Optimal Distributed Controller Synthesis for Chain Structures
Applications to Vehicle Formations

Omid Khorsand, Assad Alam and Ather Gattami
Department of Automatic Control, EES, KTH-Royal Institute of Technology, Stockholm, Sweden

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Abstract: We consider optimal distributed controller synthesis for an interconnected system subject to communication constraints, in linear quadratic settings. Motivated by the problem of finite heavy duty vehicle platooning, we study systems composed of interconnected subsystems over a chain graph. By decomposing the system into orthogonal modes, the cost function can be separated into individual components. Thereby, derivation of the optimal controllers in state-space follows immediately. The optimal controllers are evaluated under the practical setting of heavy duty vehicle platooning with communication constraints. It is shown that the performance can be significantly improved by adding a few communication links. The results show that the proposed optimal distributed controller outperforms a suboptimal controller in terms of control input energy.

1 INTRODUCTION

The systems to be controlled are, in many application domains, getting larger and more complex. When there is interconnection between different dynamical systems, conventional optimal control algorithms provide a solution where centralized state information is required. However, it is often preferable and sometimes necessary to have a decentralized controller structure, since in many practical problems, the physical or communication constraints often impose a specific interconnection structure. Hence, it is interesting to design decentralized feedback controllers for systems of a certain structure and examine their overall performance.

The control problem in this paper is motivated by systems, generally referred to as vehicle platooning, involving a chain of closely spaced heavy duty vehicles (HDVs). Governing vehicle platoons by an automated control strategy, the overall traffic flow is expected to improve (Ioannou and Chien, 1993) and the road capacity will increase significantly (De Schutter et al., 1999). By traveling at a close intermediate spacing, the air drag is reduced for each vehicle in the platoon. Thereby, the control effort and inherently the fuel consumption can be reduced significantly (Alam et al., 2010). However, as the intermediate spacing is reduced the control becomes tighter due to safety aspects; mandating an increase in control action through additional acceleration and braking. Hence, it is of vast interest for the industry to find a fuel optimal control. Thus, with limited information and control input constraints, the control objective is to maintain a predefined headway to the vehicle ahead based upon local state measurements, which makes it a decentralized control problem.

Decentralized control problems are still intractable in general. One approach has been to classify specific information patterns leading to linear optimal controllers. An important result was given in (Ho and Chu, 1972) which showed that for a new information structure, referred to as *partially nested*, the optimal policy is linear in the information set. In (Rantzer, 2006), stochastic linear quadratic control problem was solved under the condition that all the subsystems have access to the global information from some time in the past. Control for chain structures in the context of platoons has been studied through various perspectives, e.g., (Alam, 2011; Bamieh et al., 2008; Barooah and Hespanha, 2005; Swaroop and Hedrick, 1996; Varaiya, 1993). It has been shown that control strategies may vary depending on the available information within the platoon. However, communication constraints have not in general been considered in control design for platooning applications.

The aim of this study is to synthesize controllers for a practical decentralized system composed of M interacting systems over a chain. We minimize a quadratic cost under the partially nested information

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structure. This problem is known to have a linear optimal policy, (Ho and Chu, 1972) and (Voulgaris, 2000). However, most existing approaches do not provide explicit optimal controller formulae and, the order of the controllers can be large (Gattami, 2007), which makes the implementation difficult. Some work has been focused on finding numerical algorithms to these problems, (Scherer, 2002) and (Zelazo and Mesbahi, 2009). Recently, state-space solutions to the so-called two-player state-feedback H_2 version of this problem have been given in (Swigart and Lall, 2010). Also, in (Shah and Parrilo, 2010), using concepts from order theory, a control architecture has been proposed for systems having the structure of a partially ordered set. In contrast, we construct conditional estimates based on the information shared among the controllers. Thereby, we show how to decompose the cost function into independent terms and hence to derive analytical forms for the controllers.

The main contribution of this paper is to introduce a simple decomposition scheme to construct optimal decentralized controllers with low computational complexity for chain structures which is applicable to intelligent transportation systems in terms of automated platooning. Derived from the characteristics of actual Scania HDVs, we present a discrete system model that includes physical coupling with a preceding vehicle and consider interconnected subsystems over a chain structure. The proposed control scheme accounts for a constrained communication pattern among the vehicles and hence reduces the communications compared to a centralized information pattern where full state information is available to each controller.

