, standard ES recombination and mutation can be used for ρ_1, ρ_3, ρ_4, and the standard GA recombination (one-point, two-point and uniform crossover) can be used for ρ_2.

The set of strategic parameters $sp, |sp| = |\rho_1| + |\rho_2| + |\rho_3| + |\rho_4|$, defines the mutation operands.

Now we have to modify the mutation operation for the ES adapting to our problems. Let $m_p^1 \in [0,1]$ be the mutation probability for every gene and Z_1 be the Bernoulli distributed random value with $P(z_1 = 1) = m_p^1$. Then

$$op_i = op_i + z_1 \cdot N(0, sp_i), \forall op_i \in R, i = \overline{1, card(op)};$$

$$sp_i = sp_i + z_1 \cdot N(0,1), i = \overline{1, card(sp)}.$$

Let m_p^2 be the mutation probability for integer gene. Let Z_2 be also the Bernoulli distributed value with $P(z_2 = 1) = m_p^2$, $Z_3 = U(0,1)$ be a random value that is uniformly distributed, and Z_u be a uniformly distributed integer random value, $P(z_u = 1) = ... = P(z_u = k) = \frac{1}{k}$. We also need a function $f_z(a,b) = \begin{cases} 0, & a < b \\ 1, & a \geq b \end{cases}$. Each gene is mutated if $f_z(sp_i, z_3) = 1$. We can now allow the strategy parameters to affect on this mutation probability:

$$op_i = (1 - z_2) \cdot op_i + z_2 \cdot z_u,$$

$$\forall op_i \in N, i = \overline{1, card(op)} \text{ and}$$

$$sp_i = \left|(1 - z_2) \cdot sp_i + z_2 \cdot N(0,1)\right|.$$

Next modification is fulfilled to avoid constraints $r_i < r_{i+1}$ satisfaction. We make a special transition from the objective parameters to the switch points: $r_i = \sum_{j=1}^{i} op_j, i = \overline{1,k}$, and change the mutation operator to $op_i = \left|op_i + z_1 \cdot \mathrm{N}(0, sp_i)\right|$, $i = 1,...k$. In this case, the objective parameters will be nonnegative and the initial population will be generated with nonnegative individuals.

The random coordinate-wise real-valued genes optimization has been implemented for the algorithm performance improvement. The optimization is fulfilled in the following way. For every N_2 randomly chosen real-valued genes for N_1 randomly chosen individuals N_3 steps in random direction with step size h_l are executed.

For the numerical experiments in our study, the parameters of the ES-based optimization procedure were set as followed: the population size is 50, the number of generation is 50, the recombination is set as discrete one with the probability 0.8, the mutation probability for every gene was set to $1/|sp|$. Local improvement parameters were set as $N_1 = 2 \cdot |op|$, $N_2 = |op|$ and $N_3 = 5$ with $h_l = 0.05$.

The proposed algorithm performance has been evaluated on tens test problems and was found to be promising.

4 TERMINATION CONTROL PROBLEM FOR THE SATELLITE MOTION ON GEOSTATIONARY ORBIT

Let us consider a system that define the motion of a satellite on the geostationary orbit:

$$f(x,t) = \left(x_2, \quad x_1 \cdot x_3^2 - \frac{1}{x_1^2}, \quad \frac{-u(t) - 2 \cdot x_2 \cdot x_3}{x_1}, \quad x_3 \right).$$

It is necessary to reach the point $x(T) = (1, \ 0, \ 1, \ T)$ from the initial point $x(0) = (1, \ 0, \ 1, \ 0.785)$ within given finite time T, so the satellite would come from one orbit to another one. Actuator works as a relay $u(t): R^+ \to U_u$, $U_u = \{-A, A\}$, where A is the engine force.

20 runs of the proposed algorithm with the given above parameters were executed for the considered problem. Results were averaged.

Let us set the initial number of switch points $k = 10$. Then for $T = 10$ and the criterion (7) we can find the solution. The control function is shown on Figure 1 and the system state is depicted on Figure 2. The system coordinates at the end point are $x(10) = (1.0007, \ -0.0064, \ 1.0069, \ 10.002)$. The

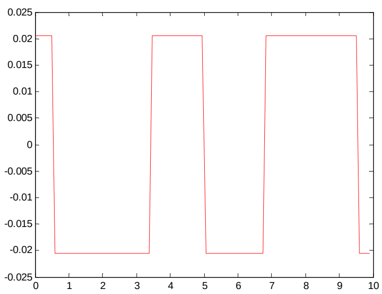

Figure 1: Control function $u(t)$.

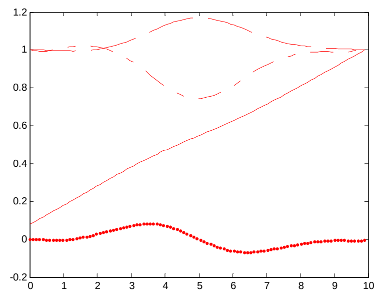

Figure 2: System state: $x_1(t), x_2(t), x_3(t), \frac{x_4(t)}{T}$.

mean of the objective function (7) is 0.0028. The objective function evaluations nimber was no more than $4 \cdot 10^4$ during every algorithm run.

For the same termination control problem but with actuator defined by the set $U_u = \{-0.005, 0, 0.005\}$, we used the criterion (5) with $T = r_k$. For $k = 20$ the following solution was found: $T = 17.2$, $x(17.2) = (1.009, \ \ -0.0065, \ \ 1.0048, \ \ 17.21)$. The mean of the objective function (5) is 0.018. The objective function evaluations number was no more than $6.25 \cdot 10^4$ in every run. Similar graphics are shown on Figures 3 and 4.

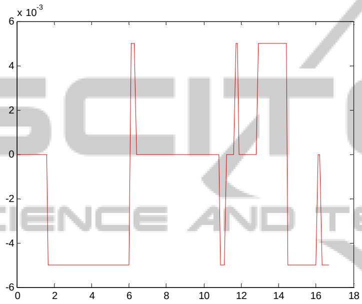

Figure 3: Control function $u(t)$. Time is unfixed. Three-position relay.

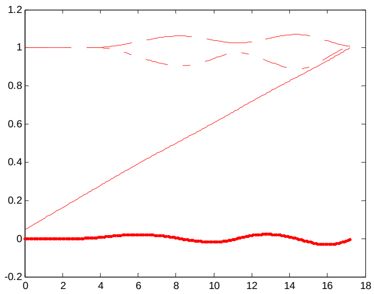

Figure 4: System state: $x_1(t), x_2(t), x_3(t), \dfrac{x_4(t)}{T}$.

As one can see, the proposed algorithm effectively solves the relay termination control problem for non-linear dynamic systems. The algorithm can find the problem solution with multilayer relay and automatically determine the number of the relay switch points.

5 CONCLUSIONS

In this study, the method solving the termination control problem with the relay actuator for different task definitions was described. The method fits if the actuator is a multilevel relay which, in other words, can be represented with a piecewise continuous function with indirect tuning of the switch points number. It is useful also if the maximum principle reduces the control function to be the ideal relay. The system can be described not only analytically, but also algorithmically.

In the future the investigation of the dependence between the number of switch points and the algorithm efficiency should be fulfilled. For today, there is also no certainty about what the optimization problem statement (constrained or unconstrained) should be chosen for the higher solution precision. It is important also to apply the proposed algorithm to the control problems with non-classical constraints that appear in real control problems.

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