EKF based Data Fusion using Interval Analysis via Covariance Intersection, ML and a Class of OGK Covariance Estimators

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Abstract: This paper addresses the comparison of robust estimation of a covariance matrix in vehicle navigation task to express the uncertainty when fusing information from multiple sensors. The EKF estimates are fused with the Interval Analysis estimates and further the results are fused using the Covariance Intersection (CI), Maximum Likelihood (ML) and a class of Orthogonal Gnanadesikan-Kettenring (OGK) estimators. The simulation results presented show that the variation between CI and OGK and the correlation between sensors are significant in the presence of outliers.

1 INTRODUCTION

The data fusion from any two different sources is traditionally used to increase the accuracy of the measurement being performed, and to overcome unreliability in sensors or uncertainty in sensor outputs, so that the resulting information is more accurate, more complete and reliable to the result of an emerging view (Carvalho and Rezende, 2004). The data fusion technique is claimed to be applicable to the fusion of sensor measurements, data estimates, or similar quantities that can be described in terms of a Gaussian probability density function. The Covariance Intersection technique is related to a more general data fusion technique that can fuse with any pair of probability density functions. The principal application of Covariance Intersection is an adjunct to Kalman filters where there is the potential for the data to be highly correlated. Hence fusion can reduce the system's dependence on invalid prior assumptions and make the system more robust.

Many researchers have proposed their algorithms by combining different sources of sensors. Smoothing approaches are also used as a viable alternative to extended Kalman filter (EKF)- based solutions. In particular, approaches have been looked at as that factorize either the associated information matrix or the measurement Jacobian into square root form. Such techniques have several significant advantages over the EKF: they are faster yet exact; and yield the entire robot trajectory at lower cost for a large class of SLAM problems. Mixing the information from diverse sources, which are obtained with a different confidence degree, is at the core of a multiple of problems in robotics. Multiple techniques for information fusion have been in use for many years in robotics, ranging from the deterministic EKF to the stochastic approaches using particle filters (for instance (Montemerlo et al., 2002), (Mourikis and Roumeliotis, 2006), (Vadakkepat and Jing, 2006), (Walter and Leonard, 2004)). At the output of the fusion process, a covariance matrix that expresses the variances of the estimation error along with the different degrees of freedom is produced.

In this paper the first mile stone is achieved by fusing the covariance matrix of the EKF estimates (Time Update step) with the Interval position of the vehicle to improve the sensor uncertainties in estimating the vehicle's accurate position. Then the Covariance Intersection (CI) algorithm given in (Julier and Uhlmann, 1997), (Ashokaraj et al., 2009) is used here as a data fusion algorithm as described in (Lazarus et al., 2008). It is further extended by employing a different data fusion estimators such as Maximum Likelihood (ML) and a class of Orthogonal Gnanadesikan-Kettenring (OGK) estimators from (Gnanadesikan and Kettenring, 1972). The obtained results are compared with one another and the change in uncertainty levels between the resultant covariance matrix of each estimators are well studied.

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2 CI, & OGK ESTIMATORS

The covariance intersection algorithm (CI) has a convex combination of the means and the covariance in the information space. CI estimates are shown that it is an optimal method when the cross correlation between the measurements being fused is unknown. This advantage of this additional information exchange is that estimates on the cross correlation between them can then be integrated in the fusion process. The class of OGK estimators used in this paper is described in Table (1). It is a variation from the method described in (Copt and Victoria-Feser, 2003) and combines both information on the covariance estimates and on the data that yielded such covariances. For the purpose of experiments this is set to $\omega = 1$. The underlying idea of the process is to extract variances accounting for the distances between the measurements. If one of the axis is much bigger than the other and the variances along this axis are small then it is likely that the measurements that induce the length of the bigger axis are but outliers.

To summarize the process, the OGK estimator starts by scaling the input data (step 2) and computing an initial covariance estimate using the Gnanadesikan-Kettering estimator (step 3). This initial estimate is used to obtain a new basis (step 4), formed with the eigenvectors, where the scaled data are projected and the new variances are computed along each of the axis of the new frame (step 5). The data are then reverted back to the original frame (step 6).

