A Hybrid Metaheuristic Approach to Solve the Vehicle Routing Problem with Time Windows

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Abstract: This paper addresses the Capacitated Vehicle Routing Problem with Time Windows, with constraints related to the vehicle capacity and time windows for customer service. To solve this problem two different metaheuristics are used: Tabu Search and Genetic Algorithms. Based on these techniques a hybrid algorithm is developed. The main goal is the development of a Hybrid Algorithm focused on the Vehicle Routing Problem which uses the intensification power of the Tabu Search and the diversification power of the Genetic Algorithms, in order to obtain good quality solutions without compromising the computational time. In the experiments are combined policies of diversification and intensification in Tabu Search and Genetic Algorithm to verify the efficiency and robustness of the proposed hybrid algorithm. Finally, the results are compared with the best heuristic and exact methods results found in the literature. The Hybrid Algorithm here proposed shows efficiency and robustness, with several optimal solutions achieved.

1 INTRODUCTION

The Vehicle Routing Problem (VRP) presents wide applications, especially to problems related to the distribution of goods and services, such as (Bräysy and Mester, 2005): School Bus (Newton and Thomas, 1974) Newspapers Distribution (Golden et al, 1977); Urban Public Transport Systems (Ceder and Stern, 1981), Food Distribution (Bartholdi et al, 1983), Distribution of Manufactured Products (Perl and Daskin, 1985); Delivery of Correspondent Banking (Malmborg and Simons, 1989), Routing of Helicopters (Timlin and Pulleyblank, 1990) Garbage Disposals (Kulca, 1996), Electronic Products Distribution (Barbarosoglu and Ozgur, 1999); Dynamic Routing Airline (Jiang and Barnhart, 2009).

The year 2009 marked the 50th anniversary of the publication of the first article on the Vehicle Routing Problem under the title "The truck dispatching problem" (Dantzig and Ramser, 1959, Laporte, 2009). Vehicle Routing Problem is the name given to a class of problems involving customers visited by vehicles and where a minimum cost for this task should be sought (Bodin, Golden, Assad, 1983; Tarantilis et al, 2005). The classic version of the VRP is known as the Capacitated Vehicle Routing Problem (CVRP) (Laporte, 1992; the Thangiah Petrovic, 1997, Ralph et al, 2001, Toth and Vigo, 2002; Tarantilis et al, 2005).

The main goal of this paper is to present a hybrid algorithm, which combines diversification and intensification properties from the Genetic Algorithm and Tabu Search metaheuristics when applied to the Capacitated Vehicle Routing Problem With Time Windows (CVRPTW). In order to verify the efficiency and robustness of the proposed hybrid method, the known best solutions for different instances of the problem will be used.

The paper is organized as follows. Section 2 presents a brief bibliographic review on the Vehicle Routing Problem with Time Windows. Section 3 shows the model considered in this paper, the mathematical formulation and its hybrid algorithm developed. Section 4 describes the experiments and finally Section 5 presents the conclusions.

2 VEHICLE ROUTING PROBLEM WITH TIME

The CVRPTW is an extension of CVRP where, beyond the capacity constraints, a time interval [a_i, b_i] associated to each customer i is it imposed. This time interval is called time window (Toth and Vigo,

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2002). Thus, there is a time window associated with each customer, and a vehicle that needs to visit a particular client, that cannot be reached after the end of the time window (Ropke and Cordeau, 2009).

In this version of the problem, the instant in which the vehicles leaving the depot, the travel time, t_{ij}, for each edge $(i, j) \in$ A, and an additional time of service for each customer i itself, are all known and deterministics. Moreover, typically, the source of cost and travel time coincide.

Since time windows induce an implicit orientation for each route, even if the original matrices are symmetric, the CVRPTW is usually modeled as an asymmetric problem. The CVRPTW consists in finding a set of very simple circuits R, with minimum cost, so that:

- Each circuit starts and finishes at the depot vertex;
- Each customer (vertex) is visited by exactly one circuit;
- The sum of the demands of the vertices visited by a circuit does not exceed the vehicle capacity Q_k;
- On each client I the vehicle stops for s_i moments of time, within the time window [a_i, b_i], to unload the product.

