

# Mechanical Modeling of an Actuated Platform for Precision Pointing Control *Via Finite-element Analysis and Normal Mode Analysis*

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**Abstract:** A large, segmented space telescope requires high precision and accuracy in its mirror shape to obtain clear images. The Structures, Propulsion, and Control Engineering (SPACE) telescope testbed at the NASA sponsored University Research Center of excellence must maintain a pointing control accuracy of 2 arc seconds. A Peripheral Pointing Architecture (PPA) has been designed to demonstrate the Testbed's pointing accuracy. A Finite Element Analysis (FEA) model of the PPA is developed. Normal mode analysis is performed to establish the PPA's natural frequencies, mode shapes, and the mass and stiffness matrices. Utilizing the H-infinity controllers developed to achieve figure maintenance, the pointing control of the Testbed structure is achieved.

## 1 INTRODUCTION

Due to an ever increasing need to "see" further into space, a new generation of space telescopes is needed. Younger objects, receding from us at an ever-faster rate, are red-shifted into the near infrared where Hubble loses sensitivity (Stockman, 1997).

To meet this requirement the National Aeronautics and Space Administration (NASA) will replace the Hubble Space Telescope with the James Webb Space Telescope (JWST), previously known as the Next Generation Space Telescope (NGST). This telescope consists of a very large light-gathering primary mirror capable of detecting faint signals from the first billion years, the period when galaxies have been formed. The JWST will be capable of detecting radiation whose wavelength lies in the range of 0.6 to 20  $\mu$ m. Furthermore, the JWST will be able to see objects 400 times fainter than those currently studied with large ground-based infrared telescopes such as the Keck Observatory.

Due to the size and weight limitations associated with current launch vehicles, the Next Generation Space Telescopes will employ segmented reflectors. Although multiple-mirror designs have many advantages, a number of major difficulties are associated with this technique. Due to atmospheric disturbances, the mirrors can be easily misaligned and figure maintenance, as well as, precision

pointing of the telescope cannot be achieved. Therefore, to accomplish figure maintenance and precision pointing of the large segmented structure, design of sophisticated controllers is necessary to study the control of such large segmented optical systems, in 1994, NASA established the SPACE Laboratory at California State University, Los Angeles (CSULA). One of the major goals of this Laboratory is to design and fabricate a testbed that resembles the complex dynamic behavior of a segmented space telescope, (Boussalis et al., 1995), (Boussalis, 1994), (Boussalis et al., 1996). The SPACE center team of students and faculty work on the development of control algorithms which will accomplish the figure maintenance and precision pointing of the control oriented SPACE testbed, (Boussalis, 2002). Due to the symmetry of the structure and the nature of the interconnections among its subsystems, decentralized control techniques are utilized.

The SPACE testbed utilizes an actuated laser platform to demonstrate accomplishment of the precision pointing and figure maintenance of the complex structure, (Boussalis, 2005). Previous work has shown the achievement of 1 micron RMS figure maintenance of the segmented reflector. To accomplish this requirement, a decentralized H-infinity controller has been developed, (Morales et al., 1999), and (Lim, 2011). To achieve the pointing

of the SPACE testbed with accuracy of 2 arc seconds, the team is investigating the use of the already established controllers (Morales et al., 1999), (Lim, 2011). To utilize the existing control law, first, the actuated laser platform is modeled using finite element techniques.

## 2 BACKGROUND

### 2.1 SPACE Testbed

The SPACE testbed shown in Fig. 1 emulates a Cassegrain telescope of 2.4-meter focal length with performance comparable to an actual space-borne system. The system's top-level requirements include figure maintenance of the primary mirror to within 1 micron RMS distortion with respect to a nominal shape of the primary mirror, and precision pointing with accuracy of 2 arc seconds, (Boussalis et al., 1996), (Boussalis, 2005), (Desai et al., 2011).

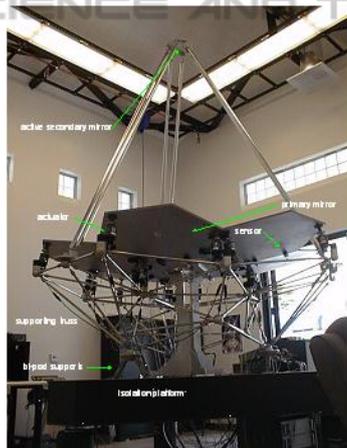


Figure 1: SPACE Testbed.

