

# Neuro-fuzzy Sliding Mode Control for a Two Link Flexible Robot

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**Abstract:** In most robotic applications, trajectory tracking control and vibration suppression in flexible link manipulator is a recurring problem, due to the unknown nonlinearities and strong coupling often caused by the presence of flexibility in the links. In order to solve this problem, a new sliding mode controller using neural networks and fuzzy logic is presented in this paper. The stability of the proposed controller is proved with the Lyapunov function method. The neural network is used to compensate the highly nonlinear system uncertainties. The fuzzy logic is used to eliminate the chattering effect caused by the robust conventional sliding mode control. The effectiveness of this control system will be compared to the performance obtained with a second order sliding mode control which is the super twisting algorithm. Comparative simulations show the superiority of the proposed controller regarding the second order sliding mode controller and confirm its robustness with bounded disturbance and its ability to suppress the flexible link manipulator vibrations.

## 1 INTRODUCTION

In the last few years, the dynamic proprieties and control techniques for flexible link manipulators are being intensively studied (Sanz and Etxebarria, 2006). They exhibit many advantages with respect to the rigid manipulators, such as payload-to-arm ratio, operation speed and energy consumption. But the use of structurally flexible robotic manipulators requires the inclusion of deformation effects due to the flexibility in the dynamic equations which complicates the analysis and the control design.

In a robot system, there are many uncertainties, such as dynamic parameters, dynamic effects and unmodeled dynamics. These uncertainties should be taken into consideration in the control algorithm. So, the controller of flexible manipulator must achieve the same motion objectives as a rigid manipulator, and it must also stabilize the vibrations. A large number of reports have been presented, employing the hybrid control scheme (Ho Lee and Won Lee, 2002), the radial basis function network (Tang and Sun, 2005), the impedance control (Hui Jiang, 2005), inversion techniques (De Luca et al., 1989), adaptive control (Yang et al., 1997) (Lin and Yeh, 1996), and VSC (variable structure control) (Fung

and Lee, 1999) (Singh and Nathan, 1991). Sliding modes are the primary form of VSSs. The sliding mode control is a well known approach to the control of uncertain systems. It has received much attention due to its ability to reject disturbances while tracking a desired trajectory. However, standard sliding modes are characterized by a high-frequency switching of control, which causes problems in practical applications (so-called chattering effect). To avoid this drawback, higher order sliding mode (HOSM) can be used. The HOSM concept emerged in 1980s with the motivation of tackling the chattering phenomenon. HOSM controllers have the capability of stabilizing around zero in finite time not only the sliding variable, but also a number of its time derivatives. A lot of HOSM approaches have been studied in (Kunusch et al., 2009) (Khan et al., 2003) (Boiko and Fridman, 2005) (Levant and Alelishvili, 2004) (Levant, 2000) (Jimenez, 2004). In order to reduce the chattering, other methods can be applied such as boundary layer approach (Yeung and Chen, 1988), fuzzy sliding mode control (Wang, 2009) and neural network sliding mode control (Peng et al., 2006).

This paper presents the design of neuro fuzzy sliding mode controller for flexible robotic trajectory

and vibration suppression. The controller integrates the merits sliding mode, fuzzy logic and neural networks, in order to compensate for unmodeled dynamics, eliminate the chattering and save the robustness. This controller is compared with the super twisting algorithm, which is a second order sliding mode control. It has been developed and analysed for systems with relative degree one. Stability of the control system is analyzed. Simulations are carried out to demonstrate the effectiveness and higher performance of the proposed control method which is characterized by robustness to parameter variations and insensitivity to disturbances.

## 2 DYNAMIC MODELING

We consider a flexible robot manipulator consisting of  $n$  flexible links driven by  $n$  rigid joints. In order to obtain the dynamic model, it becomes necessary to introduce of a convenient kinematic description of the manipulator, including the deformation of the links.

Let  $\theta \in R^n$  be the joint variable vector,  $\delta \in R^n$  be the link flexible displacement vector. We assume that the robot has no redundant degree of freedom on its joints, and define vector  $P \in R^n$  to describe the end-effector position and orientation in the  $n$ -dimension workspace. The kinematical relationship among  $P$ ,  $\theta$  and  $\delta$  is nonlinear, and can be given as:  $P = f(\theta, \delta)$ .