Notation. We denote a matrix partitioned into blocks by $A = [A_{ij}]$, where A_{ij} denotes the block matrix of A in block position (i, j). The submatrix of A formed by row partitions i through j and column partitions k through l will be denoted by $A[i : j, k : l]$:

$$A[i:j,k:l] = \begin{bmatrix} A_{ik} & A_{i(k+1)} & \cdots & A_{il} \\ A_{(i+1)k} & A_{(i+1)(k+1)} & \cdots & A_{(i+1)l} \\ \vdots & \vdots & \cdots & \vdots \\ A_{jk} & A_{j(k+1)} & \cdots & A_{jl} \end{bmatrix}.$$

The expected value of a random variable x is denoted by $\mathbf{E}\{x\}$. The conditional expectation of x given y is denoted by $\mathbf{E}\{x|y\}$. The trace of a matrix A is denoted by $\mathbf{Tr}\{A\}$, and the sequence $x(0)$, $x(1)$,..., $x(t)$, is denoted by $x(0:t)$.

Figure 1: The figure shows a platoon of M heavy duty vehicles, where each vehicle is only able to communicate with the preceding vehicles.

2 SYSTEM MODEL AND PROBLEM STATEMENT

In this section we present the physical properties of the system that we are considering, the linear discrete system model structure for a heterogeneous HDV platoon and its associated cost function. Finally, the problem formulation is given.

2.1 System Model

We consider an HDV platoon as depicted in Figure 1. The velocities do not deviate significantly for the vehicles with respect to the lead vehicle's velocity in an automated HDV platoon. Thus, a linearized model should give a sufficient description of the system behavior. The discrete HDV platoon model with respect to a set reference velocity, an engine torque which maintains the velocity, a fixed spacing between the vehicles, and a constant slope is hence given by,

$$x(t+1) = Ax(t) + Bu(t) + w(t), \tag{1}$$

where

$$A = \begin{bmatrix} \Theta_1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \delta_2 & \Theta_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_3 & \Theta_3 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \Theta_{M-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \delta_M & \Theta_M \end{bmatrix},$$

$$B = \begin{bmatrix} k_{u_1} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & k_{u_2} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & k_{u_3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & k_{u_M} \end{bmatrix}, \quad x = \begin{bmatrix} v_1 \\ d_{12} \\ v_2 \\ d_{23} \\ v_3 \\ \vdots \\ v_{M-1} \\ d_{(M-1)M} \\ v_M \end{bmatrix}, \tag{2}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_M \end{bmatrix}, \quad \begin{aligned} \Theta_1 &= T_s(1 - 2k_d v_0), \\ \Theta_i &= -T_s 2k_d \Phi(d_0) v_0, \quad i = 2, \ldots, M, \\ \delta_i &= -T_s \kappa_1 k_d v_0^2, \end{aligned}$$

where $\Phi(d)$ is the linearized air drag function, and δ_i and Θ_i denote physical dynamics constants (Alam, 2011). The derived model in (2) has a lower block triangular structure, which can generally be stated as

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ \vdots \\ x_M(t+1) \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 & \cdots & 0 \\ A_{21} & A_{22} & 0 & \cdots & 0 \\ 0 & A_{32} & A_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & A_{MM} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_M(t) \end{bmatrix} + \begin{bmatrix} B_1 & 0 & 0 & \cdots & 0 \\ 0 & B_2 & 0 & \cdots & 0 \\ 0 & 0 & B_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & B_M \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ \vdots \\ u_M(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \\ \vdots \\ w_M(t) \end{bmatrix} \quad (3)$$

where the corresponding vehicle states for each subsystem are

$$x_1(t) = v_1(t), \quad x_i(t) = \begin{bmatrix} d_{i-1,i} \\ v_i \end{bmatrix}, \quad i = 2, \ldots, M.$$

