

# Compensation of Unknown Input Dead Zone using Equivalent-Input-Disturbance Approach

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**Keywords:** Compensation of Nonlinearity, Dead Zone, Distortion Factor, Equivalent Input Disturbance (EID), Nonlinearity.

**Abstract:** This paper considers the problem of the compensation of an unknown dead zone in the input of a plant. A new compensation method is presented based on the estimation of an equivalent input disturbance (EID). Unlike other methods, this method does not require the exact information of a dead zone. First, we consider the dead zone as an input-dependent disturbance and employ an EID estimator to estimate it. Then, we incorporate the estimate in the control input and compensate the effect of the dead zone almost completely. Simulation results demonstrate the validity of the method.

## 1 INTRODUCTION

Dead zone appears in many mechatronic systems, for example, in a motor drive, a photoelectric sensor, etc. This nonlinearity has a direct effect on the accuracy of a controlled output and leads to the deterioration of system performance (Hung et al., 2008). The compensation of dead zone has attracted considerable attention over the last few decades.

Many studies focused on a system with an unknown dead zone in the control input. Since it is very hard to precisely acquire the parameters of the dead zone, it is difficult to completely compensate it, and it causes a fundamental problem in high precision control. To handle this nonlinearity, many methods have been proposed for the case in which a dead zone is measurable (Recker et al., 1991; Wang et al., 2003). Tao and Kokotovic proposed a method based on the construction of an adaptive dead zone inverse for a system with an unmeasurable dead zone (Tao and Kokotovic, 1994). However, their method requires that the output of a dead zone is within a known compact set. And the inverse model of a dead zone is usually difficult to calculate. To avoid the construction of an inverse model for a dead zone, Ma and Yang (2008) proposed a new

adaptive control strategy. Since an adaptive control method often causes the problem of instability, many intelligent methods have been used to solve this problem (Semile and Lewis, 2000; Boulkroune and M'saad, 2011). Sliding mode control was also used to deal with nonlinearities in the control input by making use of its fast switching speed (Tong and Li, 2003). But these methods are usually computationally expensive and also have to meet the matching condition.

In this paper, we present a new compensation method for a plant with an unknown input dead zone. Unlike other methods, this method is based on the idea of an equivalent input disturbance (EID), which was first presented by She et al. (2008) to deal with the problem of disturbance rejection in a linear servo system. The main advantages of this method are

- 1) It does not require any information of the dead-zone.
- 2) The compensation effect is satisfactorily.
- 3) A robust control system can easily be designed for the complemented plant using advanced control theory, and high-precision tracking can easily be achieved.

## 2 PROBLEM FORMULATION

We consider a single-input-single-output (SISO) nonlinear plant. The linear part of the plant is

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu_d(t), \\ y(t) = Cx(t), \end{cases} \quad (1)$$

where  $x(t) \in R^n$  is the state,  $y(t) \in R$  is the output, and  $u_d(t) \in R$  is the control input of the linear part.  $A \in R^{n \times n}$ ,  $B \in R^{n \times 1}$ , and  $C \in R^{1 \times n}$ .  $u_d$  is the output of the input dead zone. It is described as

$$u_d(t) = \begin{cases} u(t) - b_r, & \text{if } u(t) > b_r, \\ 0, & \text{if } b_l \leq u(t) \leq b_r, \\ u(t) - b_l, & \text{if } u(t) < b_l, \end{cases} \quad (2)$$

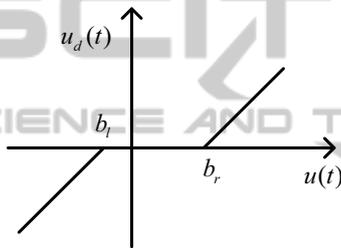


Figure 1: Dead zone.

where ( $b_l < 0$ ) and ( $b_r > 0$ ) are the breakpoints of the dead zone on the left- and right-half planes, respectively (Fig. 1). And  $u(t)$  is the control input of the plant.