The SPACE testbed consists of a primary mirror, a secondary mirror and a lightweight flexible truss structure. The primary mirror (mounted on the support truss) consists of seven hexagonal panels, each 101 cm in diameter. The six peripheral panels are actively controlled in the three degrees-of-freedom by 18 linear electromagnetic actuators (3 actuators per active panel), and the seventh panel is used as a reference. A set of 18 edge sensors are used to provide measurements of relative displacement and angle of the panels (3 sensors per active panel). The Testbed's active secondary mirror is a six-sided pyramidal mirror used to reflect the light from the primary mirror to the central plane and is attached to the primary by a tripod. The entire

Testbed is supported by a triangular isolation platform made of aluminum honeycomb core with stainless steel top and bottom skin, (Morales, 2001).

#### 2.1.1 Figure Maintenance

Unlike a monolithic primary mirror, a segmented reflector, such as the JWST or the SPACE testbed, requires an active control system to maintain the desired optical performance. Active control of the Testbed reflector panels was achieved using a set of decentralized H-infinity controllers. Fig. 2 shows the closed loop response results, (Morales et al., 1999), using this control scheme under decentralized control. It is anticipated that system model being developed will allow the development of a new H-infinity controller for pointing precision that can be incorporated with the current figure maintenance controller as a 7<sup>th</sup> local subcontroller to demonstrate both figure maintenance and pointing precision simultaneously.

It is evident that figure maintenance is achieved using the decentralized control scheme described in (Boussalis et al., 1996), (Boussalis, 2002), (Lim, 2011).

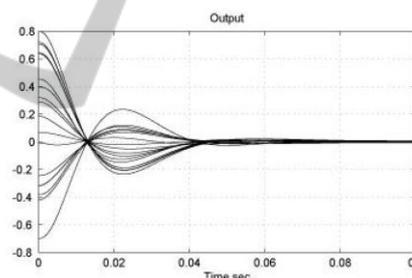


Figure 2: Closed loop response.

#### 2.1.2 Peripheral Pointing Architecture

The SPACE telescope testbed is required to perform precision pointing while maintaining the parabolic shape of the primary mirror. In order to achieve precision pointing of the Testbed with accuracy of 2 arc seconds, a Peripheral Pointing Architecture (PPA) has been designed. The PPA structure is shown in Fig. 3 (left).

The PPA uses an assembly of six lasers as shown in Fig. 3 (right) to simulate the object of study. Each laser corresponds to its own separate panel and optical detector. The laser assembly sits on a motorized tip/tilt platform and the laser source coincides with the rotation, or gimbal, point of the platform.

Since the laser source lies on the rotation point of the platform, there is no translation or displacement

of the source. When the motorized platform is tipped or tilted, the source is stationary, while only the laser beams direction are affected.

Using the distance from the platform's rotation point to each actuator, ( $a_x$  and  $a_y$  for the actuator on each respective axis), and the actuator displacement, it is calculated that the platform moves an angle  $\theta$  from its zero position on the y-axis. The platform's normal vector also moves the same angle  $\theta$  from the positive z-axis.

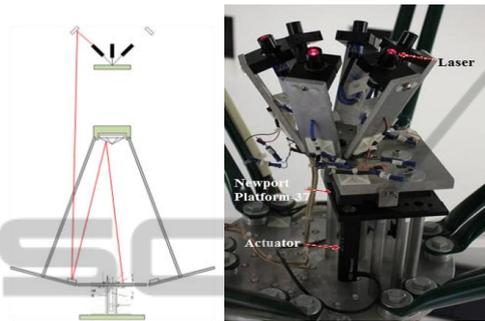


Figure 3: SPACE Testbed PPA Laser Path (left) and physical assembly (right).

### 3 FINITE ELEMENT MODEL DEVELOPMENT

#### 3.1 SolidWorks Model

A geometric model of the PPA is developed using SolidWorks. The PPA structure is composed of six lasers, six laser holders, a hexagonal plate, the top platform, the Newport 37 platform, two Newport linear actuators, and nine supporting rods.

