Global Optimal Solution to SLAM Problem with Unknown Initial Estimates

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Abstract: The paper presents a practical approach for finding the globally optimal solution to SLAM. Traditional methods are based upon local optimization based strategies and are highly susceptible to local minima due to non-convex nature of the SLAM problem. We employed the nonlinear global optimization based approach to SLAM by exploiting the theoretical limit on the numbers of local minima. Our work is not reliant on good initial guess, whereas existing approaches in SLAM literature assume good starting point to avoid local minima problem. The paper presents experimental results on different datasets to validate the robustness of the approach, finding the global basin of attraction with unknown initial guess.

1 INTRODUCTION

Simultaneous Localization and Mapping (SLAM) has drawn significant interests in robotics communities, as it enables the robotic vehicles to be deployed in a fully autonomous way for various applications. SLAM literature is mostly categorized into two main streams: Filtering based (Bailey and Durrant-Whyte, 2006) and Maximum likelihood based (Lu and Milios, 1997) approaches. The first stream consists of Extended Kalman filter and Information filter (S. Huang and Dissanayake, 2008) which requires linearization of process and measurement models with a cost of potential divergence and inconsistency. FastSLAM (A. Doucet and Russel, 2002) is based upon factorization of posterior but, due to limited numbers of particles, it is unable to represent the trajectory posterior in the long run.

In graph-based SLAM approaches measurements acquired during robot motions are modeled as constraints. The goal of these approaches is to estimate the configuration of parameters that maximally explain a set of measurements affected by Gaussian noise (minimizes the nonlinear least square error). The pioneering work in graph-based SLAM is by (Lu and Milios, 1997) in which brute force technique for range scan alignment was proposed. With the assumption of known rotation (T. Duckett and Shapiro, 2002) introduced a Gauss-Seidel relaxation. The work was improved by (U. Frese and Duckett, 2005) solving a network at different level of resolution. (Dellaert and Kaess, 2006) came up with QR factorization of information matrix to solve the full SLAM problem.

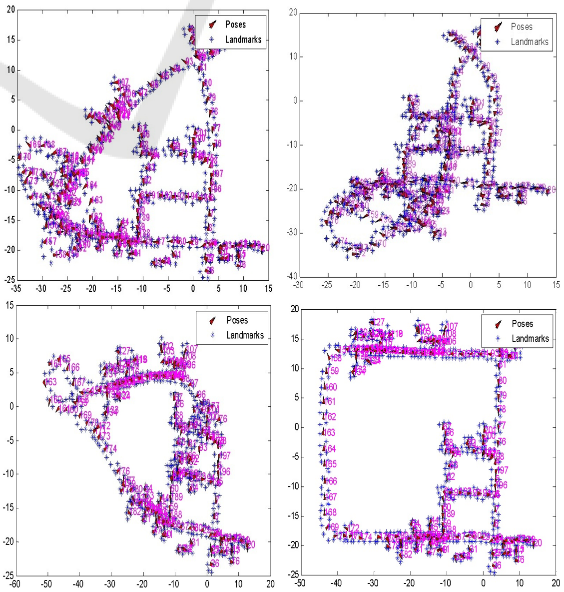

Figure 1: Examples of local solution for DLR dataset (J. Kurlbaum, 2010) with different random initial guess.

(E. Olson and Teller, 2006) uses stochastic gradient descent (SGD) algorithm (E. Olson and Teller, 2006) to solve the pose only SLAM problem by addressing each constraint individually and surprises many researcher as the algorithm can converge to the correct solution with poor initial values. Recent re-

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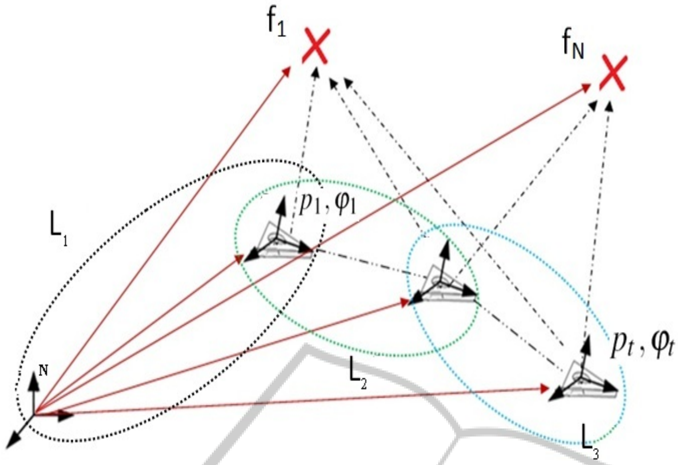


