A Proposal of Multiobjective Fuzzy Regulator Design for State Space Nonlinear Systems

Rafael J. M. Santos and Ginalber L. O. Serra
Federal Institute of Education, Science and Technology,
Av. Getúlio Vargas 04, Monte Castelo, CEP: 65030-005, São Luís, Maranhão, Brazil

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Abstract: This paper proposes a Takagi-Sugeno (TS) fuzzy regulator design methodology for nonlinear dynamic systems. The Linear Quadratic Regulator (LQR) and Pole Placement (PP) techniques are combined in a TS fuzzy structure in order to guarantee an optimal controller with satisfactory transient response based on poles allocated properly. The definition and analysis of the multiobjective feasible region, considering the influence of the desired poles on the weighting matrices **Q** and **R** in the quadratic cost function, are presented. Lyapunov based stability analysis and simulations results on fuzzy regulator design for a robotic manipulator illustrates the efficience of the proposed methodology.

1 INTRODUCTION

The actual control problems have natural tendency to increase its complexity due to the multiobjective performance criterion and its satisfaction with high precision and accuracy. In this context, the classical control theory, characterized by the input-output representation has limitations because it does not allow full access to all variables of the plant to be controlled, which has motivated the search for new control strategies, exploring the state space representation, characterizing the modern control theory. Since then, several methods have been proposed to deal with linear plants (in this case, the nonlinearities and uncertainties are considered negligible) as well as to develop more advanced controllers taking into account the nonlinearities, uncertainties and time varying parameters of the plant (Choi, 2007; Li and Tsai, 2007; Chen et al., 2007a; Zhu and Ma, 2006; Ghosh et al., 2009; Nie and Tan, 2011; Zhao et al., 2004).

The search for new model based control strategies from computational intelligence techniques, considering the impact of fuzzy systems, among others, has allowed succesfull applications in modeling and control of complex plants with promising results (Chen et al., 2007b; Mohammadian et al., 2003; Pedrycz and Gomide, 2007; Liu et al., 2011; Mumford and Jain, 2009; Gorji and Menhaj., 2008; Eberhart and Shi, 2007; Gui-juan et al., 2010; Ko and Jatskevich, 2007; Luo et al., 2006). The fuzzy systems theory, in particular, has received great attention from researchers in the this area to deal effectively with uncertainties and nonlinearities through its functional structure (Babuška, 1998; Khanesar et al., 2011; Boulkrounea et al., 2010; Nie and Tan, 2011; Preitl et al., 2008; Abbas et al., 2011; Jiang et al., 2008). This research interest has grown in recent years by the possibility of incorporating in the fuzzy inference structure the expert knowledge as well as the mathematical formalism from the modern control theory, resulting in fuzzy control systems with high degree of transparency, interpretation, robustness and stability (Eltamaly et al., 2010; Yuana et al., 2010; Liu et al., 2010; Berrios et al., 2011; Li, 2010; Ko and Jatskevich, 2007; Torres-Pinzón and Leyva, 2009; Gheysari and Mashoufi, 2011; Shi et al., 2011). In (Márquez et al., 2009), a general methodology that uses fuzzy logic to systematically and formally synthesize stable nonlinear control systems design is proposed. Although this methodology is based on Lyapunov theory, it avoids searching for Lyapunov functions. This allows the synthesis procedure to be systematic as well as formal and, especially, independent of heuristics. In (Zhao et al., 2009), a novel robust fuzzy controller design method is proposed to stabilize a class of chaotic (hyperchaotic) systems with uncertain parameters based on their equivalent TS fuzzy models. In this method, the interval system theory is

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applied to deal with the parametric uncertainty firstly, and then a fuzzy state feedback controller is designed to stabilize the equilibrium of the uncertain chaotic (hyperchaotic) systems robustly based on Exact Linearization (EL) theory and Parallel Distributed Compensation (PDC) technique. The designed controller with simple structure and rapid response can stabilize many types of uncertain chaotic or hyperchaotic systems. In (Zhao et al., 2004), after a study on synthesis of neural network and fuzzy logic based controllers for optimally controlling uncertain nonlinear systems linear in control, three different types of hierarchical controller architectures are proposed, which include a hierarchical neuro-fuzzy controller architecture, a hierarchical fuzzy-neuro controller architecture and a hierarchical fuzzy logic controller architecture. The study concludes the proposed neural-network and fuzzy logic based control schemes are useful for nonlinear system applications.

In this paper is proposed a Takagi-Sugeno (TS) fuzzy state space control strategy for nonlinear dynamic systems based on two methodologies: Pole Placement and Linear Quadratic Regulator. The pole placement technique is used to design the state feedback gain matrix which determines the transient response characteristics by allocating the desired closed loop poles. The Linear Quadratic Regulator, in turn, is used to compute the state feedback gain matrix considering the optimization of a quadratic cost function. The proposal in this paper is to combine these methodologies, in a fuzzy context, in order to design an optimal regulator for nonlinear plants with desired transient response performance. This paper is structured as follow: Section 2 presents the fundamental structure of the proposed methodology. Section 3 presents the main computational results of the fuzzy multiobjective regulator design for a robotic manipulator according to multiobjective feasible region defined by LQR and pole placement techniques. Section 4 presents conclusions and proposals for future work.