We made the following assumptions.

**Assumption 1:** The linear part of the plant is a minimum-phase system.

**Assumption 2:** The linear part of the plant is controllable and observable.

**Assumption 3:** The output of the dead zone is not measurable. The parameters,  $b_l$  and  $b_r$ , are unknown.

Due to Assumption 3, we cannot construct an inverse of the dead zone. In this study, we treat the dead zone as an input-dependent disturbance

$$u_d(t) = u(t) + d(u(t)), \quad (3)$$

where

$$d(u(t)) = \begin{cases} -b_r, & \text{if } u(t) \geq b_r, \\ -u(t), & \text{if } b_l \leq u(t) \leq b_r, \\ -b_l, & \text{if } u(t) \leq b_l. \end{cases} \quad (4)$$

(3) decomposes  $u_d(t)$  into a linear part,  $u(t)$ , and a nonlinear part,  $d(u(t))$ . And  $d(u(t))$  is an artificial disturbance introduced in this study. Submitting (3) into the state equation of (1) yields

$$\dot{x}(t) = Ax(t) + B[u(t) + d(u(t))] \quad (5)$$

To suppress the influence of the dead zone on the output, we devise a mechanism to automatically estimate and compensate  $d(u(t))$  by employing an EID estimator (She *et al.*, 2008) in the next section.

## 3 DESIGN OF EID-BASED COMPENSATOR

The configuration of the EID-based compensator for dead zone is shown in Fig. 2. It has three parts: the plant, a state observer, and an EID estimator.

### 3.1 Estimation of EID

To obtain an EID with high precision, we first construct the following observer to reproduce the state of the plant:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu_f(t) + L[y(t) - C\hat{x}(t)], \quad (6)$$

where  $L$  is the observer gain.

Taking the error state to be

$$\Delta x = \hat{x}(t) - x(t) \quad (7)$$

and substituting it into (5) yield

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + Bd(u(t)) + \Delta\dot{x}(t) - A\Delta x(t) \quad (8)$$

We find a control input  $\Delta d_e(t)$  that satisfies

$$\Delta\dot{x}(t) - A\Delta x(t) = B\Delta d_e(t) \quad (9)$$

Combining (8) and (9) and denoting

$$\hat{d}(t) = d(u(t)) + \Delta d_e(t) \quad (10)$$

Give

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B[u(t) + \hat{d}(t)] \quad (11)$$

From (6) and (11), we have

$$\hat{d}(t) = B^+ [LC(x(t) - \hat{x}(t))] + u_f(t) - u(t), \quad (12)$$

where  $B^+ := \frac{B^T}{B^T B}$ .  $\hat{d}(t)$  is an estimation of the actual EID. From the above equations, we know that, if the state of the observer is exactly equal to the state of the actual plant, the estimated EID asymptotically converges to the actual EID.

A low-pass filter

$$F(s) = \frac{1}{Ts+1} \quad (13)$$

is used to select the angular-frequency band for the disturbance estimate,  $\hat{d}(t)$ . In (13),  $T$  is the time constant of the filter. The filtered disturbance estimate is  $\tilde{d}(t)$ . The relationship between  $\tilde{d}(t)$  and  $\hat{d}(t)$  is

$$\tilde{D}(s) = F(s)\hat{D}(s), \quad (14)$$

where  $\tilde{D}(s)$  and  $\hat{D}(s)$  are the Laplace Transform of  $\tilde{d}(t)$  and  $\hat{d}(t)$ , respectively.

