

Trajectory Tracking Control of Nonholonomic Wheeled Mobile Robots

Combined Direct and Indirect Adaptive Control using Multiple Models Approach

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Abstract: This paper presents a novel methodology for the trajectory tracking control of nonholonomic wheeled mobile robots using multiple identification models. The overall control system includes two stages. In the first stage, a kinematic controller developed by using kinematic model provides the required linear and angular velocities of the robot for tracking a reference trajectory. In the second stage, the required velocities are taken as the inputs to an adaptive dynamic controller which uses multiple adaptive models for the parameter identification. The proposed adaptive dynamic controller is developed using a combined direct and indirect adaptive control approach where both prediction and tracking errors are used for identification. Simulation results show the effectiveness of the proposed combined direct and indirect control scheme and multiple models approach.

1 INTRODUCTION

Tracking control of a wheeled mobile robot (WMR) is one of the most attractive research areas for the several decades. Many WMR models and control schemes have been presented. Generally, the aim of such schemes is either to utilize a kinematic trajectory tracking controller or to construct and integrate kinematic and dynamic controllers to track a desired trajectory. Yutaka et al. (1990) proposed a control rule to determine reasonable linear and rotational velocities for a stable tracking control. An integrated kinematic controller and a torque controller with a dynamic extension for a nonholonomic mobile robot have been presented by Fierro and Lewis (1995). Yun and Yamamoto (1992) have studied feedback linearization of a WMR and its dynamic system. A complete dynamic model of a WMR which makes it suitable to consider rotational and translational velocities as control signals has been given by De La Cruz and Carelli (2006).

For the tracking control of a WMR, there are also adaptive control frameworks in literature. Felipe et al. (2008) have proposed an adaptive controller to

guide a WMR during trajectory tracking. In this study reference velocities are generated using a kinematic model, and then these values are processed to compensate for the robot dynamics. An adaptive trajectory tracking controller for a nonholonomic WMR with a nonlinear control law based on input-output feedback linearization has been proposed by Khoshnam et al. (2010). Cao et al. (2011) has proposed an adaptive kinematic controller to generate the command of velocity based on backstepping method, and then Zhengcai et al. (2011) has proposed adopting the reference model with a dynamic adaptive controller. Similarly, a new kinematic adaptive controller integrated with a torque controller for the dynamic model of a nonholonomic WMR has been proposed by Takanori et al.(2000). Pourboghraat and Karlsson (2002) has used adaptive control rules for the dynamics level of nonholonomic WMRs with unknown dynamic parameters and a fixed posture backstepping technique for tracking a reference trajectory and stabilization. Petrov (2010) has proposed an adaptive dynamic based path control for a differential drive mobile robot.

The studies previously mentioned provide the schemes of trajectory tracking, but they did not

focus on the transient behaviour. However, when the parameter errors are very large, the transient response of the system may include unacceptably large peaks. Although the system is asymptotically stable, the adaptive control approach may be inapplicable for some systems due to the transient peaks. To overcome this difficulty, the enhancement of the transient response using multiple models and switching has been proposed for the linear systems by Narendra and Balakrishnan (1997). Some approaches using multiple models and switching for nonlinear systems have been presented in several studies. Narendra and George (2002) have presented a multiple model, switching and tuning methodology which improves the transient performance for a class of nonlinear systems. A novel approach which makes use of multiple identification models and switching based on direct adaptive control scheme has been proposed by Cezayirli and Ciliz (2007). Besides composite approach where both prediction and tracking errors are used in a combined direct and indirect adaptive control framework has been studied by (Ciliz and Narendra, 1995) and (Ciliz and Cezayirli, 2004). Ye (2008) has proposed a multiple model adaptive controller for nonlinear systems in parametric-strict-feedback form. An adaptive control of a class of single-input single-output (SISO) nonlinear systems considering transient performance improvement by using multiple models and switching has been considered by Cezayirli and Ciliz (2006 and 2008). Ciliz and Narendra (1994), Ciliz and Tuncay (2005) have used a scheme consisting of multiple models, switching and tuning for the adaptive control of robotic manipulators.

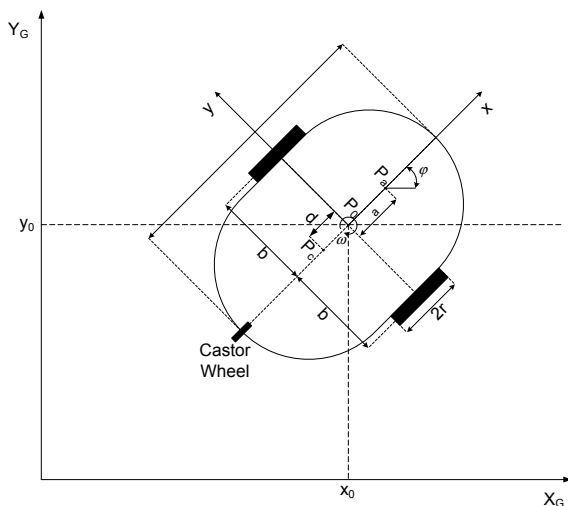


Figure 1: Nonholonomic WMR.