2 POLE PLACEMENT BASED LINEAR QUADRATIC FUZZY REGULATOR

This section presents the analitical formulation of the multiobjective fuzzy regulator design from the fusion of pole placement and *LQR* methods to garantee a desired transient response with optimal control action for nonlinear systems.

2.1 State Space Takagi-Sugeno Fuzzy Modeling

2.1.1 State Space Takagi-Sugeno Fuzzy Inference System

The Takagi-Sugeno fuzzy inference system uses in the consequent proposition of its IF-THEN rule base a functional expression of the linguistic variables in the antecedent proposition. The $i|^{[i=1,2,\ldots,l]}$-th rule, where l is the number of rules, is given by

$$R^i: \quad \textbf{IF } \tilde{x}_1 \text{ is } F^i_{j|\tilde{x}_1} \textbf{ AND } \ldots \textbf{ AND } \tilde{x}_n \text{ is } F^i_{j|\tilde{x}_n} \quad \textbf{THEN } \dot{\mathbf{x}}(t) = A_i\mathbf{x}(t) + B_i\mathbf{u}(t) \tag{1}$$

The vector $\tilde{\mathbf{x}} \in \mathbb{R}^n$ contains the linguistic variables of the antecedent proposition. Each variable has its own linguistic universe of discourse $U_{\tilde{x}_1},\ldots,U_{\tilde{x}_n}$ partitioned by fuzzy sets representing the corresponding linguistic terms. The variable $\tilde{x}_t|^{t=1,2,\ldots,n}$ belongs to the fuzzy set $F^i_{j|\tilde{x}_t}$ with a value $\gamma^i_{F_{j|\tilde{x}_t}}$ defined by a membership function $\gamma^i_{\tilde{x}_t}: \mathbb{R} \to [0,1]$, with $\gamma^i_{F_{j|\tilde{x}_1}} \in \left\{\gamma^i_{F_{1|\tilde{x}_1}}, \gamma^i_{F_{2|\tilde{x}_1}}, \ldots, \gamma^i_{F_{p_{\tilde{x}_t}|\tilde{x}_1}}\right\}$, where $p_{\tilde{x}_t}$ is the number of partitions of the universe of discourse associated with the linguistic variable $\tilde{x}_t$. The matrices $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times 1}$ represent the parameters of the i-th local state space model of the nonlinear plant on its i-th operating point; $\mathbf{x}(t) \in \mathbb{R}^{n\times 1}$, is the state vector of the plant and $u(t) \in \mathbb{R}$, is the input vector of the plant. The fulfillment degree h_i for the rule i is given by the t-norm operator:

$$h_i = \gamma^i_{F_{j|\tilde{x}^*_1}} \wedge \gamma^i_{F_{j|\tilde{x}^*_2}} \wedge \ldots \wedge \gamma^i_{F_{j|\tilde{x}^*_n}} \tag{2}$$

where $\tilde{x}^*_t$ is any point in $U_{\tilde{x}^*_t}$. The normalized fulfillment degree for the i-th rule is defined by:

$$\lambda_i(\tilde{\mathbf{x}}) = \frac{h_i(\tilde{\mathbf{x}})}{\sum_{r=1}^{l} h_r(\tilde{\mathbf{x}})} \tag{3}$$

This normalization implies that

$$\sum_{i=1}^{l} \lambda_i(\tilde{\mathbf{x}}) = 1 \tag{4}$$

The TS fuzzy model response is a weighted sum of the consequents, i.e., a convex combination of the local state space models:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{l} \lambda_i(\tilde{\mathbf{x}}) \left(A_i\mathbf{x}(t) + B_i\mathbf{u}(t)\right) \tag{5}$$

The TS fuzzy model can be considered as a mapping from space of the antecedent proposition (input) to the convex region (polytope) in the space of local submodels defined by the functional expressions in the consequent proposition. This property simplifies the analysis in the context of robust systems for identification and controllers design (Serra and Bottura, 2009; Bergsten, 2001).

2.2 Fuzzy Regulator Design

Consider a SISO linear plant, representing a local submodel for i-th rule, described by

$$\dot{\mathbf{x}} = \mathbf{A}_i\mathbf{x} + \mathbf{B}_i u \tag{6}$$

where $\mathbf{x} \in \mathbb{R}^{n\times 1}$, is the state vector of the plant; $u \in \mathbb{R}$, is the input vector of the plant; $\mathbf{A}_i \in \mathbb{R}^{n\times n}$, is the state matrix; $\mathbf{B}_i \in \mathbb{R}^{n\times 1}$, is the input matrix.