The relationship among the velocities  $\dot{\theta}$ ,  $\dot{\delta}$  and  $\dot{P}$  is linear and can be analyzed as follows:

$$\dot{P} = J_\theta \dot{\theta} + J_\delta \dot{\delta} \tag{1}$$

Where  $J_\theta \in R^{n \times n}$  and  $J_\delta \in R^{n \times k}$  are the Jacobian matrices of  $f$  with respect to  $\theta$  and  $\delta$  that are defined as  $J_\theta = \frac{\partial f}{\partial \theta}$  and  $J_\delta = \frac{\partial f}{\partial \delta}$

By taking derivation of  $\dot{P}$  with respect to time, we have the end-effectors' acceleration

$$\ddot{P} = J_\theta \ddot{\theta} + J_\delta \ddot{\delta} + \dot{J}_\theta \dot{\theta} + \dot{J}_\delta \dot{\delta} \tag{2}$$

We derived the dynamic model of the flexible using Lagrange approach and FEM (Finite Element Method).

The physical model of two-link flexible manipulator is shown in Figure 1

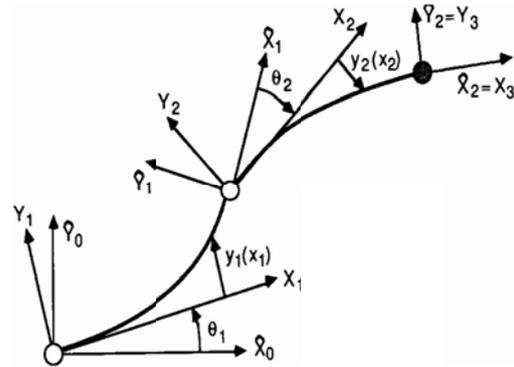


Figure 1: A two-link flexible manipulator.

The dynamic equation of a two-link flexible robotic manipulator with rigid joints (Benosman and Le Vey, 2004) (Talebi et al., 2002) (Subudhi and Morris, 2002) can be given as:

$$M_{6 \times 6}(\theta, \delta) \begin{Bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{Bmatrix} + C_{6 \times 1}(\theta, \dot{\theta}, \delta, \dot{\delta}) + \begin{Bmatrix} 0 \\ D_{4 \times 4} \dot{\delta} + K_{4 \times 4} \delta \end{Bmatrix} + F_d(\theta, \delta) = \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} = u \tag{3}$$

where  $M$  is the inertia matrix, which is symmetric and positive definite,  $C$  is effect of coriolis and centrifugal forces,  $D$  is the diagonal and positive semi definite link damping matrix,  $F_d$  is friction terms and external disturbances and  $K$  is the diagonal stiffness matrix that only affects to the flexible modes. Although,  $F_d(\theta, \delta)$  cannot be modeled very accurately. The generalized coordinate vector consists of link positions ( $\theta_1, \theta_2$ ) and modal displacements ( $\delta_{11}, \delta_{12}, \delta_{21}, \delta_{22}$ ).

$\tau = \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix}$  are the torques applied by rotor-1 and rotor-2 respectively.

We take

$$\begin{Bmatrix} 0 \\ D_{4 \times 4} \dot{\delta} + K_{4 \times 4} \delta \end{Bmatrix} = H \tag{4}$$

The parameter matrices are introduced in reference (De Luca and Siciliano, 1991).

The control of a manipulator formed by flexible elements bears the study of the robot's structural flexibilities. The control objective is to move the manipulator within a specific trajectory but attenuating the vibrations due to the elasticity of some of its components.

## 3 SLIDING MODE CONTROL

Tracking error is defined as:

$$e = \theta - \theta_d \quad (5)$$

Where  $\theta_d$  is desired joint trajectory vector.  
The sliding surface variable is defined by:

$$s = \dot{e} + \lambda e \quad (6)$$

Where  $\lambda = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_n]$  in which  $\lambda_i$  is a positive constant for  $i=1, 2, \dots, n$ .

The control goal is to guarantee the state trajectories convergence to sliding surface  $s=0$ , and keep them on the sliding surface, that is  $\dot{s} = 0$ .