The purpose of this paper is to present an integrated kinematic and dynamic controller for the trajectory tracking of a WMR that includes parametric uncertainties in the dynamics. A composite approach, in which both prediction and tracking errors are used in a combined direct and indirect adaptive control framework with multiple identification models and switching, is used. There are a few works which make use of the multiple models approach for the control of the WMRs. De La Cruz et al. (2008) has proposed a switching control for a novel tracking adaptive control of WMRs. Another method that uses multiple models of the robot for its identification in an adaptive and learning control framework has been presented by D'Amico et al. (2006).

2 KINEMATICS AND DYNAMICS

Consider the WMR model given by (1). The parameters are given in Table 1 and the system is shown in Figure 1. The system is subjected to m constraints:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = B(q)\tau + A^T(q)\lambda \quad (1)$$

where $q \in R^n$ is generalized coordinates, $\tau \in R^r$ is the input vector, $\lambda \in R^m$ is the vector of constraint forces, $M(q) \in R^{n \times n}$ is a symmetric positive-definite inertia matrix, $C(q, \dot{q}) \in R^{n \times n}$ is coriolis matrix, $B(q) \in R^{n \times r}$ is the input transformation matrix, and $A(q) \in R^{m \times n}$ is the matrix associated with the constraints.

Table 1: Model Parameters of WMR.

Parameter	Description
r	Driving wheel radius
$2b$	Distance between two wheels
d	Distance point P_c from point P_0
a	Distance from P_0 to P_a
m_c	The mass of the platform without the driving wheels and the rotors of the DC motors
m_w	The mass of each driving wheel plus the rotor of its motor
I_c	The moment of inertia of the platform without the driving wheels and the rotors of the motors about a vertical axis through P_c
I_w	The moment of inertia of each wheel and the motor rotor about the wheel axis
I_m	The moment of inertia of each wheel and the motor rotor about a wheel diameter

Assuming that the velocity of P_0 is in the direction of x -axis of the local frame and there is no side slip, and considering $q = [x_0 \ y_0 \ \varphi]^T$, the following

constraint with respect to P_0 is obtained

$$\dot{x}_0 \sin \varphi - \dot{y}_0 \cos \varphi = 0 \quad (2)$$

By writing this constraint in matrix form, matrices $A(q)$ and $S(q)$ are given by

$$A(q) = \begin{bmatrix} \sin \varphi & -\cos \varphi & 0 \end{bmatrix}, \quad S(q) = \begin{bmatrix} \cos \varphi & 0 \\ \sin \varphi & 0 \\ 0 & 1 \end{bmatrix} \quad (3)$$

Therefore, it can be written as

$$A(q) \cdot S(q) = 0 \quad (4)$$

It is possible to write the kinematic equation of the wheeled mobile robot motion in terms of the pseudo velocities vector $v(t) \in R^{n-m}$ as

$$\dot{q} = S(q) \cdot v(t), \quad (5)$$

where $v(t) \in [v(t) \ \omega(t)]^T$ is made up of linear and angular velocities. Taking the time derivative of (5)

$$\ddot{q} = \dot{S}(q) \cdot v + S(q) \cdot \dot{v} \quad (6)$$

Next, by replacing (5) and (6) into (1), multiplying the result by S^T and considering (4), the following equation can be obtained

$$\bar{M}\dot{v}(t) + \bar{C}(v)v(t) = \bar{B}(q)\tau, \quad (7)$$

where $\bar{M} = S^T M S$, $\bar{C} = S^T (M \dot{S} + C S)$ and $\bar{B} = S^T B$.

By denoting $\bar{B}(q)\tau$ as $\bar{\tau}$

$$\bar{M}\dot{v}(t) + \bar{C}(v)v(t) = \bar{\tau}, \quad (8)$$

The matrices \bar{M} and \bar{C} are obtained as follows:

$$\bar{M} = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 & m_c d \dot{\varphi} \\ -m_c d \dot{\varphi} & 0 \end{bmatrix} \quad (9)$$

where $m = m_c + 2m_w$ and $I = I_c + 2I_m + m_c d^2 + 2m_w b^2$.

There is a parametric vector θ on dynamics that satisfies

$$\bar{M}\dot{v}(t) + \bar{C}(v)v(t) = Y(q, \dot{q}, v, \dot{v})\theta, \quad (10)$$

where the parameters $\theta_i, i=1, \dots, 4$ are bounded and defined as follows

$$\theta_1 = m, \quad \theta_2 = I, \quad \theta_3 = m_c d \quad (11)$$

3 CONTROLLER DESIGN

3.1 Kinematic Controller

In the proposed control scheme, a kinematic controller is used (Felipe et al., 2008). The design of the kinematic controller is based on the kinematic model of the WMR. The WMR's kinematic model is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos \varphi & -a \sin \varphi \\ \sin \varphi & a \cos \varphi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad (12)$$

where x, y are the coordinates of the point of interest P_a , and the outputs. By assuming

$$h = [x, y]^T, \quad \dot{h} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos \varphi & -a \sin \varphi \\ \sin \varphi & a \cos \varphi \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} = T \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad (13)$$

where

$$T = \begin{bmatrix} \cos \varphi & -a \sin \varphi \\ \sin \varphi & a \cos \varphi \end{bmatrix}. \quad (14)$$

The inverse of the matrix T is

$$T^{-1} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\frac{1}{a} \sin \varphi & \frac{1}{a} \cos \varphi \end{bmatrix}. \quad (15)$$