The quadratic optimal regulator problem consists in minimizing the objective function given by:

$$J = \int_0^\infty \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + u^T\mathbf{R}u\right) dt \tag{7}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices to be selected by designer related to the state $\mathbf{x}$ vector and control action u, respectively. The matrices $\mathbf{Q}$ and $\mathbf{R}$ must also be nonnegative definite, which is most easily accomplished by picking the $\mathbf{Q}$ and $\mathbf{R}$ to be diagonal with all diagonal elements positive or zero. The solution of this objective function, for the i-th rule, results in a state feedback optimal control gain matrix $\mathbf{K}_i$, given by:

$$\mathbf{u}(t) = -\mathbf{K}_i\mathbf{x}(t) \tag{8}$$

Considering the i-th functional expression in the consequent proposition of the Takagi-Sugeno fuzzy inference model, according to equation (1), the closed loop state feedback fuzzy system is given by:

$$\dot{\mathbf{x}} = \mathbf{A}_i\mathbf{x} - \mathbf{B}_i\mathbf{K}_j\mathbf{u} = (\mathbf{A}_i - \mathbf{B}_i\mathbf{K}_j)\mathbf{x} \tag{9}$$

Assuming that the matrix $\mathbf{A}_i - \mathbf{B}_i\mathbf{K}_j$ is stable, it is possible to obtain a state feedback fuzzy control gain matrix $\mathbf{K}_j$ from the solution of the algebraic Riccati equation:

$$\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i - \mathbf{P}\mathbf{B}_i\mathbf{R}^{-1}\mathbf{B}_i^T\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{10}$$

for the matrix $\mathbf{P}$. Therefore, the gain $\mathbf{K}_i$ is given by:

$$\mathbf{K}_i = \mathbf{R}^{-1}\mathbf{B}_i^T\mathbf{P}\big|^{[j=1,2,\ldots,l]} \tag{11}$$

Equation (11) provides the following optimal feedback fuzzy control action:

$$\mathbf{u}(t) = -\mathbf{K}_i\mathbf{x}(t) = -\mathbf{R}^{-1}\mathbf{B}_i^T\mathbf{P}\mathbf{x}(t) \tag{12}$$

Consider, the consequent proposition represented by a second order functional expression, in the controller canonical form:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -a_2^i & -a_1^i \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u} \tag{13}$$

The matrices $\mathbf{Q}$ and $\mathbf{R}$ are defined as

$$\mathbf{Q} = \begin{bmatrix} \mu & 0 \\ 0 & 1 \end{bmatrix} \tag{14}$$

$$\mathbf{R} = \begin{bmatrix} \beta \end{bmatrix} \tag{15}$$

where $\mu \geq 0$ and $\beta > 0$.

The matrix $\mathbf{P}$ to be obtained, is defined by:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{16}$$

where $p_{12} = p_{21}$. Substituting the matrix $\mathbf{A}_i$ in (13) and the matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{P}$ defined in (14), (15), (16), respectively, in equation (10), results

$$\begin{bmatrix} 0 & -a_2^i \\ 1 & -a_1^i \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -a_2^i & -a_1^i \end{bmatrix} - \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}[\beta]^{-1}\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{17}$$

Developing the equation (17), it yields:

$$\begin{bmatrix} -2a_2^i p_{12} - \frac{(p_{12})^2}{\beta} + \mu & p_{11} - a_1^i p_{12} - a_2^i p_{22} - \frac{p_{12}p_{22}}{\beta} \\ p_{11} - a_1^i p_{12} - a_2^i p_{22} - \frac{p_{12}p_{22}}{\beta} & 2p_{12} - 2a_1^i p_{22} - \frac{(p_{22})^2}{\beta} + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{18}$$

From equation (18), the following equations system is obtained:

$$\begin{aligned} -2a_2^i p_{12} - \frac{(p_{12})^2}{\beta} + \mu &= 0 \\ p_{11} - a_1^i p_{12} - a_2^i p_{22} - \frac{p_{12}p_{22}}{\beta} &= 0 \\ 2p_{12} - 2a_1 p_{22} - \frac{(p_{22})^2}{\beta} + 1 &= 0 \end{aligned} \tag{19}$$

For the solution of (19), the values of p_{11}, p_{12} and p_{22} are given by:

$$
\begin{aligned}
p_{11} &= a_1^i p_{12} + a_2^i p_{22} + \frac{p_{12}p_{22}}{\beta} \\
p_{12} &= -a_2^i\beta + \sqrt{\left(a_2^i\right)^2\beta^2 + \mu\beta} \\
p_{22} &= -a_1^i\beta + \sqrt{\left(a_1^i\right)^2\beta^2 + 2\left(-a_2^i\beta + \sqrt{\left(a_2^i\right)^2\beta^2+\mu\beta}\right)\beta + \beta}
\end{aligned}
\tag{20}
$$