The PPA is composed of two main parts which consist of a controllable and a fixed part. The controllable part is the top half section of the structure where the actuators push upwards in a linear fashion and where the Newport 37 platform has its gimbal point. For simplicity and ease of computation the fixed part of the PPA structure was removed. To develop the PPA model, first the model for each component is developed. The designed geometry is saved as a parasolid file and imported into Finite Element Modeling and Post Processing (FEMAP).

#### 3.2 FEMAP Model

Subsequently, a mesh analysis is performed by importing the SolidWorks model into FEMAP. The analysis performed on the model resulted with

250000 nodes and 150000 elements. The minimum number of nodes and elements achieved while keeping reasonably accurate results using solid elements is 146000 nodes and 91000 elements..

In using plate and beam elements, as opposed to solid elements, the number of nodes and elements of the model can be reduced significantly. Using plate elements and increasing the growth ratio and mesh density the model is reduced to 5795 nodes and 4993 elements.

The PPA model is constrained at three points. Two points for the linear actuators are constrained in the x-axis and y-axis for translation and in the x, y, and z axes for rotation. The gimbal point has a pinned constraint meaning that it cannot translate.

### 4 RESULTS

An initial analysis shows the frequency range for the first 100 modes ranged from 56.7 Hz to 13.017 kHz as shown in Fig. 4. A second analysis yielded similar results, but were truncated to the first 20 modes (approximately 1 kHz), to reduce the size of the eventual state-space model, since it was determined that higher frequencies would not be within the operational range.

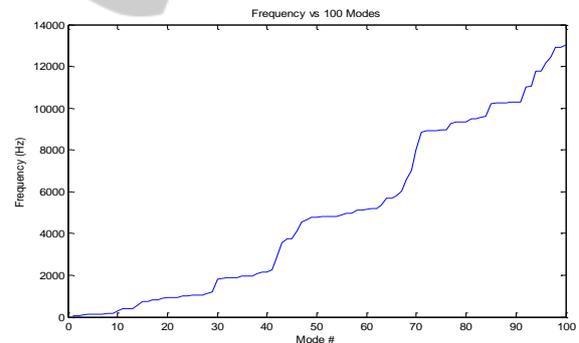


Figure 4: Frequency of first 100 modes of platform.

The first group of mode shapes corresponds to deformations of the laser holders followed by the deformation of the platform. The deformations of mode 4 at 126.2326 Hz and mode 10 at 261.7744 Hz are shown in Fig. 5. This figure shows the deformations of the laser holders in the first group of frequencies followed by the more rigid platform at higher frequencies.

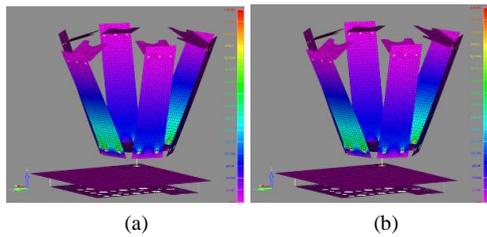


Figure 5: Mode 4, 126.23 Hz (a). Mode 10, 261.77 Hz (b).

## 5 CONTROL DESIGN

Consider the linear time-invariant system given by the following state equations,

$$\begin{aligned} \dot{x} &= Ax + \sum_{i=1}^v B_i u_i \\ y_i &= C_i x, \quad i = 1, \dots, N \end{aligned} \quad (1)$$

Where  $x \in \mathfrak{R}^n$ ,  $u_i \in \mathfrak{R}^{m_i}$  and  $y_i \in \mathfrak{R}^{p_i}$  represent the state, input and output respectively of the  $i$ th local control station.  $A$ ,  $B_i$  and  $C_i$  are real, constant matrices. The results of the modal analysis are used to determine the matrices  $A$ ,  $B_i$  and  $C_i$  that will describe the dynamics of the PPA structure.