Therefore, the inverse kinematics is given by

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\frac{1}{a} \sin \varphi & \frac{1}{a} \cos \varphi \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}, \quad (16)$$

and the proposed kinematic controller is given by

$$\begin{bmatrix} v_{ref} \\ \omega_{ref} \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\frac{1}{a} \sin \varphi & \frac{1}{a} \cos \varphi \end{bmatrix} \begin{bmatrix} \dot{x}_d + I_x \tanh\left(\frac{k_x}{I_x} \tilde{x}\right) \\ \dot{y}_d + I_y \tanh\left(\frac{k_y}{I_y} \tilde{y}\right) \end{bmatrix} \quad (17)$$

Here, $\tilde{x} = x_d - x$, and $\tilde{y} = y_d - y$ are the current position errors in the direction of x -axis and y -axis, respectively. $k_x > 0$ and $k_y > 0$ are the gains of the controller, $I_x \in R$, and $I_y \in R$ are saturation constants, and (x, y) and (x_d, y_d) are the current and desired coordinates of the point of interest, respectively. The purpose of the kinematic

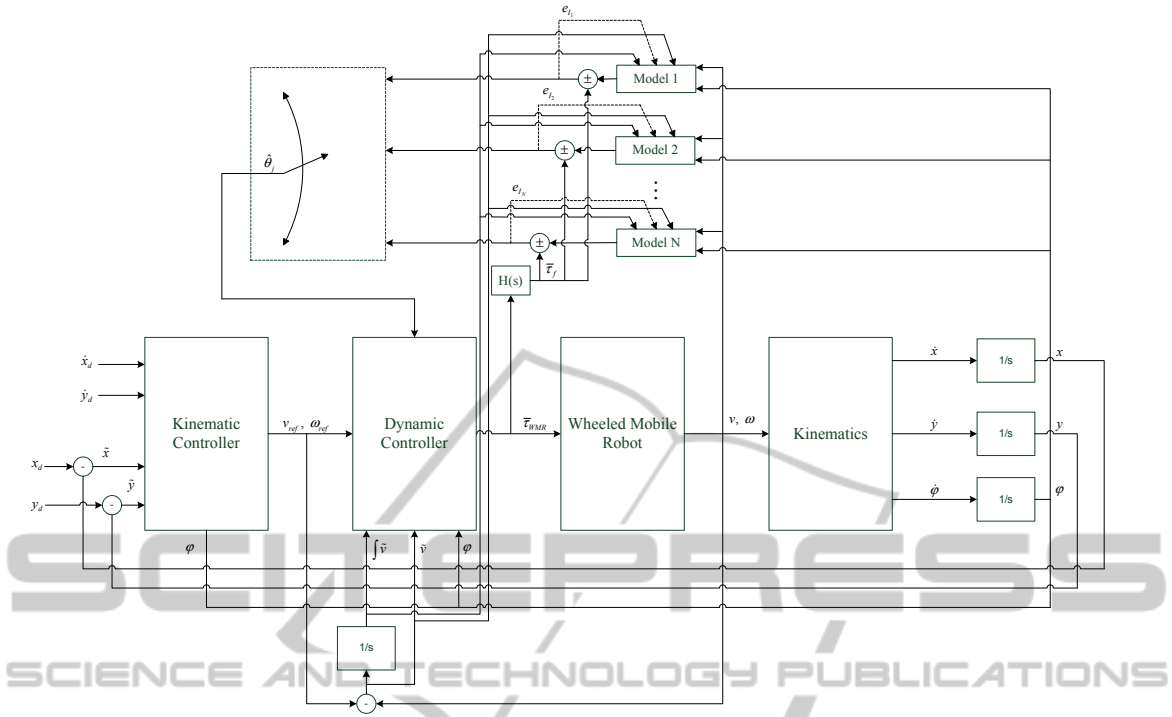


Figure 2: Block diagram of the control architecture.

controller is to generate the reference linear and angular velocities for the dynamic controller as shown in Figure 2.

3.2 Adaptive Dynamic Controller

A Proportional-Integral (PI) filtered velocity tracking error signal is given as (Wilson and Robinett, 2001)

$$s = e_v + \lambda \int e_v dt \quad (18)$$

where λ is a positive definite control gain and velocity tracking error is defined as

$$e_v = v_d - v. \quad (19)$$

where $v_d = [v_d \ \omega_d]^T$ is the vector of the desired linear and rotational velocities. Taking the derivative of (18),

$$\dot{s} = \dot{e}_v + \lambda e_v \quad (20)$$

can be obtained. Considering (8) and adding the PI filtered error terms yields

$$\bar{M}\dot{s} + \bar{C}(v)s = Y_2(v_d, \dot{v}_d)\theta - \bar{\tau} \quad (21)$$

$$Y_2(v_d, \dot{v}_d)\theta = \bar{M}(\dot{v}_d + \lambda e_v) + \bar{C}(v)(v_d + \lambda \int e_v dt) \quad (22)$$

To determine the control law and adaptive parameter update rule, consider the following Lyapunov-like function (Lewis et al., 2004)

$$V = \frac{1}{2} s^T \bar{M}s + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad (23)$$

and differentiating the function with respect to time

$$\dot{V} = \frac{1}{2} s^T \dot{\bar{M}}s + s^T \bar{M}\dot{s} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \quad (24)$$

By taking $\bar{M}\dot{s}$ from (21) and adding to (24), the following equation can be obtained

$$\begin{aligned} \dot{V} = & s^T (Y_2(v_d, \dot{v}_d)\theta - \bar{\tau} - \bar{C}(v)s) \\ & + \frac{1}{2} s^T \dot{\bar{M}}s + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \end{aligned} \quad (25)$$