Figure 2: SLAM is recasted as a map-joining problem: One is a growing global map (navigation coordinate with solid lines) and the other is a new local map L (represented with ellipses). The dashed lines indicate the odometry and feature observations.

search (R. Kummerle and Burgard, 2011) has focused on making these algorithms more efficient and robust showing that its online implementation is feasible. Although these approaches perform efficiently in practice, very little attention has been paid on the convergence condition and none of them can guarantee a global minimum over different initial guesses. Fig. 1 shows the results of a Gauss-Newton approach on publicly available dataset (J. Kurlbaum, 2010) with different random initial guesses, resulting in different local solutions.

Global optimal solutions to highly nonlinear problems has been shown to NP-hard (Freund and Jarr, 2001). In structure from motion research, the guaranteed global optimal solution is investigated with known rotation framework for L_∞ (K. Astrom and Hartley., 2007) and branch and bound based approaches (C. Olsson and Oskarsson, 2008) whereas in our work no such assumption of known rotation is considered (typically the nonlinearity in measurements of mobile robot applications is due to robot orientation). In recent work (Iser and Wahl, 2010) proposed a swarm optimization based approach to estimate (almost) optimal maps. Their work is based upon meta-heuristic optimization approach which is similar to our work but they present map as a tree of fragments/maps with particle filtered based sampling approach and finally conducted an ant colony search to obtain (almost) optimal solution.

In this paper we provide an approach to get global optimal solution for SLAM problem, which has never been proposed before to the best of author's knowledge. Feature based SLAM problem is decomposed as a problem of joining submaps. Necessary and sufficient condition for the existence of at most two local optimal (S. Huang and Dissanayake, 2012) is exploited by a meta-heuristic approach called GRASP (greedy randomized adaptive search procedure) (Resende and Ribeiro., 2003) which is combinatorial optimization to obtain global optimal solution. Meta-heuristic approaches optimize by iteratively refining the candidate solution by combining randomness with local search methods (C.Blum and A.Roli, 2003). Unknown landmark positions and vehicle pose are considered as initial guess in a planar environment (3DOF case).

The outline of this paper is as follows: Section 2 and 3 will provide nonlinear least square formulation and number of local minima in SLAM problem. Section 4 and 5 will provide detailed discussions on nonlinear global optimization based approach and greedy search strategy. Section 6 will briefly explain global optimal approach to map-joining. Results and discussions will be presented in Section 7 followed by Conclusion.

2 NONLINEAR LEAST SQUARE FORMULATION

The dimension of the SLAM problem is very high when it is formulated as a nonlinear least square (K. Ni, 2007) because all vehicle poses and feature locations are considered as parameters to be determined. The decomposition of SLAM problem into submaps not only helps to reduce the computational complexity but also helps to improve consistency by decreasing the nonlinearity of the system (S. Huang and Dissanayake, 2008; S. Huang and Frese, 2008). The assumption we undertake in this research is that, every SLAM problem can be decomposed into local maps and then solved for global optimal solution. The relative relation of local maps has fluid behavior whereas the internal structure of each local map is well known and can be optimized independently with respect to local coordinate (K. Ni, 2007). The nonlinear least square formulation for local map joining is to minimize an objective function as follows:

$$F(x) = \arg\min_x \sum(||E_p||_U^2 + ||E_f||_V^2), \tag{1}$$

where the state vector is $x = \{X, M\}$ with X being the poses (position and orientation) of local maps and M the features positions in absolute coordinate frame. U and V are corresponding covariance matrices of pose and feature observations respectively. The poses are composed of $\{p_1, \varphi_1, \ldots, p_t, \varphi_t\}$ where the end pose of each local map is the start pose of next local map. The N map features are defined as $M = \{f_1, \ldots, f_N\}$. SLAM recasted as a map-joining problem is shown in Fig. 2.