#### 4 SUPER TWISTING ALGORITHM

The super twisting algorithm is one of the popular algorithms among the second order sliding mode algorithms (Boiko et al., 2008), (Kunusch et al., 2009). The super twisting algorithm defines the control law  $u(t)$  as a combination of two terms (Khan et al., 2003). The first is defined in terms of discontinuous time derivative  $u_1(t)$ , while the second is a continuous function of the sliding variable  $u_2(t)$ . The super twisting algorithm is defined as follows:

$$u(t) = u_1(t) + u_2(t) \quad (7)$$

where

$$\dot{u}_1 = \{-u, |u| > 1\} \quad (8)$$

$$\dot{u}_1 = -\omega \text{sign}(s), |u| \leq 1 \quad (9)$$

$$u_2 = -\lambda |s_0|^\rho \text{sign}(s), |s| > s_0 \quad (10)$$

$$u_2 = -\lambda |s|^\rho \text{sign}(s), |s| \leq s_0 \quad (11)$$

And sufficient conditions for finite time convergence are:

$$\omega > \frac{\Phi}{\Gamma_m} > 0 \quad (12)$$

$$\lambda^2 = \frac{4\Phi\Gamma_M(\omega + \Phi)}{\Gamma_m^3(\omega - \Phi)} \quad (13)$$

where  $\omega$ ,  $\lambda$  and  $\rho$  are variable controller parameters,  $\Phi$  is positive norm bound on the smooth uncertain  $\Phi$ ,  $\Gamma_M$  and  $\Gamma_m$  are lower and upper positive bounds on the smooth uncertain function,  $\gamma$ . The choice of  $\rho = 0.5$  assures that sliding order 2 is achieved (Levant, 1993).

The super twisting algorithm in equation (7) can be simplified as follows:

$$u(t) = -\lambda |s|^\rho \text{sign}(s) + u_1 \quad (14)$$

$$\dot{u}_1 = -\omega \text{sign}(s) \quad (15)$$

This control algorithm does not need any information on the time derivatives of the sliding variable nor any explicit knowledge of other system parameters.

#### 5 DESIGN OF NEURO FUZZY SLIDING MODE CONTROL

We define a Lyapunov function:

$$V = \frac{1}{2} s^T M s \quad (16)$$

$$\dot{V} = s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s \quad (17)$$

$$\text{Since } s^T [\dot{M} - 2C] s = 0 \quad (18)$$

Then

$$\dot{V} = s^T (M \dot{s} + C s)$$

$$= s^T [(u - M(\ddot{q}_d - \lambda \dot{e}) - C(\dot{q}_d - \lambda e) - H - F_d)]$$

$$= s^T (u + (M\lambda \dot{e} + C\lambda e) - F_d - M\ddot{q}_d - C\dot{q}_d - H) \quad (19)$$

$$u \text{ is chosen as: } u = -\mu - k_c \text{sign}(s) \quad (20)$$

where

$$\mu = M\lambda \dot{e} + C\lambda e \quad (21)$$

$$B = M\ddot{q}_d + C\dot{q}_d + H \quad (22)$$

Then

$$\dot{V} = s^T (-F_d - B - k_c \text{sign}(s)) \quad (23)$$

The sliding condition  $\dot{V} < 0$  can be satisfied if  $k_c$  is selected such that:

$$k_c > |F_d + B| \quad (24)$$

In order to guarantee that the system tracking error is quickly convergent  $k_c$  should be chosen sufficiently large.

When  $s > 0$ ,

$$s^T (-F_d - B - k_c \text{sign}(s)) < 0 \quad (25)$$

When  $s < 0$ ,

$$s^T (-F_d - B - k_c \text{sign}(s)) < 0 \quad (25)$$

Thus

$$\dot{V} = s^T (-F_d - B - k_c \text{sign}(s)) < 0 \quad (27)$$

This guarantees that hitting condition is satisfied.

In this paper, a neuro-fuzzy is used to compensate the uncertainty  $F_d$  in the robot system reel-time. A five layer neuro-fuzzy structure is applied. It can be described in detail as below:

Where  $x = (x_1, x_2)$  is the input of the neuro-fuzzy.

$y = (y_1, y_2)$  is the output of the neuro-fuzzy.