By choosing the control law

$$\bar{\tau} = Y_2(v_d, \dot{v}_d)\hat{\theta} + K_v s \quad (26)$$

and adding (26) into the (25), the following equation can be obtained

$$\begin{aligned} \dot{V} = & s^T Y_2(v_d, \dot{v}_d)\tilde{\theta} - s^T K_v s \\ & + \frac{1}{2} s^T (\dot{\bar{M}} - 2\bar{C}(v))s + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \end{aligned} \quad (27)$$

Reader should note that the matrix $\dot{\bar{M}} - 2\bar{C}(v)$ is a skew-symmetric matrix. By choosing the parameter update rule as

$$\dot{\tilde{\theta}} = -\Gamma (Y_1^T(q, \int v, v)\tilde{\tau}_f + Y_2^T(v_d, \dot{v}_d)s) \quad (28)$$

with an identification error model

$$\tilde{\tau}_f = Y_1(q, \int v, v) \tilde{\theta} \quad (29)$$

and inserting (28) into (27)

$$\begin{aligned} \dot{V} = & s^T Y_2(v_d, \dot{v}_d) \tilde{\theta} - s^T K_v s \\ & + \frac{1}{2} s^T (\dot{\bar{M}} - 2\bar{C}(v)) s \\ & + \tilde{\theta}^T \Gamma^{-1} \left(-\Gamma \left(Y_1^T(q, \int v, v) \tilde{\tau}_f + Y_2^T(v_d, \dot{v}_d) s \right) \right) \end{aligned} \quad (30)$$

where $Y_1(q, \int v, v)$ is the filtered regressor matrix and $\tilde{\tau}_f$ is the filtered torque term (Ciliz and Narendra, 1994). Rearranging (29)

$$\begin{aligned} \dot{V} = & -s^T K_v s + \frac{1}{2} s^T (\dot{\bar{M}} - 2\bar{C}(v)) s \\ & - \tilde{\theta}^T Y_1^T(q, \int v, v) \tilde{\tau} \end{aligned} \quad (31)$$

may be obtained. By considering the identification error model in (29) and adding into (31)

$$\begin{aligned} \dot{V} = & -s^T K_v s + \frac{1}{2} s^T (\dot{\bar{M}} - 2\bar{C}(v)) s \\ & - \tilde{\theta}^T Y_1^T(q, \int v, v) Y_1(q, \int v, v) \tilde{\theta} \end{aligned} \quad (32)$$

may be obtained. For the proof of stability, the same procedures should be followed (Lewis et al., 2004). It should be noted that \dot{V} is negative definite. It can be stated that V in (23) is upper bounded and that $\bar{M}(q)$ is a positive definite matrix it can be stated that s and $\tilde{\theta}$ are bounded. Standard linear control arguments can be used to state that e_v and $\int e_v$ are bounded. Since $e_v, \int e_v, s, \tilde{\theta}$ are bounded it can be shown that \dot{s} and \dot{V} are also bounded. The reader should note that since $\bar{M}(q)$ is lower bounded, it can be stated that V is also lower bounded. Since \dot{V} is lower bounded, V is negative definite and \dot{V} is bounded, the Barbalat's Lemma can be used to state that

$$\lim_{t \rightarrow \infty} \dot{V} = 0 \quad (33)$$

which means that by Rayleigh-Ritz Theorem

$$\lim_{t \rightarrow \infty} \lambda_{\min} \{K_v\} \|s\|^2 = 0 \text{ or } \lim_{t \rightarrow \infty} s = 0 \quad (34)$$

Using the standart linear control arguments the following can be written

$$\lim_{t \rightarrow \infty} \int e_v = 0 \text{ and } \lim_{t \rightarrow \infty} e_v = 0 \quad (35)$$

3.3 Adaptive Dynamic Controller with Multiple Models

Identification models have the following structure

$$\hat{\tau}_j = \hat{M}_j \dot{v}(t) + \hat{C}_j(v) v(t) = Y(q, \dot{q}, v, \dot{v}) \hat{\theta}_j \quad (36)$$

where $j=1, \dots, N$, $\hat{\theta}_j$ denoting the parameter estimate vector and $Y(q, \dot{q}, v, \dot{v})$ is the non-linear regressor matrix. The regressor matrix common to all models, but the parameter vector $\hat{\theta}_j$ has different initializations chosen from a given compact parameter set. Using the filtering technique previously mentioned nonlinear regressor matrix without acceleration signal can be obtained and will be denoted as $Y_1(q, \int v, v)$. Each model is updated using simple gradient algorithm as it is in single model case:

$$\dot{\hat{\theta}}_j = -\Gamma(Y_1^T(q, \int v, v) \tilde{\tau}_f + Y_2^T(v_d, \dot{v}_d) s) \quad (37)$$

based on the error model which is defined as,

$$\tilde{\tau}_f = e_{\tau_j} = \tau_j - \hat{\tau}_f = Y_1(q, \int v, v) \tilde{\theta}_j \quad (38)$$

where $\tilde{\tau}_f$ is the filtered torque prediction error. $Y_2(v, \dot{v}_d)$ is the regressor matrix common to all models which is given in (22). The torque vector τ_j of j th identification model is given as:

$$\tau_j = Y_2(v_d, \dot{v}_d) \hat{\theta}_j + K_v s. \quad (39)$$

Adding the equations and (21) into (8), the closed loop dynamics can be obtained as:

$$\bar{M}_j \dot{s} + \bar{C}_j(v) s + K_v s = \bar{M}_j (\dot{v}_d + \lambda e_v) + \bar{C}_j(v) (v_d + \lambda \int e_v) \quad (40)$$

which can further be written as

$$\bar{M}_j \dot{s} + \bar{C}_j(v) s + K_v s = Y_2(v_d, \dot{v}_d) \tilde{\theta}_j \quad (41)$$

At any instant, the identification errors of the N models are available, but only one of the torque vectors τ_j is chosen as the input to the WMR.