Let there be a local map l defined as $X^l = \{p^l, \varphi^l\}$ with n features $M^l = \{f_1^l \ldots, f_n^l\}$ present in the local coordinate frame. The local map pose X^l is the observation of relative pose of p^l, φ^l from the global state vector X pose p_{l-1}, φ_{l-1} as

$$E_p = X^l - H_{odo}(X), \tag{2}$$

and the observation model for odometry is defined as

$$H_{odo}(X) \quad = \quad \left[\begin{array}{c} R(\varphi_{l-1})^T(p_l - p_{l-1}) \\ \varphi_{l-1} - \varphi_l \end{array} \right]. \tag{3}$$

The $SO(2)$ rotation matrix is defined as

$$R(\varphi) = \left[\begin{array}{cc} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{array} \right]. \tag{4}$$

The feature positions $M^l = \{f_1^l \ldots, f_n^l\}$ in local map l are observation of relative position of features and are related (assumed data association) to global state vector. The relative feature position error E_f is defined as

$$E_f = M^l - H_{feat}(X, M), \tag{5}$$

whereas the observation model for relative position of features is a function of M and X

$$H_{feat}(X, M) = \left[\begin{array}{c} R(\varphi_{l-1})^T(f_{l1} - p_{l-1}) \\ \ldots \\ R(\varphi_{l-1})^T(f_{ln} - p_{l-1}) \end{array} \right]. \tag{6}$$

The Mahalanobis distance for both relative errors in the cost function with zero-mean Gaussian noise with covariance U, V can be written as

$$F(x) = \sum(X^l - H_{odo}(X))^T U^{-1}(X^l - H_{odo}(X)) + \sum(M^l - H_{feat}(X, M))^T V^{-1}(M^l - H_{feat}(X, M)). \tag{7}$$

The measurement and process models are both nonlinear functions, and thus the nonlinear objective function is linearized multiple times to reach local minima. Generally local approaches solve the objective function $F(x)$ as

$$F(x + \delta x) = F(x) + J\delta x, \tag{8}$$

where $J = \partial F(x)/\partial(x)$. Let $\varpi = (F(x + \delta x) - F(x))$, then it becomes

$$J\delta x = \varpi, \tag{9}$$

The solution can be found using pseudo-inverse of J

$$J^T J\delta x = J^T \varpi$$
$$\delta x = (J^T J)^{-1} J^T \varpi \tag{10}$$

By including the covariance estimates (U, V) as ξ, eq. 10 becomes

$$\delta x = (J^T \xi^{-1} J)^{-1}(J^T \xi^{-1})\varpi. \tag{11}$$

When the eq. 11 is solved only ones by an optimization based approach i.e. (Dellaert and Kaess, 2006), it yields a similar result like the EKF or Extended information filter (Bailey and Durrant-Whyte, 2006). The advantage in optimization based approach comes with repeatability of solution for eq. 11 with different linearization points.

3 NUMBER OF LOCAL MINIMA IN MAP JOINING SLAM

The SLAM formulation as linearized version of nonlinear objective function assumes local convexity, that is with a reasonable initial estimate (near the global basin of attraction), the algorithm will converge to global minima. SLAM is an incremental process and at each step, small number of new parameters need to be estimated. However when the odometry and feature observation are not consistent with each other, then the local optimizer can struck in local minima.

A close lookup at the observation model in equation 3 and 6 reveals that nonlinearity is only due to orientation. Recently (S. Huang and Dissanayake, 2012) provides a theoretical bound on local minima in map-joining SLAM problem, by deriving a nonlinear equation depending only on orientation error under the assumption that covariance matrices are spherical matrices. The research findings in their work suggest

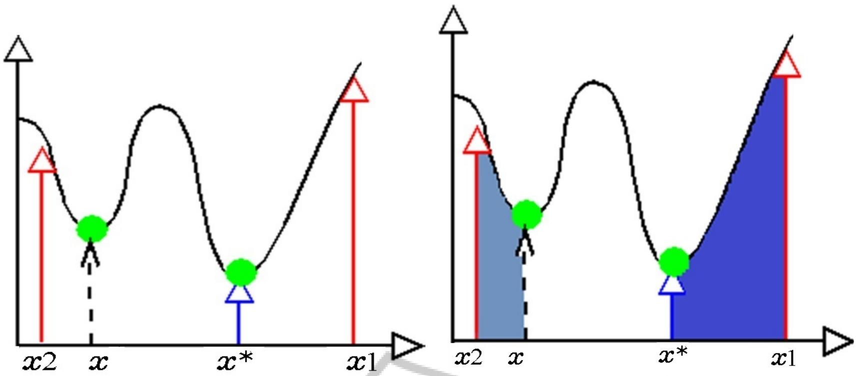


Figure 3: 1D problem with 2 local minima revealing possible feasible solutions from local optimization.

at most two and at least one local minima can occur. The approach, proposed in this paper, exploits the upper theoretical bound under noisy observations to obtain global minima by a meta-heuristic approach (discussed in following section) hence obtaining a global optimal solution.