From equation (11), it yields:

$$
\mathbf{K}_i = [\beta]^{-1}\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} \frac{p_{12}}{\beta} & \frac{p_{22}}{\beta} \end{bmatrix} \tag{21}
$$

or

$$
\mathbf{K}_i = \begin{bmatrix} k_1^i & k_2^i \end{bmatrix} \tag{22}
$$

where

$$
\begin{aligned}
k_1^i &= -a_2^i + \frac{\sqrt{\left(a_2^i\right)^2\beta^2+\mu\beta}}{\beta} \\
k_2^i &= -a_1^i + \frac{\sqrt{\left(a_1^i\right)^2\beta^2 + 2\left(-a_2^i\beta + \sqrt{\left(a_2^i\right)^2\beta^2+\mu\beta}\right)\beta + \beta}}{\beta}
\end{aligned}
\tag{23}
$$

The characteristic equation of the state feedback system can be determined after obtaining the gain matrix $\mathbf{K}_j$, as follow:

$$
\begin{aligned}
&|s\mathbf{I}_2 - \mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j| = \\
&s^2 + \left(\frac{\sqrt{\beta\left(\left(a_1^i\right)^2\beta - 2a_2^i\beta + 2\sqrt{\beta\left(\left(a_2^i\right)^2\beta+\mu\right)} + 1\right)}}{\beta}\right)s + \\
&+ \frac{\sqrt{\beta\left(\left(a_2^i\right)^2\beta+\mu\right)}}{\beta} = 0
\end{aligned}
\tag{24}
$$

Based on the linear quadratic regulator theory and from (23) and (24), it is possible to assign poles for each functional expression defined in the i-th rule of the Takagi-Sugeno fuzzy inference model and design a respective state feedback gain matrix $\mathbf{K}_i$ ensuring the optimal conditions defined by matrices $\mathbf{Q}$ and $\mathbf{R}$ in terms of μ and β, and desired transient response as well. In the Appendix, the consistency of the characteristic equation coefficients is shown according to Caley-Hamilton Theorem.

2.2.1 Multiobjective Feasible Region Analysis

According to the linear quadratic regulator theory and the equations (23) and (24), it is possible to choose poles for each submodels and design the corresponding state feedback gain matrix $\mathbf{K}_i$ to garantee the existence of the matrices $\mathbf{Q}$ and $\mathbf{R}$ in terms of μ and β. Consider two generic poles s_1 and s_2. Assuming the submodels are causal, the real part of the poles will be ever negative, in order to ensure stability. Based on equation (24), it yields:

$$
\frac{\sqrt{\beta\left(\left(a_2^i\right)^2\beta+\mu\right)}}{\beta} = (s_1 s_2) \tag{25}
$$

and

$$
\frac{\left(\left(a_2^i\right)^2\beta+\mu\right)}{\beta} = (s_1 s_2)^2
$$

And the following relation can be obtained:

$$
\frac{\mu}{\beta} = \left((s_1 s_2)^2 - \left(a_2^i\right)^2\right) \tag{26}
$$

Similarly, it yields:

$$
\frac{\sqrt{\beta\left(\left(a_1^i\right)^2\beta - 2a_2^i\beta + 2\sqrt{\beta\left(\left(a_2^i\right)^2\beta+\mu\right)} + 1\right)}}{\beta} = -(s_1+s_2) \tag{27}
$$

and

$$
\frac{\left(\left(a_1^i\right)^2\beta - 2a_2^i\beta + 2\sqrt{\beta\left(\left(a_2^i\right)^2\beta+\mu\right)} + 1\right)}{\beta} = (s_1+s_2)^2
$$

$$
\left(a_1^i\right)^2 - 2a_2^i + 2\underbrace{\frac{\sqrt{\beta\left(\left(a_2^i\right)^2\beta+\mu\right)}}{\beta}}_{s_1 s_2} + \frac{1}{\beta} = (s_1+s_2)^2
$$

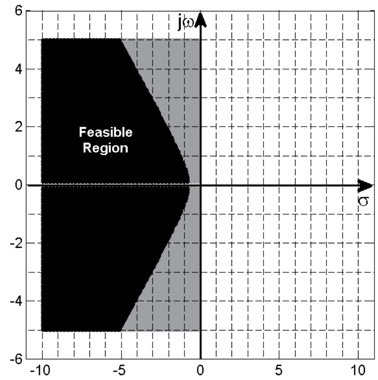
$$
\frac{1}{\beta} = s_1^2 + s_2^2 - 2a_2^i + \left(a_1^i\right)^2
$$

$$
\beta = \frac{1}{s_1^2 + s_2^2 - 2a_2^i + \left(a_1^i\right)^2} \tag{28}
$$

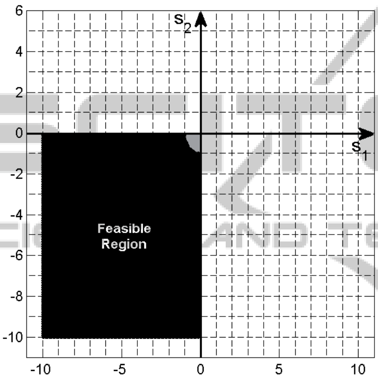
Substituting (28) in equation (26):

$$
\mu = \frac{(s_1 s_2)^2 - \left(a_2^i\right)^2}{s_1^2 + s_2^2 - 2a_2^i + \left(a_1^i\right)^2} \tag{29}
$$