In order to choose a switching criterion, first a permissible switching sequence and a switching rule must be given (Ciliz and Narendra, 1994 & 1995). A finite or infinite sequence $T_i : T_i \in R^+$ is defined as a switching sequence if $T_0 = 0$ and $\forall i, T_i < T_{i+1}$. Additionally, if there is a number $T_{\min} > 0$ such that $\forall i, T_{i+1} - T_i \geq T_{\min}$, then the sequence is called permissible switching scheme.

A switching rule is a function of time that takes values in the set $1, \dots, N$ is constant in $[T_i, T_{i+1})$ and is continuous from right. In other words, a function $h(t) : R_+ \rightarrow 1, \dots, N$ is called switching rule, if there exists a switching sequence T_i such that if $t \in [T_i, T_{i+1})$ for some $i < \infty$, then $h(t) = h(T_i)$. With this definition torque input in (21) can be defined as:

$$\bar{\tau}(t) = \bar{\tau}_{h(t)} \quad t \geq 0. \quad (42)$$

The torque vector combined with a permissible switching rule given as

$$\bar{\tau}_{h(t)} = Y_{2_{h(t)}}(v_d, \dot{v}_d) \hat{\theta}_j + K_v s \quad (43)$$

For the proof of stability, the same procedure will be followed as in the single model case. The additional requirement is that under any permissible switching rule, all signals should remain bounded. We have a Lyapunov-like function

$$V_j = \frac{1}{2} s^T \bar{M}_j s + \tilde{\theta}_j^T \Gamma^{-1} \tilde{\theta}_j \quad (44)$$

The derivative of (44) can be obtained as in the following equation

$$\begin{aligned} \dot{V}_j = & -s^T K_v s + \frac{1}{2} s^T (\dot{\bar{M}}_j - 2\bar{C}_j(v)) s \\ & - \tilde{\theta}_j^T Y_1^T(q, \int v, v) Y_1(q, \int v, v) \tilde{\theta}_j \end{aligned} \quad (45)$$

\dot{V}_j is negative definite. It can be stated that V_j in (44) is upper bounded and that $\bar{M}_j(q)$ is a positive definite matrix, it can be stated that s and $\tilde{\theta}_j$ are bounded. Standard linear control arguments can be used to state that e_v and $\int e_v$ are bounded. Since $e_v, \int e_v, s, \tilde{\theta}_j$ are bounded it can be shown that \dot{s} and \dot{V}_j are also bounded. The reader should note that since $\bar{M}_j(q)$ is lower bounded, it can be stated that V_j is also lower bounded. Since \dot{V}_j is lower bounded, V_j is negative definite and \dot{V}_j is bounded, the Barbalat's Lemma can be used to state that

$$\lim_{t \rightarrow \infty} \dot{V}_j = 0, \quad (46)$$

which means that by Rayleigh-Ritz Theorem

$$\lim_{t \rightarrow \infty} \lambda_{\min} \{K_v\} \|s\|^2 = 0 \text{ or } \lim_{t \rightarrow \infty} s = 0. \quad (47)$$

Using the standard linear control arguments as in single model case the following can be written

$$\lim_{t \rightarrow \infty} \int e_v = 0 \text{ and } \lim_{t \rightarrow \infty} e_v = 0. \quad (48)$$

3.4 Proof of Stability for the Kinematic Controller

In order to understand the rest of the proof, the reader may read (Martins et al., 2008). By considering (13) and (14):

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{bmatrix} + \begin{bmatrix} I_x & 0 \\ 0 & I_y \end{bmatrix} \begin{bmatrix} \tanh\left(\frac{k_x}{I_x} \tilde{x}\right) \\ \tanh\left(\frac{k_y}{I_y} \tilde{y}\right) \end{bmatrix} = \begin{bmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \end{bmatrix} \quad (49)$$

One can see that the error vector \mathcal{E} can also be written as Te , where e is the velocity tracking error and matrix T is defined before. Rewriting (49)

$$\dot{\tilde{h}} + L(\tilde{h}) = Te,$$

$$L(\tilde{h}) = \begin{bmatrix} I_x & 0 \\ 0 & I_y \end{bmatrix} \begin{bmatrix} \tanh\left(\frac{k_x}{I_x} \tilde{x}\right) \\ \tanh\left(\frac{k_y}{I_y} \tilde{y}\right) \end{bmatrix} \quad (50)$$