3 IA BASED POSITION ESTIMATION

This section describes the vehicle's position estimation using interval analysis (Kieffer et al., 2000) with the sensor readings from the laser sensors. The main advantage of this method is that, it guarantees on the bounds and that makes the system less sensitive to the problem of consistency of typical filters such as the Extended Kalman Filters. Also it handles the problem without any linearization. The interval position estimation using SIVIA (Set Inversion Via Interval Analysis) (Jaulin and Walter, 1993) reported in (Ashokaraj, 2004) to estimate the interval position of the vehicle. The detailed description of this algorithm is described in (Lévêque et al., 1997) and (Jaulin et al., 2001). SIVIA assumes that it has a large initial search box $[\mathbf{x}]_0$ which is guaranteed to include $\overline{\mathbf{A}}$. The SIVIA basically has four steps and the two subpavings $\underline{\mathbf{A}}$ and $\overline{\mathbf{A}}$ which are initialized with empty box so as to identify the exact interval position within that box. The four basic steps of SIVIA based on the inclusion functions are given below:

- $[\mathbf{x}]$ does not belong to $\mathbf{A}$, if $[\mathbf{f}]([\mathbf{x}])$ has an empty intersection with $\mathbf{B}$ i.e $[\mathbf{f}]([\mathbf{x}]) \cap \mathbf{B} = \emptyset$.
- $[\mathbf{x}]$ is included in the solution subpaving $\mathbf{A}$, if $[\mathbf{f}]([\mathbf{x}])$ is completely inside $\mathbf{B}$ and therefore they are stored in $\underline{\mathbf{A}}$ and $\overline{\mathbf{A}}$.
- $[\mathbf{x}]$ is judged to be undetermined, thus implying that $[\mathbf{x}]$ may only include part of the solution set if $[\mathbf{f}]([\mathbf{x}])$ has a non-empty intersection with $\mathbf{B}$, but is not completely inside $\mathbf{B}$. If $[\mathbf{x}]$ has a width w greater than ε the pre-specified precision parameter, then the box is bisected generating two offsprings and the tests are again applied recursively for these offsprings.
- Finally if the box has a width smaller than the pre-specified precision parameter ε and at the same time they are found to be undetermined, then it is considered to be small enough to be stored in the outer approximation $\overline{\mathbf{A}}$ of $\mathbf{A}$.

4 EKF BASED DATA FUSION

As described earlier, in this paper the three different data fusion estimators are presented namely, CI, ML, OGK. The vehicle is assumed that it operates in a partially known environment where the surrounding obstacles are known to some extent. It is also assumed that the vehicle velocity is assumed to be constant and the vehicle is equipped with INS sensors, laser and GPS receiver. An INS with a nonlinear extended Kalman filter is used to estimate the heading angle and the position of the vehicle. The INS provides the information about the vehicle's position, velocity and heading angle (yaw angle). To estimate the errors in the INS states a Kalman filter is used which utilizes the measurements from other estimated sources (X & Y position from IA). The Kalman filter uses an INS error model which gives the optimal Kalman gain. This Kalman gain is used with the innovations (X & Y positions) to estimate the errors in the INS estimates (Lazarus et al., 2008).

As the vehicle moves, based on the measurements received from the INS senors the time-update stepin the Kalamn filter is updated. When the vehicle passes through any of the known surrounding obstacles then the fusion algorithm is implemented. In order to accomplish this task, initially the vehicle's interval position is estimated using IA based SIVIA method which is relative to the known obstacle. Basically this fusion algorithm using EKF represents the infertilities

Table 1: The OGK covariance estimator (ω is the class parameter).

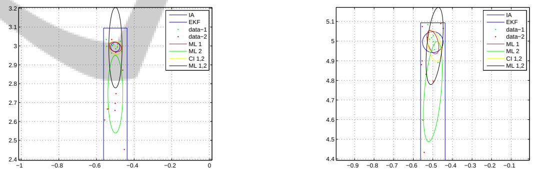
0	Let $\sigma(\cdot)$ be a standard deviation function applied to its argument
1	Let $X = (X_1, \ldots, X_p) \in \mathbb{R}^{n \times p}$ be the set of n observations, each of dimension p
2	Let $D = \mathrm{diag}\{(\}\sigma(X_j)), j = 1, \ldots, p$ and define $Y = XD^{-T}$
3	Compute $U = [u_{jk}] = \begin{cases} \frac{1}{4}(\sigma(X_j + X_k)^2 - \sigma(X_j - X_k)^2) & j \neq k \\ \sigma(X_j)^2 & j = k \end{cases}$
4	Compute E such that $U = E\Lambda E^{-1}$ with Λ the diagonal matrix with the eigenvalues of U
5	Let $Z = YE$ and $A = \omega ED^{-1}$, with ω a scalar, and $\Gamma = \mathrm{diag}(\sigma(Z_j)^2), j = 1, \ldots, p$
6	The covariance estimate is $\Sigma = A^T \Gamma A$

from two different sensors. The fist source of information is obtained from the INS sensors and its uncertainties are represented in the form of P_k matrix (i.e., the covariance matrix in EKF in time step). The second source of information is obtained from the Interval analysis method where the uncertainties are represented in the form of interval (i.e., Interval box of the vehicle's position using laser range sensor). For the sake of simplicity in this work a set of random data are generated within this interval by using the mid point of this interval box thereby a covariance is constructed which represents the uncertainty in the interval position of the vehicle.