**Layer 1:**

The nodes in this layer represent membership functions.

$$O_j^1 = \mu_{Ai}(x), \text{ for } i=1 \dots 3, j=1 \dots 3. \quad (28)$$

$$O_j^1 = \mu_{Bi-3}(x), \text{ for } i=4 \dots 6, j=1 \dots 3. \quad (29)$$

Where:  $\mu_A$  and  $\mu_B$  are triangular fuzzy sets.

**Layer 2:**

$$O_j^2 = \mu_{Ai}(x_1)\mu_{Bi}(x_2) = W_j, j = 1 \dots 9. \quad (30)$$

**Layer 3:**

$$O_j^3 = \frac{W_j}{\sum_1^9 W_j} = V_j \quad (31)$$

**Layer 4:**

$$O_j^4 = \frac{1 - e^{-O_j^3}}{1 + e^{-O_j^3}} = Z_j \quad (32)$$

**Layer 5:**

$$O_k^5 = \sum_1^9 T_{kj} O_j^4 \quad (33)$$

$k=1 \dots 2$

$T_{kj}$  is the connection weight

$$T = \begin{bmatrix} T_{11} & T_{12} & \dots & T_{19} \\ T_{21} & T_{22} & \dots & T_{29} \end{bmatrix} \quad (34)$$

The output of the five layer neuro fuzzy can be rewritten as follows:

$$Y = TZ \quad (35)$$

The system uncertainty  $F_d$  can be described as follows:

$$F_d = TZ + \varepsilon \quad (36)$$

$\varepsilon$  is the approximation error.

If the neuro fuzzy algorithm satisfies:

$$\dot{T} = -\gamma s Z^T \quad (37)$$

Where  $\gamma > 0$ .

The output of the controller is designed as:

$$u = -s - \mu + (1 + \gamma)TZ - B - k_c \text{sign}(s) \quad (38)$$

But  $k_c$  can cause chattering due to the sign function. In order to eliminate the chattering, we replace the control  $k_c \text{sign}(s)$  by a fuzzy gain  $k_{cfuzzy}$ . Then,

$$\dot{V} = s^T(-F_d - B - k_{cfuzzy}) \quad (39)$$

In order to make  $\dot{V} < 0$  and guarantee the sliding mode condition, the fuzzy rules can be decided as follows:

IF  $s$  is NB THEN  $k_{cfuzzy}$  is NB

IF  $s$  is N THEN  $k_{cfuzzy}$  is N

IF  $s$  is Z THEN  $k_{cfuzzy}$  is Z

IF  $s$  is P THEN  $k_{cfuzzy}$  is P

IF  $s$  is PB THEN  $k_{cfuzzy}$  is PB

Then

$$u = -s - \mu + (1 + \gamma)TZ - B + k_{cfuzzy} \quad (40)$$

## 6 SIMULATION RESULTS

In order to demonstrate the superior performance of the two methods, a simulation example of a two-link flexible robotic manipulator is also considered. The function of the desired trajectories can be expressed as:

$$\theta(t) = \theta_0 + \frac{(\theta_F - \theta_0)}{2\pi} \left( \frac{2\pi t}{t_{f1}} - \sin\left(\frac{2\pi t}{t_{f1}}\right) \right) \quad (41)$$

Where  $\theta(t)$  is the desired tracking curve.  $\theta_0$  is the initial value of  $\theta(t)$ .

We assume the disturbance as:

$$d(t) = w(t) \sin(2\pi t) \quad (42)$$

Where  $w(t)$  is a Gaussian distributed random signal with mean zero and standard deviation  $\sigma$ .

The figures compare the results obtained with the super twisting algorithm and neuro fuzzy sliding mode control for tip position control when the flexible manipulator was commanded to move from an initial position of 0 rad to a target tip position of 0.5 rad. From the tip deflection trajectories shown in figures (2) and (3), it can be seen that deflection is less with the neuro fuzzy sliding mode control than Super Twisting algorithm. The first and second mode of vibration has smaller amplitude with the neuro fuzzy sliding mode compared to the super twisting. Even more important, it should be noted that the oscillations of elastic modes are attenuated quickly with the neuro fuzzy sliding mode control.

Control profiles of the controllers are shown in

figure (4). Initially, the control torque rises to a maximum of 0.6 and 0.8 respectively, and in all cases, the control torque eventually becomes zero when the desired tip displacement is achieved and the vibration is completely damped out.