Now considering Lyapunov candidate function and its derivative

$$\begin{aligned} V &= \frac{1}{2} \tilde{h}^T \tilde{h}, \\ \dot{V} &= \tilde{h}^T \dot{\tilde{h}} = \tilde{h}^T (Te - L(\tilde{h})) \end{aligned} \quad (51)$$

and a sufficient condition for $\dot{V} < 0$ can be expressed as

$$\tilde{h}^T L(\tilde{h}) > |\tilde{h}^T Te| \quad (52)$$

For small values of the control error \tilde{h} following can be written

$$L(\tilde{h}) \approx K_{xy} \tilde{h}, \dots K_{xy} = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \quad (53)$$

Now the sufficient condition for $\dot{V} < 0$ can be written as

$$\begin{aligned} \tilde{h}^T K_{xy} \tilde{h} &> |\tilde{h}^T Te|, \\ \min(k_x, k_y) \|\tilde{h}\|^2 &> \|\tilde{h}\| \|Te\|, \\ \|\tilde{h}\| &> \frac{\|Te\|}{\min(k_x, k_y)} \end{aligned} \quad (54)$$

It is shown that e tend to zero as $t \rightarrow \infty$, which implies that condition in (43) is verified for any value of \tilde{h} . Thus, $\tilde{h}(t) \rightarrow 0$ as $t \rightarrow \infty$.

3.5 Switching Criterion

A cost function is considered in the form

$$J_j(t) = e_j(t)^T G_1 e_j(t) + \int_0^t e^{-\lambda(t-\tau)} e_j(\tau)^T G_2 e_j(\tau) d\tau \quad (55)$$

where J_j is the cost function of the j^{th} model, e_j is

the identification error associated with the j^{th} model, $G_1, G_2 \in R^{n \times n}$ are positive (semi)-definite weight matrices and $\lambda \geq 0$ is a scalar forgetting factor. J_a is denoted as the cost function of the current model. If $J_a(t) > J_j(t)$ with defined switching sequence, it means that adaptive model must be switched to the j^{th} model according to the switching criterion.

4 SIMULATIONS

In the simulations, the WMR should track an eight-shaped trajectory given by

$$\begin{aligned} x_r &= x_g + R \sin(2\omega_r t), \\ y_r &= y_g + R \sin(\omega_r t) \end{aligned} \quad (56)$$

where $x_g = 2.5$, $y_g = 5.5$, $\omega_r = 0.04$ and $R = 7.5$.

Initially robot $x_0 = 2$ and $y_0 = 6.5$ and robot has zero velocities and $\varphi = -\pi/6$.

The parameters of the WMR are taken as $I_m = 0.0025 \text{ Kg.m}^2$, $I_c = 15.625 \text{ Kg.m}^2$, $r = 0.15 \text{ m}$, $b = 0.75 \text{ m}$, $a = 0.3 \text{ m}$, $d = 0.3 \text{ m}$, $L = 0.1 \text{ m}$, $m_w = 1 \text{ Kg}$, $m_c = 36 \text{ Kg}$, $I_w = 0.005 \text{ Kg.m}^2$, $K_v = 10$, $\lambda = \text{diag}(10, 10)$, $\Gamma = \text{diag}(2, 2, 2)$, $\alpha = 1$, $k_x = 10$, $k_y = 10$, $I_y = 1$, $I_x = 1$. The switching sequence has a time step of 5 ms.

The real values of the unknown parameters are $\theta = [38 \ 19.95 \ 10.8]^T$, and the initial estimates for the parameters are $\hat{\theta} = [20 \ 7 \ 3]^T$.

In order to show effectiveness of the developed solution ten identification model has been chosen as

$$\begin{aligned} \hat{\theta}_1 &= [29 \ 11 \ 5]^T, \hat{\theta}_2 = [32 \ 14 \ 7]^T, \\ \hat{\theta}_3 &= [35 \ 17 \ 9]^T, \hat{\theta}_4 = [38 \ 20 \ 11]^T, \\ \hat{\theta}_5 &= [41 \ 23 \ 13]^T, \hat{\theta}_6 = [44 \ 26 \ 15]^T, \\ \hat{\theta}_7 &= [47 \ 29 \ 17]^T, \hat{\theta}_8 = [50 \ 32 \ 19]^T, \\ \hat{\theta}_9 &= [53 \ 35 \ 21]^T, \hat{\theta}_{10} = [56 \ 38 \ 23]^T \end{aligned}$$

It can be seen from the figures that proposed control approach enhances the performance of both velocity tracking and trajectory tracking. In Fig. 3, there is a trajectory tracking results for both single model and multiple model cases. The controller provides the reference trajectory tracking with a similar performance for two cases. However, if one focus on the trajectories for the first five seconds as seen in Fig. 4, he can see the differences. Also, Fig. 5 shows the tracking errors on the x and y axis. In Fig.

6 and 8, there are linear and rotational velocity errors, respectively. In order to show the enhancement of the transient behaviour, Fig. 7 and 9 shows the linear and rotational errors for the first 5 seconds of the simulation. Similarly, Fig. 10-13 show the results for the integral of the linear and rotational velocities. Fig. 14 shows the switching between models during the simulation.