2.2 Performance Criteria

The performance criteria of an HDV platoon can be mapped into quadratic costs. Hence, we formulate the weight parameters for a quadratic cost function based upon performance and safety objectives. The objective of the lead vehicle is to minimize the fuel consumption and control input, while maintaining a set reference velocity. The objective of the follower vehicles in addition, is to follow the preceding vehicles velocity, while maintaining a set intermediate spacing. The intermediate spacing reference could be constant or, as in this case, time varying. It is determined by setting a desired time gap τ s, which in turn determines the spacing policy as

$$d_{ref}(t) = \tau v(t).$$

Thereby, the vehicles will maintain a larger intermediate spacing at higher velocities. Hence, the weights for a discrete M HDV platoon can be set up as

$$\begin{aligned} J(u^*) &= \min_u \sum_{t=0}^{N-1} \Big(\sum_{i=2}^{M} w_i^{\tau}(d_{(i-1)i}(t) - \tau v_i(t))^2 \\ &\quad + w_i^{\Delta v}(v_{i-1}(t) - v_i(t))^2 \\ &\quad + w_i^{d} d_{(i-1)i}^2(t) + \sum_{i=1}^{M} w_i^{v} v_i^2(t) + w_i^{u_i} u_i^2(t) \Big) \quad (4) \\ &= \min_u \sum_{t=0}^{N-1} \sum_{i=2}^{M} \begin{bmatrix} v_{i-1}(t) \\ d_{(i-1)i}(t) \\ v_i(t) \end{bmatrix}^T Q_i \begin{bmatrix} v_{i-1}(t) \\ d_{(i-1)i}(t) \\ v_i(t) \end{bmatrix} + R_i u_i^2(t) \\ &\quad + w_1^{v} v_1^2(t) + w_1^{u_1} u_1^2(t) \end{aligned}$$

where

$$Q_i = \begin{bmatrix} w_i^{\Delta v} & 0 & -w_i^{\Delta v} \\ 0 & w_i^{d} + w_i^{\tau} & -\tau w_i^{\tau} \\ -w_i^{\Delta v} & -\tau w_i^{\tau} & \tau^2 w_i^{\tau} + w_i^{\Delta v} + w_i^{v} \end{bmatrix}, \quad (5)$$
$$R_i = w_i^{u_i}.$$

The weights in (4) give a direct interpretation of how to enforce the objectives for a vehicle traveling in a platoon. The value of w_i^{τ} determines the importance of not deviating from the desired time gap. Hence, a large w_i^{τ} puts emphasis on safety. $w_i^{\Delta v}$ creates a cost for deviating from the velocity of the preceding vehicle, and $w_i^{u_i}$ punishes the control effort which is proportional to the fuel consumption. The following terms, w_i^{d}, w_i^{v}, put a cost on the deviation from the linearized states. Note that the main objective is to maintain a set intermediate distance, while maintaining a fuel efficient behavior. Therefore, $w_i^{\tau}, w_i^{\Delta v}$ and $w_i^{u_i}$ must be set larger than the remaining weights. The weights are chosen such that Q is positive semidefinite and R is positive definite.

2.3 Problem Formulation

Although the approach used in this paper is applicable for systems over general acyclic graphs, for simplicity we will concentrate on a simple chain structure, which we refer to as the three-vehicle chain. The aim is to synthesize controllers under imposed communication constraints.

For this problem, the system matrices are given by

$$A = \begin{bmatrix} A_{11} & 0 & 0 \\ A_{21} & A_{22} & 0 \\ 0 & A_{32} & A_{33} \end{bmatrix}, B = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix}. \quad (6)$$

Here, $\{w(t)\}$ is a Gaussian disturbance vector with covariance given by

$$\mathbf{E}\{w(k)w^T(l)\} = \begin{bmatrix} W_1 & 0 & 0 \\ 0 & W_2 & 0 \\ 0 & 0 & W_3 \end{bmatrix} \delta(k-l).$$

In this system, the dynamics of subsystem i (Vehicle i) propagates to subsystem j (Vehicle j) only if $i < j$. If all subsystems have access to the global state measurements the information structure would be classical, and the optimal linear controller could be obtained from the linear quadratic control theory. However, in the practical setting of HDV platooning the lead vehicle only has its own state information, whereas the follower vehicle can also measure the states of the preceding vehicle through radar sensors. Therefore, we consider the case in which u_2 has access to the measurement history of subsystems, 1 and 2, while u_1 has access to its own measurements. Also, to obtain partially nestedness u_3 must have access the global state measurements. Let $\mathbb{I}_i^t$ denote the information set of controller i at time t. Then

$$\begin{aligned}&\mathbb{I}_1^t = \{x_1(0:t)\}, \ \ \mathbb{I}_2^t = \{x_1(0:t), x_2(0:t)\},\\ &\mathbb{I}_3^t = \{x_1(0:t), x_2(0:t), x_3(0:t)\}.\end{aligned} \tag{7}$$