4 GREEDY RANDOM ADAPTIVE SEARCH PROCEDURE

GRASP is a multi-start meta-heuristic approach to solve combinatorial problems (Resende and Ribeiro., 2003). Previously, it has been successfully deployed in traveling sales man problem and firstly proposed here for SLAM problem. GRASP basically consist of two phases: local search and feasible solution construction. The construction phase builds a feasible solutions (using greedy approach), whose neighborhood is searched by a local search phase to find local optima. By using different feasible solutions as starting points for local search in a multi-start procedure will usually lead to good, though, most often, suboptimal solutions. While in our problem at worst case, we encountered two local minima when joining two local maps, so the upper bound is searched by multi-start until global optimal solution is returned, which is the optimal solution.

The pseudo code in algorithm 1, details the working of GRASP approach, where local search is performed by the gauss-newton formulation of eq.11.

5 RANDOMIZED GREEDY ALGORITHM

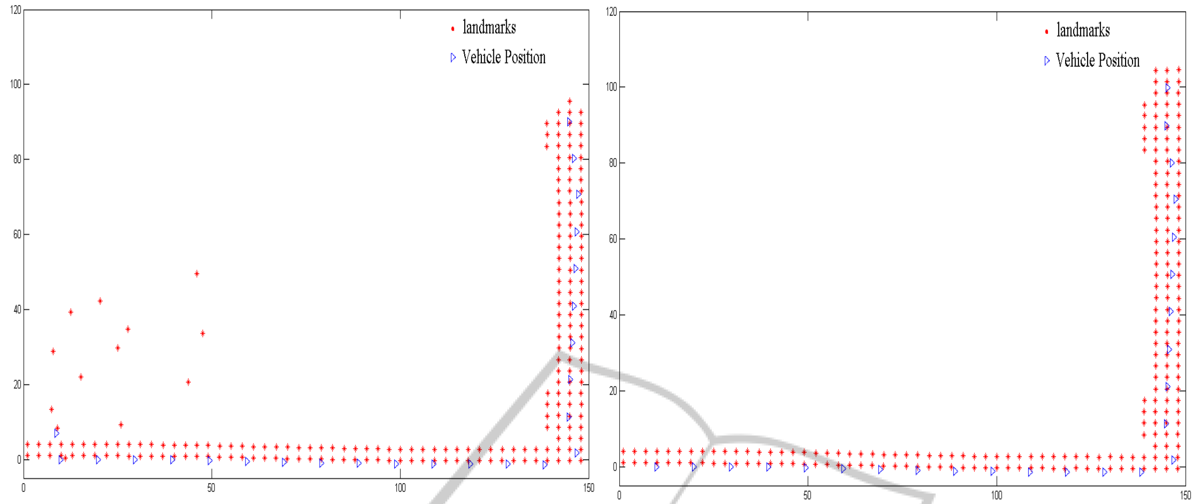
We proposed long-term memory based greedy algorithm to determine a feasible solution. Fig. 3 (left) details a simple example on 1D in which at most two local minima are considered.The two feasible solutions (x_1, x_2) are generated by GRASP approach and among them two minima are found whereas x_1 reveals the global optimal solution x^*.

Algorithm 1: GRASP-Algorithm: Determination of Optimal Solution.

$inputs : observations = \{X^l, M^l\}, x = \{pose, landmark\}$
$x^* = \{EMPTY\}$
while two local minima or max iteration are not reached **do**
  $x \leftarrow RandomizedGreedyAlgo(.) \rightarrow Algo2$
  $x \leftarrow LocalSearch(x, observations)$
  if (f(x) < f(x*)) **then**
    $x^* = x$
  end if
end while
return x^*

The generation of feasible solution is the key to obtain the optimal solution, in timely manner from a high dimensional search space of parameters. Fig. 3 (right) describes a search space reduction mechanism in which the initial guess x_1 is first hypothesized, which determines a local optimal x^* (the intermediate traversal solution of the local optimizer are stored in long-term memory for search space x_1 to x^*). The next hypothesis of initial guess is being made by greedy algorithm in consideration with the already traversed solution space in long-term memory (the goal is to avoid making an initial guess from already traversed space). The selection of new initial guess is based upon absolute distance criteria (greedy approach), in which the new initial guess for local optimizer is not from the already considered search space. The greedy algorithm helps to reduce the search space and improves the time efficiency as against the exhaustive brute force search in solution space.