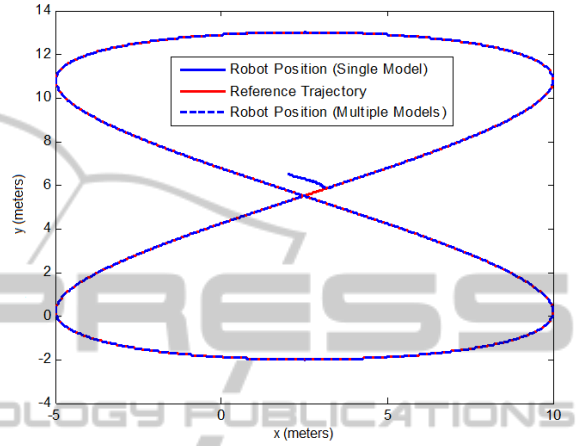


Figure 3: Robot position in single model case and multiple model case vs. Reference Trajectory.

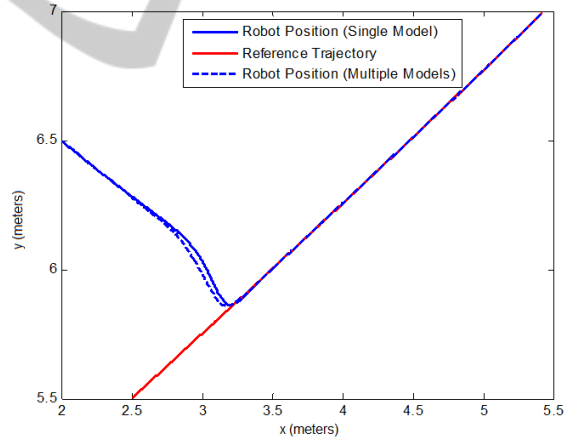


Figure 4: Robot position in single model case and multiple model case vs. reference trajectory (five seconds to see the effect).

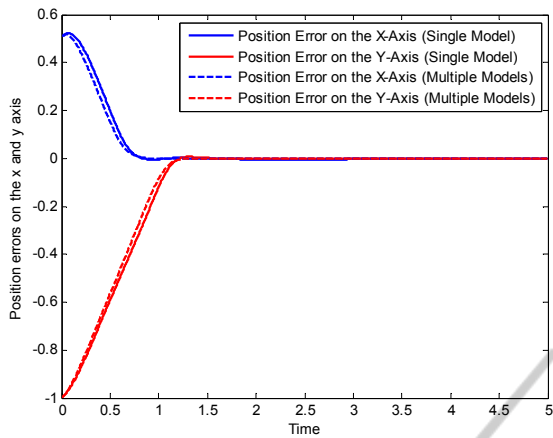


Figure 5: Position Errors on the x and y axis.

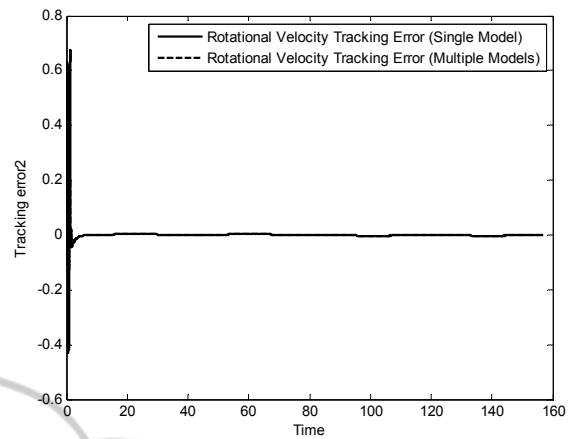


Figure 8: Rotational velocity tracking error in single model case and multiple models case.

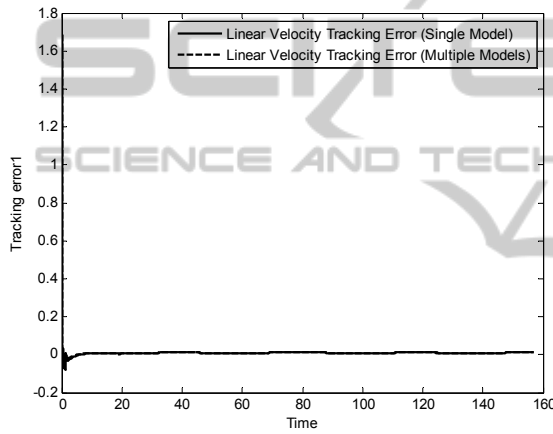


Figure 6: Linear velocity tracking error in single model case and multiple models case.

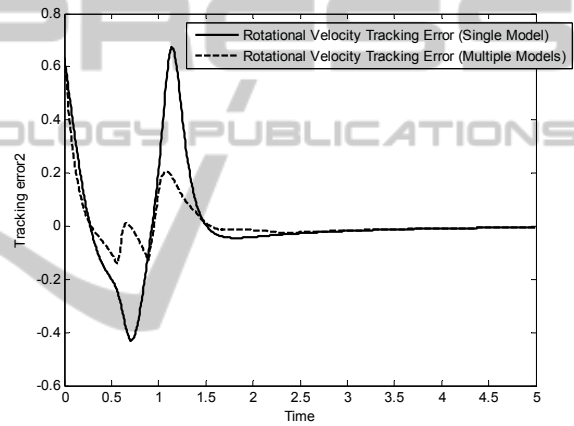


Figure 9: Rotational velocity tracking error in single model case and multiple models case (five seconds to see the effect).