Equations (28) and (29) provide a direct relationship between the desired poles to be allocated and the parameters of the linear submodels in the consequent proposition of the state space Takagi-Sugeno fuzzy model. The multiobjective optimal condition, based on the pole placement method and linear quadratic regulator problem, slightly restricts the area of the poles to be chosen adequately, which becomes a subregion in the left half of the complex plane, so called as *feasible region*. The feasible regions for complex and real poles are shown in Figure 1(a) and 1(b), respectively. As example, the complex poles $s_1 = -0.1 + j0.1$ and $s_2 = -0.1 - j0.1$, implies to $\mu = -4 \times 10^{-4}$ and $\beta = -1$, meaning these poles



(a) Complex pole placement



(b) Real pole placement

Figure 1: Multiobjective feasible regions for the pole placement problem.

are out of the feasible region, once the matrices **Q** and **R** from equations (14) and (15) define $\mu \geq 0$ and $\beta > 0$. The real poles $s_1 = -0.5$, $s_2 = -0.7$, implies to $\mu = -0.4712$ and $\beta = -3.8462$, meaning these poles are inside of the feasible region.

The methodological procedure for the multiobjective fuzzy regulator design is shown in Table 1. The desired closed loop poles are chosen to be allocated for each submodel in the consequent proposition of the Takagi-Sugeno fuzzy model, which represents the nonlinear plant. Next, the characteristic equation according to desired poles are determined. The characteristic equation is compared with equation (24) and solved in terms of the variables μ and β. Once that positive real solutions of μ and β for all submodels exist, then the state feedback gain matrix can be determined from the equation (22). Otherwise, it is not possible to allocate the desired poles due to restriction from the linear quadratic optimal control estrategy.

2.2.2 Stability Conditions based on Lyapunov Approach

Since fuzzy systems are essentially nonlinear systems, stability analysis methods for fuzzy control systems, in particular, are based on nonlinear stability theory. In the literature, some stability analysis methods based on Lyapunov approach are proposed (Lendek et al., 2009; Tanaka et al., 1996; Sheikholeslam and Shekaramiz, 2011).

Table 1: The methodological procedure for the multiobjective fuzzy regulator design.

Step	Procedure
1	Obtain the matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ from linear submodels
2	Choose the poles s_1 and s_2 from feasible regions in Figure 1
3	Determine μ and β from solution of $s^2+\left(\frac{\sqrt{\beta\left(\left(a_1^i\right)^2\beta-2a_2^i\beta+2\sqrt{\beta\left(\left(a_2^i\right)^2\beta+\mu\right)}+1\right)}}{\beta}\right)s+\frac{\sqrt{\beta\left(\left(a_2^i\right)^2\beta+\mu\right)}}{\beta} = (s-s_1)(s-s_2)$
4	Design the state feedback gain matrix $\mathbf{K}_j\vert^{[j=1,2,\ldots,l]}$ from (22)

Consider a continuous fuzzy control system (*CFS*) described by:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \sum_{i=1}^{l}\sum_{j=1}^{l}\lambda_i(\mathbf{x}(t))\lambda_j(\mathbf{x}(t))\left[\mathbf{A}_i-\mathbf{B}_i\mathbf{K}_j\right]\mathbf{x}(t) \\ &= \sum_{i=1}^{l}\lambda_i(\mathbf{x}(t))\mathbf{G}_{ii}\mathbf{x}(t)+ \\ &\quad +2\sum_{i<j}^{l}\lambda_i(\mathbf{x}(t))\lambda_j(\mathbf{x}(t))\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)\mathbf{x}(t) \end{aligned} \quad (30)$$

where,

$$\mathbf{G}_{ij} = \mathbf{A}_i - \mathbf{B}_i\mathbf{K}_j \quad (31)$$

Stability conditions for this *CFS* are established by following theorem (Tanaka et al., 1996; Michels et al., 2006):

Theorem 1. *The equilibrium of a* CFS *is asymptotically stable in the large if there exists a common positive definite matrix* **P** *such that*

$$\mathbf{G}_{ii}^T\mathbf{P}+\mathbf{P}\mathbf{G}_{ii} < \mathbf{0} \quad (32)$$

$$\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)^T\mathbf{P}+\mathbf{P}\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right) < \mathbf{0}, \quad i<j, \quad (33)$$

for all i and j excepting the pairs (i,j) such that $\lambda_i(\mathbf{x}(t))\lambda_j(\mathbf{x}(t)) = 0$ for all t.