The pseudo code in algorithm 2, describes the greedy algorithm, in which the long-term memory of



(a) Before joining (b) After joining

Figure 4: Intermediate results of simulation dataset (S. Huang and Frese, 2008) with 50 local maps.

all the solution space is maintained. The selection of new feasible solution is determined similar to 1D explained approach on orientation space (as vehicle and feature positions are linear with respect to orientation). Selection of ε in radians decides the selection of new hypothesis for initial guess to boot the local optimizer.

Algorithm 2: GRASP-Algorithm: Randomized greedy algorithm.

while Initial guess not found | Max iter not reached **do**
  x = random(.)
  $found = 1$
  for i=1:num of elements in LTmemoryX **do**
    Oldx = LTmemoryX[i]
    if $|(x - Oldx)| < \varepsilon$ **then**
      found = 0
      Break → select another x
    end if
  end for
end while
return x

An appealing characteristic of GRASP approach is the ease of implementation by setting and tuning few parameters. The computation time of the approach does not vary much from iteration to iteration and increases linearly with the number of iterations whereas the time increases combinatorially with the increase of searched spaced parameters.

6 GLOBAL OPTIMAL SOLUTION TO MAP-JOINING

Most of the map joining approaches start with linear approximation of map states and do optimization as a post processing step to estimate the local solution (S. Huang and Frese, 2008; S. Huang and Dissanayake, 2008; K. Ni, 2007). Our approach is not dependent upon the initial guess and bound to search the optimal results by modified GRASP approach. We initialized each map randomly (unknown feature and vehicle pose) and solve the map joining problem as illustrated in pseudo code algorithm 3. The data association is assumed to be known. Two maps are considered at each time and provided to GRASP algorithm, which returns the optimal solution for those two sets of maps observation. The sequential joining is not necessary and parallel algorithm can be employed for faster computation, if the running time is bottleneck in a large scale environment. The input to algorithm 1 will be set of observations (relative feature and odometry information of local maps l) and global map state vector X, M to obtain x^*.

Algorithm 3: MAP Joining Optimal: GRASP variant for Map Joining.

x* = first local map
while Fuse local map k+1(l) into x* **do**
  Data Association assumed to be known
  Initialize random pose/landmark for l into x*
  x*= GRASP-Algorithm (x*, Local Map l)
end while
return x*

7 RESULTS AND DISCUSSION

The performance of the proposed algorithm is tested on simulated (S. Huang and Frese, 2008) as well on