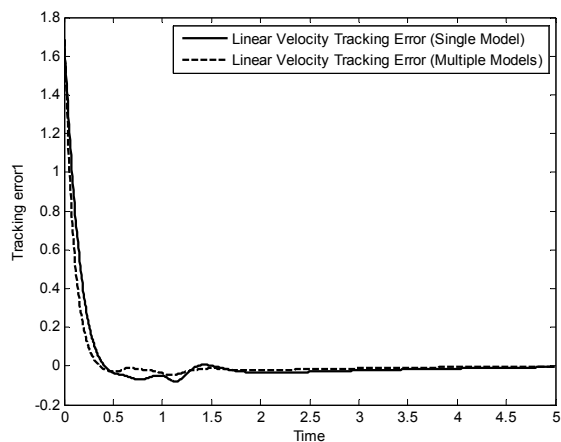


Figure 7: Linear velocity tracking error in single model case and multiple models case (five seconds to see the effect).

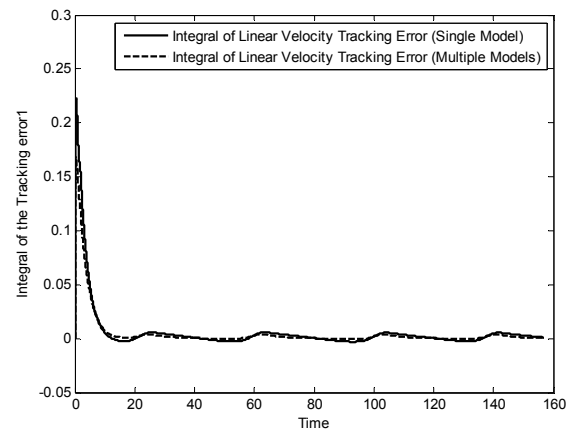


Figure 10: Integral of linear velocity tracking error in single model case and multiple models case.

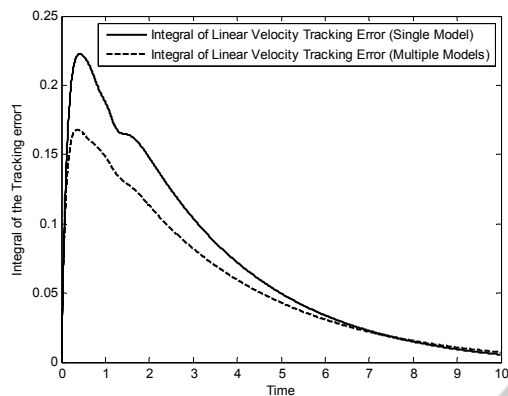


Figure 11: Integral of linear velocity tracking error in single model case and multiple models case (ten seconds to see the effect).

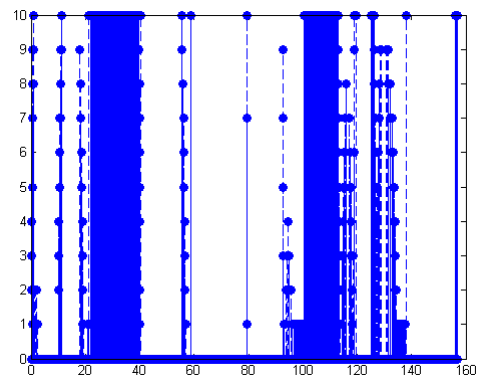


Figure 14: Switching between models.

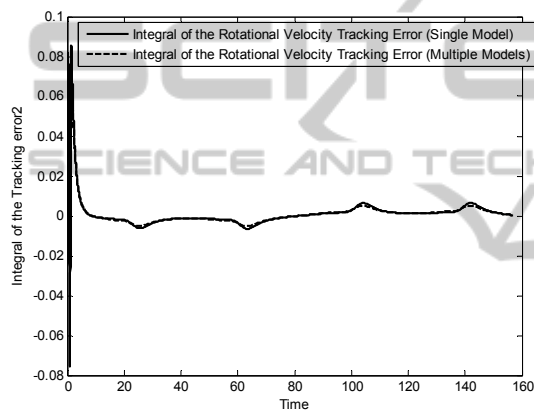


Figure 12: Integral of rotational velocity in single model case and multiple models case.

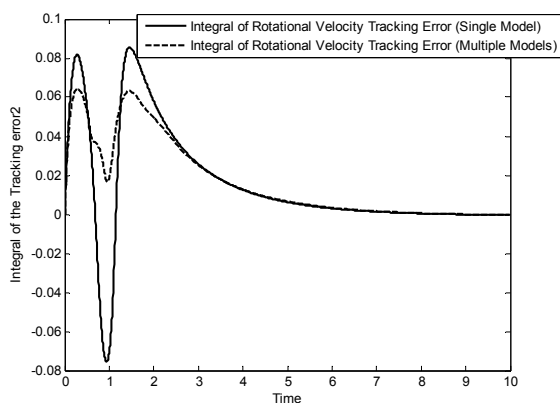


Figure 13: Integral of rotational velocity in single model case and multiple models case (ten seconds to see the effect).

5 CONCLUSIONS

An adaptive control algorithm with a multiple models approach is proposed for the trajectory tracking of a WMR. The controller uses a combined direct and indirect adaptive control approach where both prediction and tracking errors are used in identification and switches between multiple models of the WMR dynamics and the control input is applied based on the model which closely describes the WMR dynamics. This dynamic controller provides fast velocity tracking under parameter uncertainties. The proposed kinematic controller provides the velocity profile needed for the trajectory tracking of the WMR in Cartesian coordinates. The stability of the overall control system was proved. As a result, simulations show that the proposed control system is applicable to the WMR and it significantly enhances the transient behavior during the trajectory tracking.

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