3 COMPUTATIONAL RESULTS

This section presents the computational results showing the efficience of the proposed methodology from the multiobjective fuzzy regulator design for a robotic manipulator.

The differential equation of the robotic manipulator is given by:

$$ml^2\ddot{\theta} + Bl\dot{\theta} + mgl\sin(\theta) = T_c \tag{34}$$

where $B = 1\,kgm^2/s$ is the damping factor, $m = 1\,kg$ is the mass and $l = 1\,m$ is the length of the manipulator arm; $g = 9.81\,m/s^2$ is the gravitational constant, T_c is the input variable, i.e., the torque in $N.m$. The angular position θ is the output variable of the manipulator. Consider an angular position θ_0 of the robotic manipulator. Equation (34) can be formulated by:

$$\ddot{\theta} = -\psi\dot{\theta} - \delta\theta + u^* \tag{35}$$

where $\alpha = \dfrac{1}{ml^2}$, $\psi = \dfrac{B}{ml}$, $\gamma(\theta_0) = \dfrac{g}{l}[\sin(\theta_0) - \theta_0\cos(\theta_0)]$, $\delta(\theta_0) = \dfrac{g\cos(\theta_0)}{l}$ and $u^* = \alpha T_c - \gamma$. The state space representation is given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -\delta(\theta_0) & -\psi \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u^* \qquad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}(t) \tag{36}$$

where $x_1(t) = \theta$, $x_2(t) = \dot{\theta}$, $\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}$, $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ and $y(t)$ is the angular position of the robotic manipulator.

Considering operating points for the robotic manipulator $-90°$, $-60°$, $-30°$, $0°$, $30°$, $60°$, and $90°$, as shown in Figure 2, it is possible to obtain the linear submodels and group them into a TS fuzzy structure. The rule base for TS fuzzy model is shown in Table 2.

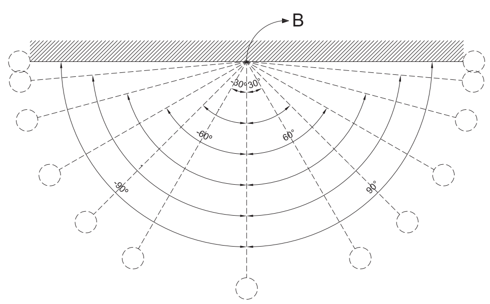


Figure 2: Operating points of the manipulator.

Table 2: Rule base of the Takagi-Sugeno fuzzy model for robotic manipulator described by the equation (34).

Model
R_1: If θ is $-90°$ then $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_1{}^*$
R_2: If θ is $-60°$ then $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -4.9050 & -1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_2{}^*$
R_3: If θ is $-30°$ then $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -8.4957 & -1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_3{}^*$
R_4: If θ is $0°$ then $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -9.81 & -1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_4{}^*$
R_5: If θ is $30°$ then $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -8.4957 & -1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_5{}^*$
R_6: If θ is $60°$ then $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -4.9050 & -1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_6{}^*$
R_7: If θ is $90°$ then $\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_7{}^*$

The rule base for TS fuzzy regulator is shown in Table 3. The fuzzy control action is given by:

$$\tilde{\mathbf{u}}(t) = -\sum_{i=1}^{7}\lambda_i\mathbf{K}_i\tilde{\mathbf{x}} + \sum_{i=1}^{7}\lambda_i u_i \tag{37}$$

where λ_i denotes the normalized fulfillment degree.

Table 3: Rule base of the Takagi-Sugeno fuzzy regulator for robotic manipulator described by the equation (34). For this rule base, $u_1 = 9.81$, $u_2 = 3.3592$, $u_3 = 0.4567$, $u_4 = 0$, $u_5 = -0.4557$, $u_6 = -3.3592$ and $u_7 = -9.81$.

Controller
R_1: If θ is $-90°$ then $\tilde{\mathbf{u}}(t) = -\mathbf{K}_1\tilde{\mathbf{x}}(t) + u_1$
R_2: If θ is $-60°$ then $\tilde{\mathbf{u}}(t) = -\mathbf{K}_2\tilde{\mathbf{x}}(t) + u_2$
R_3: If θ is $-30°$ then $\tilde{\mathbf{u}}(t) = -\mathbf{K}_3\tilde{\mathbf{x}}(t) + u_3$
R_4: If θ is $0°$ then $\tilde{\mathbf{u}}(t) = -\mathbf{K}_4\tilde{\mathbf{x}}(t) + u_4$
R_5: If θ is $30°$ then $\tilde{\mathbf{u}}(t) = -\mathbf{K}_5\tilde{\mathbf{x}}(t) + u_5$
R_6: If θ is $60°$ then $\tilde{\mathbf{u}}(t) = -\mathbf{K}_6\tilde{\mathbf{x}}(t) + u_6$
R_7: If θ is $90°$ then $\tilde{\mathbf{u}}(t) = -\mathbf{K}_7\tilde{\mathbf{x}}(t) + u_7$

The simulation diagram of the robotic manipulator and fuzzy regulator are shown in Figure 3.