### 3.2 Design of Filter and State Observer

(12) equals to

$$\hat{d}(t) = -B^+ LC\Delta x(t) + \tilde{d}(t) \quad (15)$$

If there is no nonlinearity, the plant (5) is

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (16)$$

And

$$\Delta \dot{x}(t) = (A-LC)\Delta x(t) + B\tilde{d}(t) \quad (17)$$

The transfer function from  $\tilde{d}(t)$  to  $\hat{d}(t)$  is

$$G(s) = B^+ (sI - A) [sI - (A-LC)]^{-1} B \quad (18)$$

From the small-gain theorem, we know that

$$\|GF\|_{\infty} \leq 1 \quad (19)$$

guarantees the stability of the estimation, where  $\|GF\|_{\infty} := \sup_{0 \leq \omega \leq \infty} \sigma_{\max} |G(j\omega)F(j\omega)|$  and  $\sigma_{\max}(j\omega)$  is the maximum singular value of  $G$ .

From Assumption 1, we know that the dual system  $(A^T, B^T, C^T)$  is also a minimum-phase system. The concept of perfect regulation shows that a very large weighting parameter,  $\rho$ , in a quadratic performance index ensures

$$\lim_{\rho \rightarrow \infty} B^T [sI - (A^T - C^T L_{\rho})]^{-1} = 0 \quad (20)$$

Since the left side of this equation is part of the transfer function,  $G(s)$ , (18) and (20) mean that a large enough  $\rho$  makes the condition (19) true.

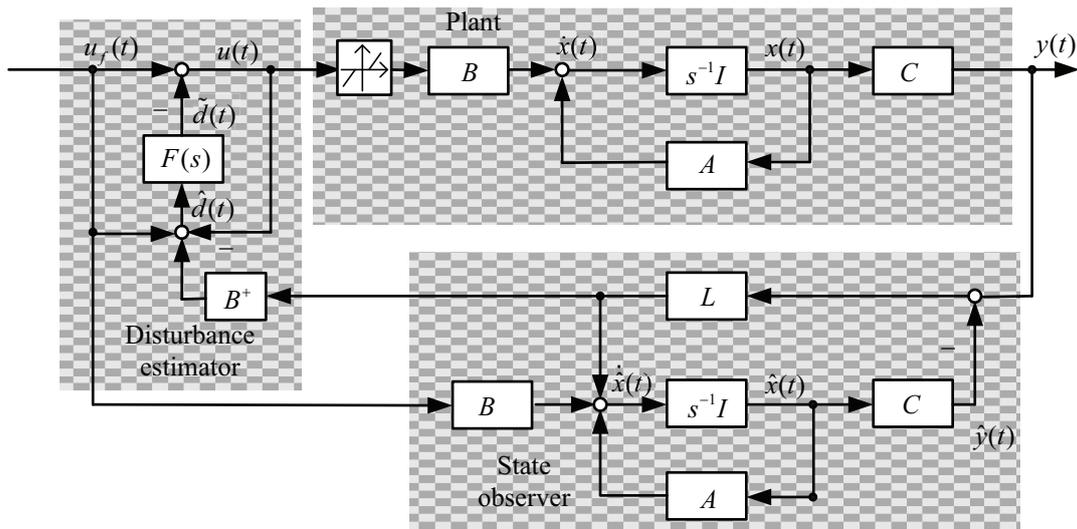


Figure 2: Configuration of EID-based compensator for dead zone.

### 4 NUMERICAL EXAMPLE

Consider the nonlinear plant, (1) and (2), with:

$$A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [0 \ 1], \quad (21)$$

$$b_l = -1, \ b_r = 1. \quad (22)$$

We chose  $T = 0.05$  s and the input as

$$u_f = 5 \sin(2\pi t / T_s), \ T_s = 2 \text{ s}. \quad (23)$$

Using MATLAB function, `lqr` to solve

$$\begin{cases} PA^T + A^T P - PC^T CP + \rho Q = 0, \\ Q = \text{diag}\{10^9, 1\}, \ \rho = 1 \end{cases} \quad (24)$$

Yielded

$$L_\rho = PC^T = [31371 \ 250] \quad (25)$$

The simulation results are shown in Figs. 3-5. It is clear from Fig. 3 that the effect of the dead zone on the output of the plant cannot be ignored. Fig. 4 shows the EID-based compensation results. Comparing Fig. 4 with Fig. 3, we can see that the EID-based compensator reduced the effect of the dead zone on the output greatly and the output of the compensated nonlinear plant is almost the same as that of the plant without the input dead zone. When the dead zone was not compensated, the largest error between the outputs of the plant without the dead zone and with the dead zone was 0.4. On the other hand, it reduced to 0.027 when the EID-based compensator was used.