Figure (5) shows the position error for the two methods. The tip position trajectory with the method of Super Twisting algorithm has a low rise time but overshoots more than the method of neuro fuzzy sliding mode control.

Figures (6) and (7) show the velocity error for the two methods. It can be seen that the tracking of the desired velocity is better with the neuro fuzzy sliding mode control.

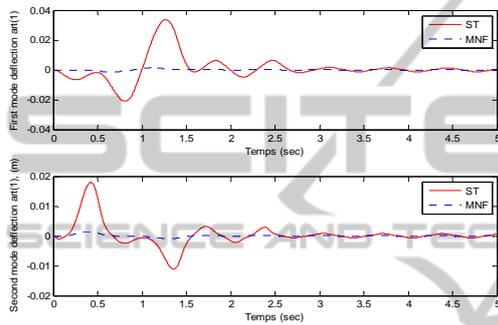


Figure 2: First mode and second mode deflection trajectories (link 1).

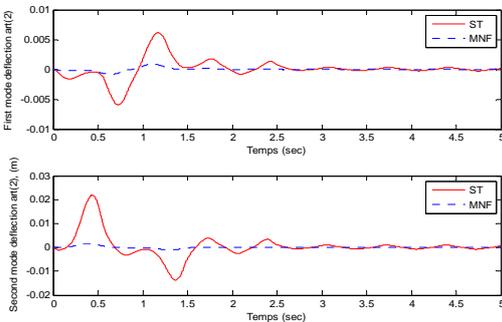


Figure 3: First mode and second mode deflection trajectories (link 2).

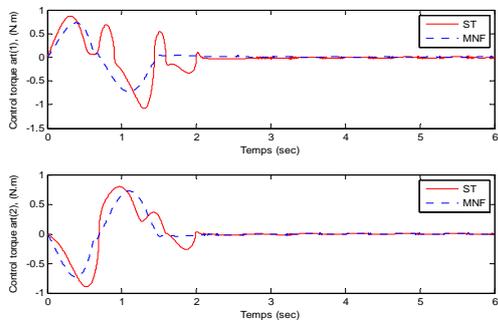


Figure 4: Control torque.

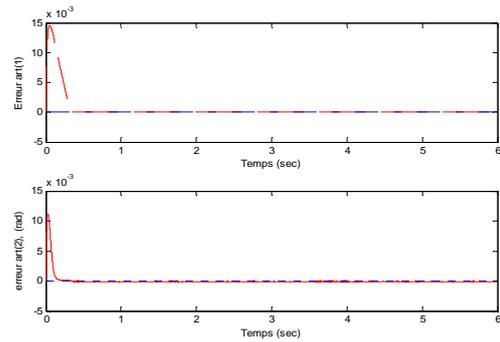


Figure 5: Position error  $\sigma = 0$ .

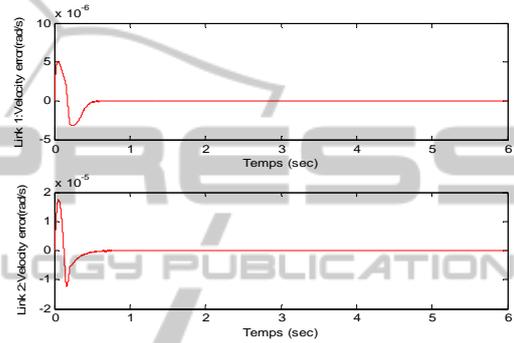


Figure 6: Velocity error with Super Twisting algorithm.

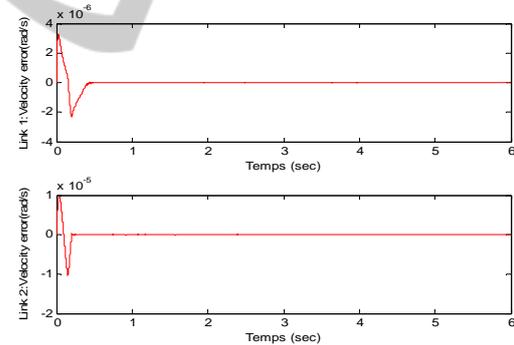


Figure 7: Velocity error with neuro fuzzy sliding mode control.