This information pattern is not classical anymore and is a simple case of a *partially nested* information structure. This is one of a few non-classical information patterns for which the optimal policy is known to be unique and linear in the information set.

Here only one communication link is needed from vehicle 1 to vehicle 3, since vehicle 2 and 3 can measure the preceding vehicle states with on-board radar sensors.

Thus, the problem that we solve is finding an analytical formulation for optimal controllers constrained to specified information sets that minimize the infinite-horizon quadratic cost

$$\lim_{N\to\infty} \frac{1}{N}\mathbf{E}\sum_{t=0}^{N-1}(x^T(t)Qx(t) + u^T(t)Ru(t)), \tag{8}$$

subject to the given system dynamics and performance objectives.

3 MAIN RESULT: THREE-VEHICLE CHAIN

The aim of this section is to present an outline of the optimal controller structures for the three vehicles chain problem. A detailed version and intuition for the general case is given in (Khorsand et al., 2012).

Theorem 1. *Assume that*

i. (A,B), $(A[2:3,2:3], B[2:3,2:3])$, *and* (A_{33}, B_3) *are stabilizable,*

ii. (Q,A), $(Q[2:3,2:3],\ A[2:3,2:3])$, *and* (Q_{33}, A_{33}) *are detectable.*

Then, the optimal controller for the three-vehicle chain is given by:

$$\begin{bmatrix}\eta_1(t+1)\\ \eta_2(t+1)\end{bmatrix} = (A - BL^1)[2:3,1:3]\begin{bmatrix}x_1(t)\\ \eta_1(t)\\ \eta_2(t)\end{bmatrix}$$

$$\eta_3(t+1) = (\tilde{A} - \tilde{B}L^2)[2,1:2]\begin{bmatrix}x_2(t)-\eta_2(t)\\ \eta_3(t)\end{bmatrix}$$

$$\begin{bmatrix}u_1(t)\\ u_2(t)\\ u_3(t)\end{bmatrix} = -L^1\begin{bmatrix}x_1(t)\\ \eta_1(t)\\ \eta_2(t)\end{bmatrix} - \begin{bmatrix}0\\ L^2\begin{bmatrix}x_2(t)-\eta_1(t)\\ \eta_3(t)\end{bmatrix}\end{bmatrix} - \begin{bmatrix}0\\ 0\\ L^3(x_3(t)-\eta_2(t)-\eta_3(t))\end{bmatrix},$$

and the optimal cost *is*

$$\mathbf{Tr}(X_{11}^1 W_1) + \mathbf{Tr}(X_{11}^2 W_2) + \mathbf{Tr}(X^3 W_3).$$

The matrices X^1, X^2, and X^3 are the positive semidefinite stabilizing solutions to the Riccati equations

$$\begin{aligned}X^1 =& A^T X^1 A + Q - A^T X^1 B(B^T X^1 B + R)^{-1} B^T X^1 A\\ X^2 =& \tilde{A}^T X^2 \tilde{A} + \tilde{Q} - \tilde{A}^T X^2 \tilde{B}(\tilde{B}^T X^2 \tilde{B} + \tilde{R})^{-1}\tilde{B}^T X^2 \tilde{A}\\ X^3 =& A_{33}^T X^3 A_{33} + Q_{33}\\ & - A_{33}^T X^3 B_3 (B_3^T X^3 B_3 + R_{33})^{-1} B_3^T X^3 A_{33}\end{aligned}$$

where $\tilde{A} = A[2:3,2:3]$, $\tilde{B} = B[2:3,2:3]$, $\tilde{Q} = Q[2:3,2:3]$ *and* $\tilde{R} = R[2:3,2:3]$. *The matrix X^1 is partitioned into blocks according to the partitions of x as*