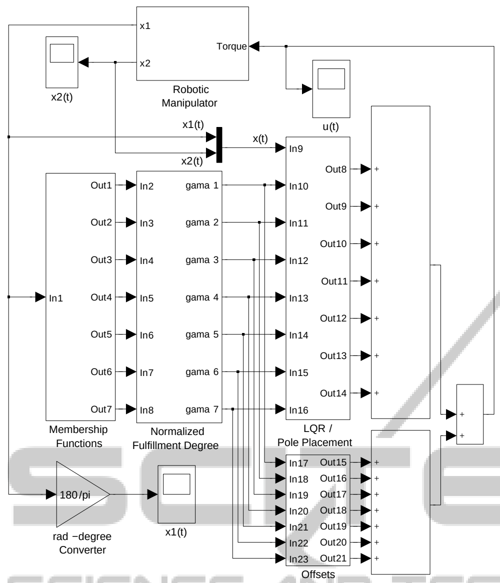


Figure 3: Simulation diagram of the multiobjective fuzzy regulator design.

The transient response and the control action of the multiobjective fuzzy regulator system for the robotic manipulator, considering some complex and real poles of the feasible and unfeasible region, from Table 4, are shown in Figure 4.

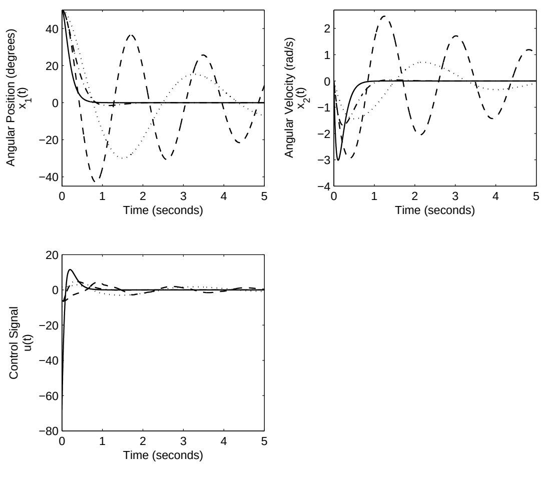


Figure 4: Multiobjective fuzzy controller performance. The poles $s_1 = -8$, $s_2 = -10$ (solid line) and $s_1 = -3 + j2$, $s_2 = -3 - j2$ (dash-dot line) are allocated in the feasible region; the poles $s_1 = -0.3$, $s_2 = -0.5$ (dotted line) and $s_1 = -0.2 + j3$, $s_2 = -0.2 - j3$ (dashed line) are allocated in the unfeasible region.

The poles placed into feasible region implies to a better transient response specifications. This is because that the proposed multiobjective methodology formulation allows the choosing the poles according to desired transient response and satifies the optimality criterion from LQR method as well.

Table 4: Relationship between performance criteria through the matrices $\mathbf{Q}$ and $\mathbf{R}$, defined by equations (14) and (15), respectively, and the pole placement method.

Poles	θ_0	μ	β	**K**
	$-90°$	08.258	0.0323	[16.000 07]
	$-60°$	05.683	0.0245	[11.095 07]
	$-30°$	03.830	0.0208	[07.504 07]
$[-4,-4]$	$0°$	03.156	0.0198	[06.190 07]
	$30°$	03.830	0.0208	[07.504 07]
	$60°$	05.683	0.0245	[11.095 07]
	$90°$	08.258	0.0323	[16.000 07]
	$-90°$	18.778	0.1111	[13.000 05]
	$-60°$	07.706	0.0532	[08.095 05]
	$-30°$	03.725	0.0385	[04.504 05]
$[-3 \pm j2]$	$0°$	02.542	0.0349	[03.190 05]
	$30°$	03.725	0.0385	[04.504 05]
	$60°$	07.706	0.0532	[08.095 05]
	$90°$	18.778	0.1111	[13.000 05]
	$-90°$	39.264	0.0061	[80.000 17]
	$-60°$	36.896	0.0058	[75.095 17]
	$-30°$	35.156	0.0056	[71.504 17]
$[-8,-10]$	$0°$	34.518	0.0055	[70.190 17]
	$30°$	35.156	0.0056	[71.504 17]
	$60°$	36.896	0.0058	[75.095 17]
	$90°$	39.264	0.0061	[80.000 17]

Below it is proved that the state feedback gain matrices $\mathbf{K}_i$ $(i = 1, 2, \cdots, l)$ obtained by proposed methodology for fuzzy regulator design satisfies the conditions of Theorem 1 for a common positive definite matrix $\mathbf{P}$. Therefore, the procedure for this demonstration is as follow:

1. Determine $\mathbf{K}_i$ from $\mathbf{A}_i$ and $\mathbf{B}_i$ using the proposed methodology;
2. Find a common $\mathbf{P}$ satisfying the conditions of Theorem 1.