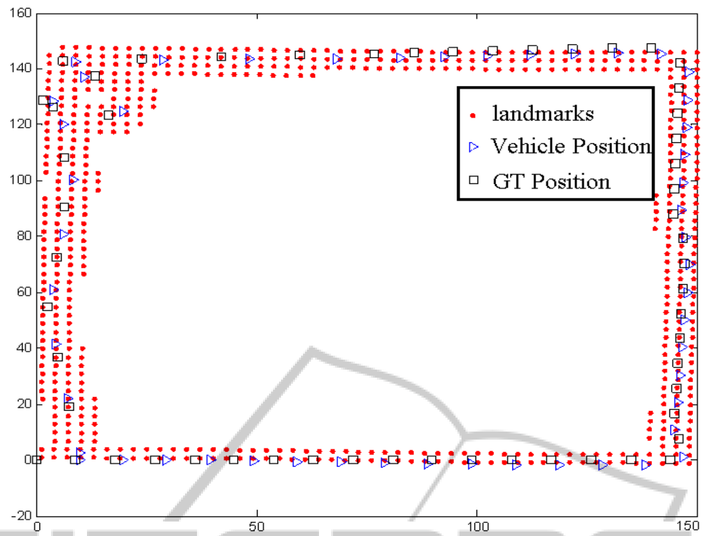


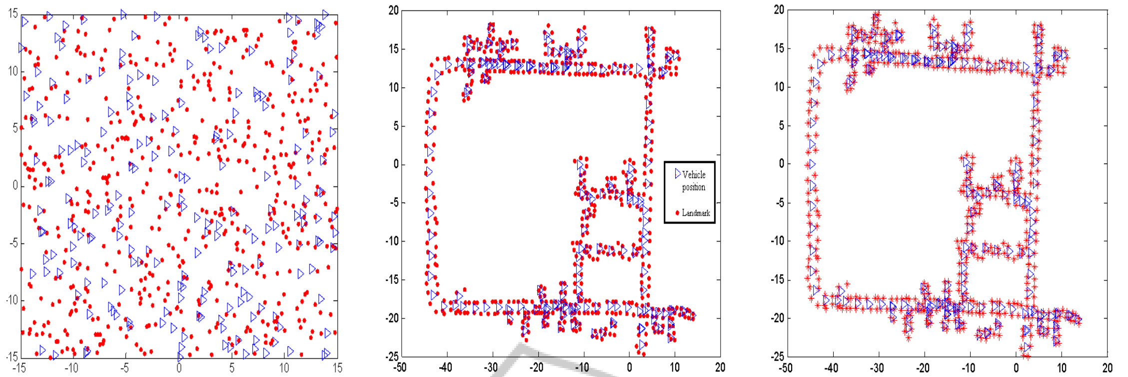
Figure 5: Ground truth and estimated vehicle/landmark positions for simulation dataset (S. Huang and Frese, 2008) with 50 local maps.

real dataset (J. Kurlbaum, 2010), which are both available publicly. The $150 \times 150m^2$ simulation environment (S. Huang and Frese, 2008) containing 2500 features uniformly space in rows and columns is tested first. The robot started from the left bottom corner of the square and followed a big loop. A sensor with a field of view of 180 degrees and a range of 6 meters was simulated to generate relative range and bearing measurements between the robot and the features. There were 50 local maps in total with 612 observed features and 1374 odometry/feature measurements were made from the robot poses. Fig. 4(a) shows the intermediate results of 25^{th} local map where new local map with random initial guesses of landmark position and pose. The GRASP based smoothing is performed and optimal results obtained is shown in Fig. 4(b). Final results with ground truth result is shown in Fig. 5, showing estimated position against the interpolated ground truth positions.

The GRASP based approach with unknown initial guess is tested on DLR dataset (J. Kurlbaum, 2010) and compared with the state of the art map-joining approach (S. Huang and Frese, 2008). DLR dataset is acquired with a camera attached on a wheeled robot and odometry. The robot moved around a building detecting scattered artificial white/black landmarks, placed on the ground. Odometry measurements and relative position of the observed landmarks are being provided. 200 local maps with 540 observed features and 1680 odometry/feature observations is considered with known data association. Fig. 6 (a) shows the random initial guess for vehicle pose and feature positions in global frame of reference, to be processed by GRASP approach. Fig. 6 (b and c) shows a visual comparison of proposed approach with (S. Huang and Frese, 2008) whereas (S. Huang and Frese, 2008) approach is booted with linear initialization and our proposed approach is not dependent upon known initial guess. Gaussian noise is introduced into the odometry observations (which makes inconsistency between odometry and feature observations) to gauge the performance of proposed approach, hence we are able to get a global solution each time, which is optimum in Maximum likelihood sense whereas local optimization based approaches results in local solution similar to Fig. 1. The performance of the experimental results validates the proposed idea and makes the SLAM problem solvable by global optimization based approach without initial guess.

8 CONCLUSIONS AND FUTURE WORKS

A practical approach for finding the globally optimal solution to SLAM is presented. Local optimization based strategies which are mostly adopted for SLAM problem are highly susceptible to local minima problem due to non-convex structure of the problem. By exploiting the theoretical limit on the number of local minima, we proposed a framework to estimate a global optima. The proposed approach is not reliant on good initial guess which is the primary condition of local optimization based approaches for global convergence. Experimental results are provided on different datasets available online to validate the robustness of approach.



(a) Random initial guess for local maps

(b) GRASP based vehicle/landmark position estimates

(c) I-SLSJF with EIF based initialization (S. Huang and Frese, 2008)

Figure 6: DLR dataset results using GRASP with 200 local maps, showing global convergence (b) with random initial guesses (a).

Future work will provide the time/memory comparison of the proposed approach and possible extension to 6DOF SLAM problem.

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