For decentralized control, it is necessary to determine  $n$  local feedback control laws that will dynamically compensate for (1) in order to stabilize the control loop, generating the following feedback controllers:

$$\begin{aligned} \dot{z}_i &= F_i z_i + G_i y_i \\ u_i &= H_i z_i + K_i y_i + v_i, \quad i = 1, \dots, N \end{aligned} \quad (2)$$

where  $z_i \in \mathfrak{R}^{n_i}$  and  $v_i \in \mathfrak{R}^{m_i}$  are the  $i$ th subcontroller and local external input and  $F_i$ ,  $G_i$ ,  $H_i$ , and  $K_i$  are real, constant matrices. The standard two-block mixed-sensitivity H-infinity technique, (Morales et al., 1999), will be applied to accomplish a pointing accuracy of 2 arc seconds to the final reduced and validated state-space model.

## 6 CONCLUSIONS

An FEA model of an actuated laser platform used for pointing control of a segmented telescope testbed is developed. Modal analysis is performed on the FEA model which calculates the natural frequencies, mode shapes, degrees of freedom, and eigenvalues of the structure. Further research is to be undertaken to define which nodes are desirable and which nodes are unnecessary in order to be able to perform

Guyan Reduction to reduce the size of the model for practical implementation. The process described here streamlines the process of modeling a motorized platform from mechanical model (CAD, SolidWorks, etc) to a preliminary, albeit enormous, state-space model for the implementation of controllers.

## REFERENCES

- H. S. Stockman, 1997. "The Next Generation Space Telescope Visiting a Time When Galaxies Were Young".
- H. Boussalis, Z. Wei, and M. Mirmirani, 1995. "Dynamic Performance Modeling and Decentralization of A Segmented Reflector Telescope," *IASTED Conference on Modeling and Simulation*, Mexico.
- H. Boussalis, 1994. "Decentralization of Large Spaceborne Telescopes", *Proc. 1994 SPIE Symposium on Astronomical Telescopes*, Hawaii.
- H. Boussalis, M. Mirmirani, A. Chassiakos, K. Rad, 1996. "The Use of Decentralized Control in Design of a Large Segmented Space Reflector", *Control and Structures Research Laboratory, California State University, Los Angeles*, Final Report.
- H. Boussalis, 2002. "The Use of Decentralized Control in Design of A Large Segmented Space Reflector", *Structure Pointing and Control Laboratory, California State University, Los Angeles*, Final Technical Report.
- H. Boussalis, K. Rad, A. Khosafian, Y. Komandyan, 2005. "Pointing Control Testbed for Segmented Reflectors".
- A. Desai, J. Alvarenga, H. Tarsaria, K. Rad, H. Boussalis, 2011. "Ray Tracing Visualization Using LabVIEW for Precision Pointing Architecture of a Segmented Reflector Testbed," *MED 2011*, Corfu, Greece.
- M. J. Morales, 2001. "Design and modeling of the SPACE Tesbted," M.S. Thesis, *Department of Electrical Engineering, California State University, Los Angeles*, Los Angeles, CA, USA.
- M.J. Morales, M. Mirmirani, H. Boussalis, 1999. "Design, Simulation & Control of a Segmented Reflector Testbed," *MED 1999*, Haifa, Israel.
- K. Lim, 2011. "Modelling and Overlapping Decentralized Control of a Large Segmented Telescope Test-Bed," *PhD Dissertation, University of Southern California*, Los Angeles, CA, USA.
- K. J. Bathe, and E. L. Wilson, 1976. *Numerical Methods in Finite Element Analysis, 2nd Ed.*, John Wiley and Sons.
- C. H. Ih, H. C. Briggs, S. J. Wang, 1990. "3D Dynamic Modeling and Simulation of a Precision Segmented Reflector Telescope", *Proceedings of the 21st Annual Pittsburgh Conference*.
- H. Ryaciotaki-Boussalis, H. C. Briggs, and C. H. Ih, 1991. "Dynamic Performance Modeling and Stability Analysis of a Segmented Reflector Telescope," *Proceedings of the 1991 Automatic Control Conference*.

- A. C. Carrier, 1990. "Modeling and Shape control of a Segmented-Mirror Telescope," Ph.D. dissertation, *Dept. Aeronautics and Astronautics, Stanford Univ., Stanford, CA.*
- R. D. Cook, 1981. *Concepts and Applications in Finite Element Analysis, 2nd Ed., John Wiley and Sons.*
- R. Bishop, and D. C. Johnson, 1981. *The Mechanics of Vibration, Cambridge University Press.*