Finally the information obtained from these two different sources are fused with the CI algorithm so as to perform the measurement-update process in the Kalman Filter. Further, the same process of generating a random set of data is applied for the EKF estimated covariances as well. then the constructed two different covariance matrices which use the random data within two different uncertainties are fused using the ML algorithm. Then, this set of two different random data is given as an input to the OGK estimator in order to fuse this two different source of information. The resultant covariance matrix is used to update the Kalman Filter's measurement-update step.

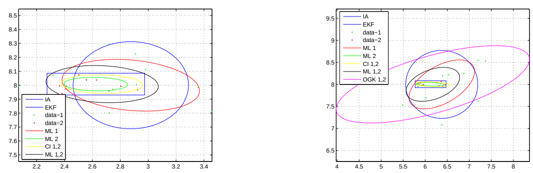
5 SIMULATION RESULTS

The simulation results of the above explained algorithm is given in this section. The task of vehicle navigation is achieved by updating the EKF at each iteration (i.e., the time and measurement updates). Initially a sequence of ten random data is generated within the interval region thereby a covariance is estimated from the IA estimated position so as to fuse this covarince matrix with the Kalman predicted covariance matrix (i.e. time-update step). Secondly these two covariances are fused using CI algorithm. Thirdly a same set of random data is also generated within the region of the Kalman estimated covariance and thereby a covariance matrix is constructed using this random data. Finally, these two set of covariances are fused using the ML and the OGK estimators. The simulation results of some of the iterations are given in figures 1 - 3. As can be observed from these fig-



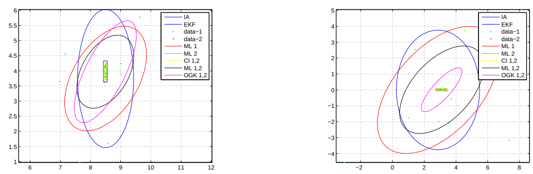
(a) Iteration 2 (b) Iteration 6

Figure 1: The resultant covariances using CI, ML, and OGK at each iterations-1.



(a) Iteration 12 (b) Iteration 19

Figure 2: The resultant covariances using CI, ML, and OGK at each iterations-2.



(a) Iteration 27 (b) Iteration 39

Figure 3: The resultant covariances using CI, ML, and OGK at each iterations-3.

ures, since the Kalamn estimated covariance is negligible (small), the OGK algorithm is not implemented in the initial iterations. As the vehicle moves, the uncertainties in the system noise will rapidly increas; this requires the necessitates the implementation of the data fusion algorithm. It is also observed that the Kalman estimated covariance is emerging bigger in each of the iterations by increasing the region of uncertainties. Whenever the data fusion algorithm is applied the region of this uncertainties is rapidly reduced by performing the measurement-update in the Kalman filter. The last stages of iterations show that the OGK algorithm is robust when the Kalman estimates are unpredictable within the assumed distributions. In nutshell, the presented algorithms are complementary in the sense that they compensate for each other's limitations, so that the resulting performance is much better than of its individual techniques, which in turn, provide more accurate position information of the vehicle.

6 CONCLUSIONS

The paper presents robust data fusion techniques via CI and a particular class of OGK covariance estimators for fusion of information from two different sources namely EKF and IA. The CI approach uses the covariance matrices of the data sources whereas the OGK uses an estimate of the joint covariance and information from the measurements themselves. The comparison between the two estimators is based on the 2-norms. This measure combines information related to the volume of the error ellipsoids and their eccentricity. The analysis on the relevant bounds for the two measures shows that, in worst case conditions, there are regions of the spectrum of the covariance matrices in which each of the estimators outperforms the other. For generic applications, a hybrid of the two estimators may then provide the best results.

The application of this OGK estimation technique to 3D needs a recalculation of the bounds involved. Still, the same basic principles apply. The ongoing work also includes the testing of classes of OGK estimators in the information fusion problem that are applied to a number of ground and aerial vehicle involved in a mapping mission.

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