The CVRPTW is NP-Hard (Toth and Vigo, 2002; Ropke and Cordeau, 2009; Dell'Amico et al, 2007), since it generalizes the CVRP, which arises when $a_i = 0$, $B = +\infty$ for each $i \in$ V.

3 THE MODEL

This VRP model used in this work is based on the classic version with capacity constraints and time windows (CVRPTW). In this version of the problem, the capacity of all vehicles is the same and they are initially placed in a single depot. The respective demand and the time window for each client are previously known. All routes must start and finish in the depot. The total demand of a route cannot exceed the capacity of the vehicle and the vehicle cannot reach the customer after the end of the correspondent time window. The goal is to create a set of routes that meets all clients at once, minimizing the costs..

3.1 Mathematical Formulation

The CVRPTW can be formally described following the model of multiproduct network flow with time windows and capacity constraints, presented by (Cordeau et al, 2003).

$$(CVRPTW)\ \min f = \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}x_{ijk} \tag{1}$$

Subject to:

$$\sum_{k\in K}\sum_{j\in\Delta^+(i)} x_{ijk} = 1; \quad \forall i \in N \tag{2}$$

$$\sum_{j\in\Delta^+(i)} x_{0jk} = 1; \quad \forall k \in K \tag{3}$$

$$\sum_{i\in\Delta^-(j)} x_{ijk} - \sum_{i\in\Delta^+(j)} x_{ijk} = 0;\ \forall k \in K,\ j \in N \tag{4}$$

$$\sum_{i\in\Delta^-(n+1)} x_{i,n+1,k} = 1; \quad \forall k \in K \tag{5}$$

$$x_{ijk}(w_{ik} + s_i + t_{ij} - w_{jk}) \le 0;\ \forall k \in K, (i,j) \in A \tag{6}$$

$$a_i \sum_{j\in\Delta^+(i)} x_{ijk} \le s_{ik} \le b_i \sum_{j\in\Delta^+(i)} x_{ijk}$$

$$\forall k \in K, j \in N \tag{7}$$

$$E \le w_{ik} \le L;\ \forall k \in K,\ i \in \{0, n+1\} \tag{8}$$

$$\sum_{i\in N} d_i \sum_{j\in\Delta^+(i)} x_{ijk} \le Q_k;\ \forall k \in K \tag{9}$$

$$x_{ijk} \ge 0;\ \forall k \in K,\ (i,j) \in A \tag{10}$$

$$x_{ijk} \in \{0,1\};\ \forall k \in K,\ (i,j) \in A \tag{11}$$

The objective function (1) expresses the total cost. The restriction (2) restricts customers to be served by exactly one vehicle, where Δ^+(i) represents the set of edges that can be covered from the vertex i. The restrictions (3)-(6) characterize the flow on the path to be followed by the vehicle where Δ^-(i) represents the set of edges arriving at vertex i. Still, the restrictions (7), (8) and (9) guarantee the feasibility of sequencing with respect to time and capacity. For a given vehicle k, the constraint (8) makes $w_{ik} = 0$ whenever the customer i is not visited by the vehicle k. Finally, the restriction (11) imposes a condition for the binary flow variables.

The condition (11) allows the condition (7) to be linearized according to equation (12):

$$w_{ik} - s_i + t_{ij} - w_{ik} \le (1 - x_{ijk})M_{ij},\ \forall k \in K, (i, j) \in A \tag{12}$$

where M_{ij} is a big constant value. In addition, M_{ij} can be replaced by $max\{b_i + s_i + t_{ij} - a_j, 0\}$, $(i, j) \in$ A, and the constraints (11) and (16) simply need to be exchanged for arcs $(i, j) \in$ A such that $M_{ij} > 0$, otherwise when you $max\{b_i + s_i + t_{ij} - a_j, 0\} = 0$, these constraints are satisfied for all values of w_{ik}, w_{jk} e x_{ijk}.