Figures (8) to (11) show the position error for the two controllers with the variation of the perturbation. The neuro fuzzy sliding mode is more robust than Super Twisting algorithm. It can be seen that the tip position exhibits better tracking of the desired trajectory with the neuro fuzzy sliding mode control. For  $\sigma = 0.1$  to 100, the error position is acceptable with the two methods. But since  $\sigma = 120$ , the desired trajectory with Super Twisting algorithm is completely divergent. For  $\sigma = 180$ , the error position with the neuro fuzzy sliding mode control start to be high.

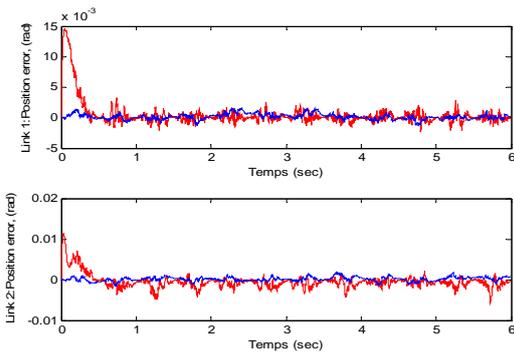


Figure 8: Position error  $\sigma = 6$ .

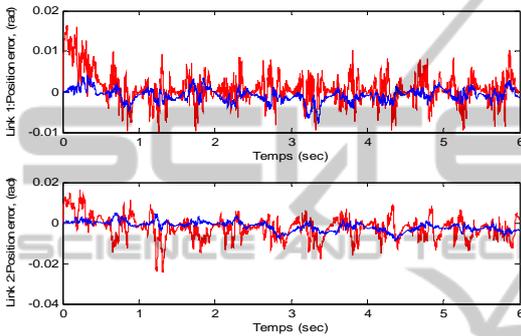


Figure 9: Position error  $\sigma = 20$ .

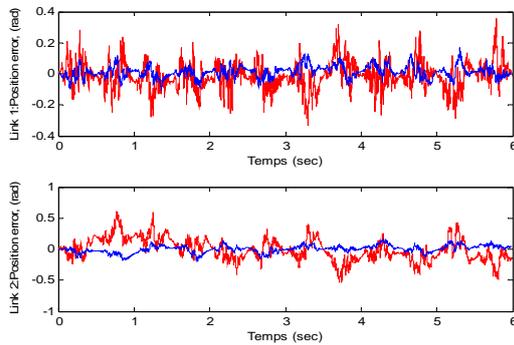


Figure 10: Position error  $\sigma = 120$ .

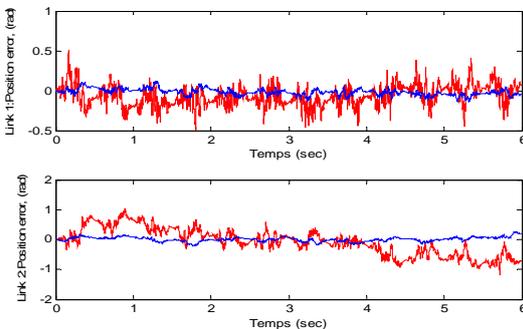


Figure 11: Position error  $\sigma = 180$ .

Table 1: Comparison between the two controllers.

Control	$\sigma$	Rise time	Precision	Robustness
Super Twisting algorithm	6	0.1	0.015	Good
	20	0.5	0.02	Good
	120	Bad	0.6	Bad
	180	Bad	Bad	Very bad
Neuro Fuzzy Sliding Mode Control	6	0.1	$10^{-7}$	Very good
	20	0.2	$10^{-5}$	Very good
	120	0.5	0.1	Good
	180	0.6	0.2	Good

## 7 CONCLUSIONS

Due to nonlinearities and uncertainties, the dynamic characteristics of flexible-link manipulator are very difficult to obtain precisely. In order to achieve high precision position control and suppress the vibrations, a combined control strategy based on the concept of sliding mode control, neural network and fuzzy logic is proposed in this paper. Neural network is employed to mimic an equivalent control law in the sliding mode control and approximate the uncertainties and disturbances; fuzzy logic is developed to eliminate the chattering phenomenon. This controller is compared with the super twisting algorithm. The simulation results show that the two methods can eliminate the phenomenon chattering greatly, and confirm that the proposed controller achieves efficient positioning and vibration suppression performances. The neuro fuzzy sliding mode controller is more robust than Super Twisting algorithm.

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