The results of the Step 1 were shown in Table 4. The results of the Step 2 are shown in Table 5 and Table 6.

The four matrices $\mathbf{P}$ obtained from algebraic Riccati equation solution are shown in Tables 5 and 6. They guarantee the multiobjective fuzzy regulator stability since satisfy the conditions established in Theorem 1 for all rules simultaneously. As can be seen in Table 5 and 6, all values of the matrices $\mathbf{P}$ ensure matrices $\mathbf{G}_{ii}^T \ \mathbf{P} + \mathbf{P}\mathbf{G}_{ii}|_{i=1,2,\cdots,7}$ and $\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)^T \mathbf{P} + \mathbf{P}\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)$ whose eigenvalues are negative.

Table 5: The matrix $\mathbf{G}_{ii}^T\mathbf{P}+\mathbf{P}\mathbf{G}_{ii}$ and its eigenvalues for all values of the matrix $\mathbf{P}$.

$\mathbf{P}$	$\mathbf{G}_{ii}^T \;\; \mathbf{P}+\mathbf{P}\mathbf{G}_{ii}\|_{i=1,2,\cdots,7}$
$\mathbf{P}=\begin{bmatrix} 8.8344 & 0.4908 \\ 0.4908 & 0.1043 \end{bmatrix}$	$\begin{bmatrix} -78.5276 & -08.3436 \\ -08.3436 & -02.7730 \end{bmatrix}$ Eigenvalues: -79.4357 and -1.8649
$\mathbf{P}=\begin{bmatrix} 8.3045 & 0.4346 \\ 0.4346 & 0.0984 \end{bmatrix}$	$\begin{bmatrix} -69.5284 & -07.3874 \\ -07.3874 & -02.6724 \end{bmatrix}$ Eigenvalues: -70.3349 and -1.8658
$\mathbf{P}=\begin{bmatrix} 7.9532 & 0.3973 \\ 0.3973 & 0.0944 \end{bmatrix}$	$\begin{bmatrix} -63.5624 & -06.7535 \\ -06.7535 & -02.6056 \end{bmatrix}$ Eigenvalues: -70.3349 and -1.8658
$\mathbf{P}=\begin{bmatrix} 7.8315 & 0.3844 \\ 0.3844 & 0.0931 \end{bmatrix}$	$\begin{bmatrix} -61.4960 & -06.5340 \\ -06.5340 & -02.5825 \end{bmatrix}$ Eigenvalues: -62.2120 and -1.8666

Table 6: The matrix $\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)^T\mathbf{P}+\mathbf{P}\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)$ and its eigenvalues for all values of the matrix $\mathbf{P}$.

$\mathbf{P}$	$\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)^T \;\; \mathbf{P}+\mathbf{P}\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)\Big\|_{i,j=1,2,\cdots,7(i<j)}$
$\mathbf{P}=\begin{bmatrix} 8.8344 & 0.4908 \\ 0.4908 & 0.1043 \end{bmatrix}$	$\begin{bmatrix} -78.5276 & -08.3436 \\ -08.3436 & -02.7730 \end{bmatrix}$ Eigenvalues: -79.4357 and -1.8649
$\mathbf{P}=\begin{bmatrix} 8.3045 & 0.4346 \\ 0.4346 & 0.0984 \end{bmatrix}$	$\begin{bmatrix} -74.0280 & -07.8610 \\ -07.8699 & -02.7227 \end{bmatrix}$ Eigenvalues: -74.8853 and -1.8654
$\mathbf{P}=\begin{bmatrix} 7.9532 & 0.3973 \\ 0.3973 & 0.0944 \end{bmatrix}$	$\begin{bmatrix} -71.0450 & -07.5411 \\ -07.5559 & -02.6893 \end{bmatrix}$ Eigenvalues: -71.8687 and -1.8657
$\mathbf{P}=\begin{bmatrix} 7.8315 & 0.3844 \\ 0.3844 & 0.0931 \end{bmatrix}$	$\begin{bmatrix} -70.0118 & -07.4304 \\ -07.4472 & -02.6778 \end{bmatrix}$ Eigenvalues: -70.8238 and -1.8658

4 CONCLUSIONS

A new fuzzy multiobjective control design methodology for nonlinear dynamic systems was proposed in this paper. In this approach, two techniques, which are pole placement and LQR methods widely used for linear systems, were combined and extended for nonlinear systems via state space Takagi-Sugeno fuzzy inference structure. Simulation results shown that the multiobjective feasible region allows the choosing the poles according to desired transient response and satifies the optimality criterion from LQR method as well. For further works, the following research interest can be considered:

- Industrial plants applications via high performance virtual/electronics instrumentation;
- Adaptive control design, once that analytical formulas for multiobjective fuzzy regulator were obtained;
- Multiobjective fuzzy regulator design for multivariable and/or time delayed nonlinear plants.

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