$$X^1 = [X_{ij}^1], \ \ i,j = 1,..,3,$$

also, X^2 is partitioned according to the dimensions of x_2 and x_3 as

$$X^2 = [X_{ij}^2], \ \ i,j = 1,2.$$

The gain matrices are given by

$$\begin{aligned}L^1 &= (R + B^T X^1 B)^{-1} B^T X^1 A,\\ L^2 &= (\tilde{R} + \tilde{B}^T X^2 \tilde{B})^{-1}\tilde{B}^T X^2 \tilde{A},\\ L^3 &= (R_{33} + B_3^T X^3 B_3)^{-1} B_3^T X^3 A_{33}.\end{aligned}$$

Proof 1. *See (Khorsand et al., 2012).*

4 NUMERICAL RESULTS

In this section, we implement the proposed controller on an $M = 3$ HDV platoon (Figure 1) and evaluate the performance through a realistic scenario that HDV platoons often face on the road. We assume that the vehicles in the platoon can only measure the velocity and relative distance of the preceding vehicle and only receive state information through wireless communication from all the preceding vehicles. This assumption is made to evaluate if a small addition in communication links could improve the system performance. Hence, a numerical comparison is made between the proposed controller and a suboptimal controller specifically designed for HDV platooning (Alam et al., 2011). The suboptimal controller uses local information, namely it only accounts for the dynamics of the preceding vehicle. Finally, we compare the proposed controller with the fully centralized linear quadratic controller.

In practice, many random disturbances such as wind variation, changing topology, or varying road properties are inflicted upon the HDV platoon. These disturbances are modeled as disturbances in state measurements. An additional disturbance of interest is a mandated deviation in the lead vehicles velocity. This often occurs due to varying traffic events that the lead vehicle must adhere to. Hence, integral action for the lead vehicle is also added as a state to the system presented in (2), to model such disturbances.

The modeled HDVs are described as traveling in a longitudinal direction on a flat road. We consider a heterogeneous platoon, where the masses are set to $[m_1, m_2, m_3] = [30000, 40000, 30000]$ kg. All the vehicles are assumed to be traveling in the steady state velocity $v_0 = 19.44$ m/s (70 km/h) at time gap $\tau = 1$ s, which gives an intermediate distance of $d_0 = 19.44$. The maximum engine and braking torque for a commercial HDV varies based upon vehicle configuration but can be approximated to be 2500 Nm and 60000 Nm/Axle respectively.

State disturbances as well as several lead vehicle deviation disturbances are imposed on the system, see Figure 2. The lead vehicle deviation disturbances can be explained by the following scenario. The platoon travels along a road where the road speed is 70 km/h. Suddenly a slower vehicle enters the lane through a shoulder path (at the 45 s time marker). The lead vehicle must therefore reduce its speed to 60 km/h, in turn forcing the follower vehicles to reduce their speed and adapt their relative distance accordingly. After a while, the slower vehicle increases its speed to the road speed of 70 km/h and no longer inhibits the platoon (120 s time marker). Hence, the lead vehicle again resumes the road speed and the follower vehicles adapt the speed and distance automatically as well. Finally, the platoon arrives at a point where the road speed is changed to 80 km/h (180 s time marker).

Figure 2 shows the velocity trajectories of three HDV platoon in the top plot and the corresponding intermediate spacings in this scenario. The trajectories obtained through the optimal decentralized controller are bold. The trajectories are also plotted, with thinner lines, for the suboptimal decentralized controller. We see that the proposed optimal controller displays a good performance. The suboptimal controller displays a slightly harsher behavior with a faster speed change, since it does not take follower vehicles into account. Hence, the required relative control input energy is much higher for the suboptimal controller compared to our proposed controller, as can been seen in Figure 3. It can also be seen that the required control input to handle the disturbances are well within the feasible physical range. By estimating the states of the follower vehicles, the mandated control input to handle the presented disturbances can be reduced significantly. Both the maximum and minimum values are lowered in the control input requirement for the optimal decentralized controller. The first two rows in Table 1 states the total control input energy required to handle the imposed disturbances. We can see that the optimal distributed controller reduces the control input energy by 10.4 % for the lead vehicle,