3.2 Hybrid Algorithm

The hybrid optimization process is represented by two distinct phases. The first phase is characterized by the Genetic Algorithm application, which provides appropriate diversity for the population, enabling the search in unexplored portions of the search space. The diversification is achieved when suitable genetic parameters are used. In the GA phase, the elitism policy (De Jong, 1975) is used, where only the best individual is copied into the next population. With this elitism strategy, individuals can overlap and cover a wider search area in the solution space. Concerning the genetic operators, the UOBX was adopted for the crossover and the Swap Mutation was adopted for the mutation. Both genetic operators were studied in Geiger (2008).

The second phase of the hybrid optimization process provides intensification, guiding the search to promising regions of the solution space. This intensification will be performed by a Tabu Search algorithm, where its parameters will be applied in order to guide the solution to the optimal point in their search space. In this process, a neighborhood structure based in four different movements is used. The first one is called Intra-Route and it is applied by selecting a random route and trying to exchange a pair of vertices. The second movement is called Inter-Route and it is applied by selecting a random vertex and removing it from its original route and trying to insert it in every other routes.

The third movement is based in changing the position of two vertices. A random vertex is selected to be changed with every other vertex in the same route. Finally, the last movement is applied by recreating a route using a nearest neighbour heuristic. It is applied in every route individually.

Furthermore, an intensification process based on a memory structure is applied. The five best individuals provided by the Tabu Search process are stored. When the current best solution is not improved after 10 consecutive generations, the best solution from the list of candidates is extracted and the search is resumed from this new solution. With this intensification process, a more consistent exploration of the search space is sought. However, in order to have an efficient process, the solution representation in the Genetic Algorithm and Tabu Search should be the same, or a conversion process must be used. In this work, the solution encoding is the same in both optimization phases. The solution encoding consists of the sequence of all customers that the route must be covered so that a new route starts when any constraint is violated. Therefore, both optimization phases may use the solutions generated by the same encoding strategy.

4 EXPERIMENTS

The experiments were performed based on the problems presented by Solomon (1987). It is noteworthy that these are minimization problems, thus, the best results are those of lower value. According to the author, these problems are divided into six sets, here separated into three classes: R, C and RC, and two series: 1 and 2.

The classes differ in the geographic data of customers. For problems of class R, the spatial data were generated randomly. In class C data were obtained from groups of customers, while RC class, presents graphs that are the combination of some clusters and some additional nodes scattered randomly. The series define the flexibility of time windows. The first series has a narrower horizon of sequencing, and time windows that allow a maximum of ten clients per route. On the other hand, in the series two, time windows have great flexibility, often not causing problems and restrictions to allow more than thirty clients to be served by a route.

Based on these three classes and two series, so a total of six sets of problems (R1, R2, C1, C2, RC1, RC2). In each set, geographic location is the same, and only the variation of the time windows is considered. The set R1 has twelve problems, ranging from R101 to R112, R2 has eleven problems, all ranging from R201 to R211. In this class, the location of customers was randomly obtained. In class C, customers were obtained through the identification of clusters. The sets C1 and C2 have nine and eight problems, ranging from C101 to C109 and C201 to C208, respectively. Finally, sets RC1 and RC2 present eight problems each, ranging from RC101 to RC108 and RC201 to RC208, respectively, where the RC class combines, randomly, geographically spread customers with grouped customers. Every problem has a total of one hundred customers.

All the experiments were performed on a computer with eight processing cores with 3.0 GHz each, and 24 GBs of RAM memory.

The Table 1 presents the average solution, the standard deviation based on the average solutions and the best solution obtained with the hybrid algorithm. The best solutions obtained are compared with the optimal solution and the best heuristic solution found on literature. The * represents that the

Table 1: The best solutions obtained.