To assess the effectiveness of the EID-based compensation method, a comparison was made between two outputs for a sine wave input. One is the output of the plant without the dead zone, and the other is the output of the EID-based compensated plant with the dead zone. Three characteristic factors (Kim & Russell, 1995) were calculated (Tab. 1). Clearly, the distorted output caused by the dead zone was compensated almost completely.

### 5 CONCLUSIONS

A dead zone often exists in mechatronic systems. It

deteriorates the control performance. In this paper, we presented an EID-based compensation method for a plant with an unknown input dead zone. We regarded the dead zone nonlinearity as an input-dependent disturbance, estimated an EID, and added it to the input channel. This method does not need any information of the dead zone. We do not need to calculate an inverse dynamics of the dead

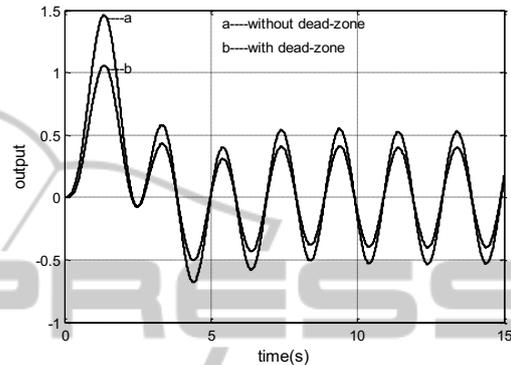


Figure 3: Outputs of plant with and without dead zone.

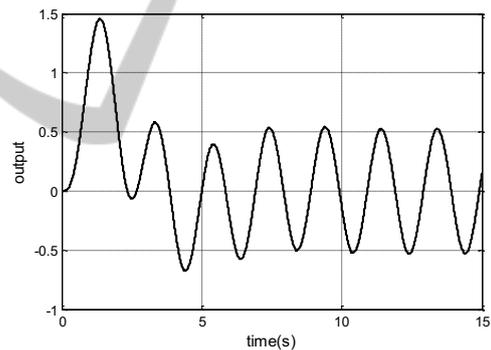


Figure 4: output of plant with dead zone using EID-based compensator.

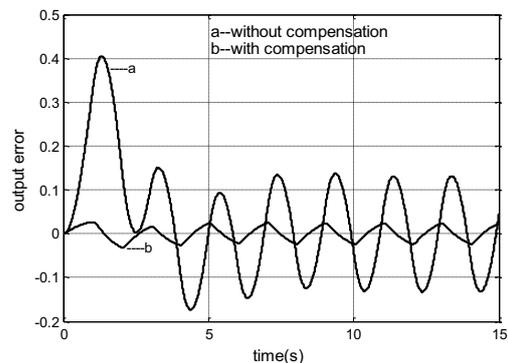


Figure 5: Output errors of plant with and without using EID-based compensator.

Table 1: Comparison of characteristic factors of outputs.

	Form factor	Crest factor	Distortion factor
Without dead zone	1.114	1.414	0
With compensation	1.113	1.415	0.021

zone as well. Simulation results show that this method provides good compensation performance.

## ACKNOWLEDGEMENTS

The work of L. Ouyang and M. Wu was supported by the National Science Foundation of China under grants 60974045 and 60674016. And the work of J. She and H. Hashimoto was supported by Casio Science Promotion Foundation.

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