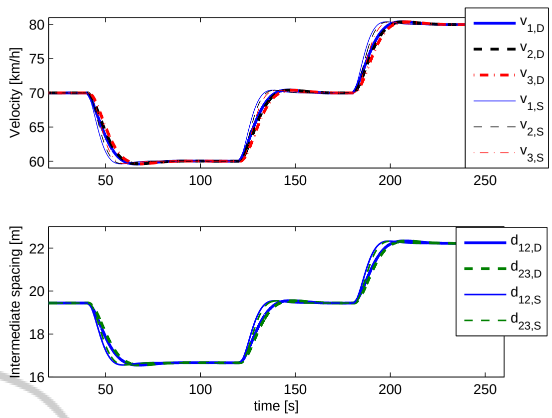

Figure 2: Three HDV platoon, where a disturbance in velocity of the lead vehicle is imposed. The top plot shows the velocity trajectories for the $M = 3$ HDV platoon and the bottom plot shows the intermediate spacings. The trajectories obtained through the optimal decentralized controller are bold and subindexed with i, D and the trajectories obtained through the suboptimal controller are subindexed with i, S, where $i = 1, 2, 3$ denote the platoon position index.

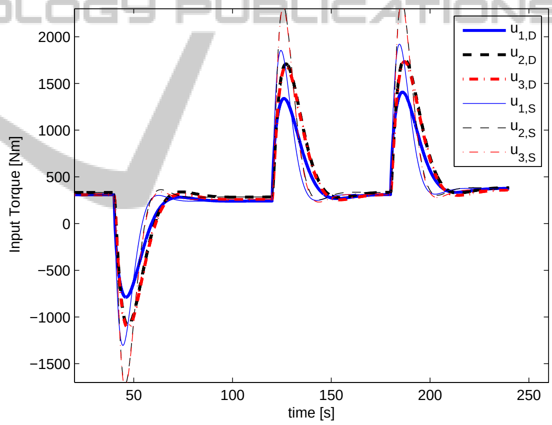

Figure 3: Corresponding input torque to handle the imposed disturbances in Figure 2. Similarly, the trajectories obtained through the optimal decentralized controller are bold and subindexed with i, D and the trajectories obtained through the suboptimal controller are subindexed with i, S, $i = 1, 2, 3$.

by 16.3 % for the second vehicle, and by 15.5 % for the third vehicle. By estimating the states of the follower vehicles, the proposed controller mimics a centralized control strategy and displays a smoother behavior. However, the reduced control energy is obtained at the cost of adding a communication link between vehicle 1 and vehicle 3, since vehicle 1's state cannot be measured through the mounted radar on vehicle 3. Furthermore, the average velocity is reduced by 2.6 %. Travel time is equally important for fleet operators. However, it is clear that there is a considerable saving in the fuel consumption at the cost of additional communication links and a much smaller

Table 1: Table of the required control input (Torque) to handle the disturbances in Figure 3.

i	1	2	3
$\|\|u_{i,D}\|\|_2$ [kNm]	81.9	100.1	80.8
$\|\|u_{i,S}\|\|_2$ [kNm]	91.4	112.6	89.0

reduction in travel time.

The proposed controller also accounts for all the states in the platoon by estimating the states of the follower vehicles, the behavior is close to the centralized controller. The computed relative differences in the cost function as well as the difference in required control inputs to handle the disturbances are minimal.

5 CONCLUSIONS

We have presented a quadratic optimal distributed control method for chain structures with applications to heterogeneous vehicle platooning under communication constraints. A procedure has been given for constructing low order optimal decentralized controllers through a simple decomposition scheme. A discrete HDV platoon model has been derived that includes physical coupling between the vehicles upon which the controllers are evaluated. The results show that the total control input energy required for the proposed controller is very close to a centralized controller where communication is needed among all the vehicles, and is significantly lower compared to a suboptimal controller which only accounts for the immediate preceding vehicle. In particular, by estimating the interaction with the follower vehicles, performance can be improved by adding a communication link from the first to the third vehicle in a three-vehicle platoon. Thus, considering preceding vehicles as well as follower vehicles is significant for fuel optimality.

A natural extension to the presented work is to derive explicit solutions for the problem of M-HDVs. It is planned for future work.

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