Problem	Hybrid Algorithm			Optimal Solution	Author	Best Heuristic Solution	Author
	Average	Standard Deviation	Best Solution Obtained				
C101	893,8	51,1	827,3*#	827,3	KDMSS[1]	828.9	RT[8]
C102	870,8	39,9	827,3*#	827,3	KDMSS[1]	828.9	RT[8]
C103	908,1	57,2	826,3*#	826,3	KDMSS[1]	828.0	RT[8]
C104	870,3	44,8	822,9*#	822,9	KDMSS[1]	824.7	RT[8]
C105	863,1	35,5	827,3*#	827,3	KDMSS[1]	828.9	RT[8]
C106	851,1	20,4	827,3*#	827,3	KDMSS[1]	828.9	RT[8]
C107	871	41,3	827,3*#	827,3	KDMSS[1]	828.9	RT[8]
C108	906,5	51,3	827,3*#	827,3	KDMSS[1]	828.9	RT[8]
C109	891,2	60,7	827,3*#	827,3	KDMSS[1]	828.9	RT[8]
RC101	1671	25,2	1636,1 #	1619,8	KDMSS[1]	1696.9	TBGGP[9]
RC102	1634,5	88,4	1461,6 #	1457,4	CR[2]+KLM[4]	1554.7	TBGGP[9]
RC103	1703,9	59,2	1267,3	1258,0	CR[2]+KLM[4]	1261.6	S98[10]
RC104	1718,5	89,9	1139,5			1135.4	CLM[11]
RC105	1730,4	57	1520,3 #	1513,7	KDMSS[1]	1629.4	BBB[12]
RC106	1699,4	88,8	1424.7			1424.7	BBB[12]
RC107	1682,5	78,4	1221,6 #	1207,8	IV[5]	1230.4	S97[13]
RC108	1681,9	82,8	1142,4	1114,2	IV[5]	1139.8	TBGGP[9]
R101	1716,1	57,1	1657,7	1637,7	KDMSS[1]	1645.7	H[14]
R102	1540,3	50,8	1480,5 #	1466,6	KDMSS[1]	1486.1	RT[8]
R103	1300,6	85	1213,9 #	1208,7	CR[2]+L[3]	1292.6	LLH[15]
R104	1002,5	13,7	980,8 #	971,5	IV[5]	1007.2	M[16]
R105	1394,9	25,7	1367,5 #	1355,3	KDMSS[1]	1377.1	RT[8]
R106	1298,4	60,7	1235,4 #	1234,6	CR[2]+KLM[4]	1251.9	M[16]
R107	1135,5	51,6	1082,5 #	1064,6	CR[2]+KLM[4]	1104.6	S97[13]
R108	975,9	85,5	983,3			960.8	BBB[12]
R109	1217	39,4	1162,9 #	1146,9	CR[2]+KLM[4]	1194.7	HG[17]
R110	1128,5	41,5	1084,5 #	1068,0	CR[2]+KLM[4]	1118.5	M[16]
R111	1153	52,6	1064,6 #	1048,7	CR[2]+KLM[4]	1096.7	RGP[18]
R112	1029,9	10,3	1018,2			982.1	GTA[19]
C201	602,6	11,4	589,1*#	589,1	CR[2]+KLM[4]	591.5	RT[8]
C202	629,2	39,2	589,1*#	589,1	CR[2]+KLM[4]	591.5	RT[8]
C203	666	24,7	588,7*#	588,7	KLM[4]	591.1	RT[8]
C204	646,3	22,9	588,1*#	588,1	IV[5]	590.6	RT[8]
C205	664,6	27,6	604,7	586,4	CR[2]+KLM[4]	588.8*	RT[8]
C206	638,4	20,3	594,9 #	586,0	CR[2]+KLM[4]	588.4*	RT[8]
C207	600,1	20,3	585,8*#	585,8	CR[2]+KLM[4]	588.2	RT[8]
C208	645	34,2	585,8*#	585,8	KLM[4]	588.3	RT[8]
RC201	1372,6	77,4	1267,7 #	1261,8	KLM[4]	1406.9	M[16]
RC202	1266	52	1096,6 #	1092,3	IV[5]+C[6]	1367.0	CC[20]
RC203	1319,7	54,5	1070,3			1049.6	CC[20]
RC204	1361,4	71,3	798.4			798.4	M[16]
RC205	1276,3	58,8	1161,4 #	1154,0	IV[5]+C[6]	1297.1	M[16]
RC206	1285,7	17,8	1265,7	1261,8	KLM[4]	1146.3	H[14]
RC207	1274,9	65,7	962,9*#	962,9	DLH[7]	1061.1	BVH[21]
RC208	1459,6	66,5	828.1			828.1	IKMUY[22]
R201	1185,1	25,5	1159,1 #	1143,2	KLM[4]	1252.3	HG[17]
R202	1341,5	73	1209,0			1191.7	RGP[18]
R203	992,7	53,1	939.5			939.5	M[16]
R204	872,9	24,4	825.5			825.5	BVH[21]
R205	1092,1	84,6	949.8*#	949,8	DLH[7]	994.4	RGP[18]
R206	1161,5	30,5	1011,8	875,9	DLH[7]	906.1	SSSD[23]
R207	895,4	35,8	794,0*#	794,0	DLH[7]	893.3	BVH[21]
R208	866,2	79,5	726.7			726.7	M[16]
R209	1042,7	82,9	909.1			909.1	H[14]
R210	955,8	52,4	900,5*#	900,5	DLH[7]	939.3	M[16]
R211	937,4	24	892.7			892.7	BVH[21]

[1] Kohl et al (1999), [2] Cook e Rich (1999), [3] Larsen (1999), [4] Kallehauge et al (2000), [5] Irnich e Villeneuve (2005), [6] Chabrier (2005), [7] Desaulniers, G., Lessard, F., Hadjar, [8] Rochat e Taillard (1995), [9] Taillard et al (1997), [10] Shaw (1998), [11] Cordeau et al (2000), [12] Berger et al (2001), [13] Shaw (1997), [14] Homberger (2000), [15] Li et al (2001), [16] Mester (2002), [17] Homberger e Gehring (1999), [18] Rousseau et al (2002), [19] Gambardella (1999), [20] Czech e Czarnas (2002), [21] Bent e Van Hentenryck (2001), [22] Ibaraki et al (2001), [23] Schrimpf et al (2000).

hybrid algorithm obtained the optimal solution and the # represents that the hybrid algorithm obtained a better or equal solution compared with the best heuristic solution known.

It is observed that the hybrid algorithm has obtained optimal solution in 19 out of 56 problems analyzed, 9 in problems of less complexity (series 1) and 10 in more complex problems (series 2).

By comparing with the best heuristic methods, the hybrid algorithm provides better solutions in 37 of the 56 problems analyzed and proved to be more efficient than other heuristic methods applied to Vehicle Routing Problem with Time Windows. Increasing the number of customers provides an increase in computational complexity, which shows the characteristics of the three techniques. The Tabu Search intensifies the search in promising regions, however cannot diversify effectively. The Genetic Algorithm has proven ineffective to intensify the search in promising regions, compared to Tabu Search, however, leads to a wider search in the search space, reaching areas not explored by Tabu Search. Combining the power of diversification of Genetic Algorithm and the power of intensification of Tabu Search, the Hybrid Algorithm promotes a broader search space without losing the power of intensification, which can be seen in the solutions obtained.

5 CONCLUSIONS

This paper addresses the Capacitated Vehicle Routing Problem with Time Windows, which must obeys the capacity constraints of the vehicle and the time windows of customer service to solve this problem. Moreover, the metaheuristics Tabu Search and Genetic Algorithms has been used in a hybrid algorithm.

Analyzing the results, it is found also that the Tabu Search obtained better solutions than the Genetic Algorithm for Capacitated Vehicle Routing Problem with Time Windows with smaller standard deviations. This is due to the intensification policy, which promotes a local search in promising regions. The experiments show that the Hybrid Algorithm has higher efficiency in obtaining better solutions, compared to Tabu Search and Genetic Algorithm, and it is still more efficient, generating minor standard deviations. Although the Hybrid Algorithm has used the characteristics of both techniques, the computational time does not undergo a significant increase, since the difference is only a few seconds relative to Tabu Search and the Genetic Algorithm.

When comparing the results obtained by different techniques, the Genetic Algorithm do not get good quality solutions, compared to Tabu Search and Hybrid Algorithm. The Tabu Search is more efficient in some cases, surpassing some results obtained by the Hybrid Algorithm in problems of less complexity. However, it looks inefficient compared to the Hybrid Algorithm, with the increase in the complexity of the problems. The Hybrid Algorithm developed shows itself flexible and efficient in obtaining good quality solutions for all types of problems analyzed. It is noteworthy that the Hybrid Algorithm obtained many solutions known a priori as optimal, and obtained better solutions for most problems compared with the best heuristic solutions.

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