

Mechatronic System Optimization based on Surrogate Models

Application to an Electric Vehicle

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Abstract: Preliminary optimization of mechatronic systems is an extremely important step in the development process of multi-disciplinary products. However, long computing time in optimization based on multi-domain modelling tools need to be reduced. Surrogate model technique comes up as a solution for decreasing time computing in multi-disciplinary optimization. In this paper, an electric vehicle has been optimized by combining Modelica modelling language with surrogate model technique. Modelica has been used to model the electric vehicle and surrogate model technique has been used to optimize the electric motor and the transmission gear ratio. Results show that combining surrogate model technique with Modelica reduces significantly computing time without much decrease in accuracy.

1 INTRODUCTION

Mechatronic Systems (MS) are interdisciplinary products with a synergistic spatial and functional integration of mechanical, electronic and software sub-systems (Craig, 2009).

The great challenge in mechatronic design lies in optimizing a complete system with various physical phenomena related to interacting heterogeneous sub-systems.

Several multi-domain modelling tools such as Bond-Graphs, VHDL-AMS, Matlab/Simulink and Modelica are used for preliminary design of MS.

For instance, Modelica (Elmqvist et al., 1998) combines object-oriented concepts with multi-port methods for modelling and simulation of physical systems. It includes a declarative mathematical description of models and provides a graphical modelling approach. Multi-domain model library of lumped parameter elements can be created and added to the default Modelica library for future use. The end results of Modelica modelling is a system of differential-algebraic equations (DAE) that represents the complete mechatronic system. So that, Modelica is considered as an ideal tool for preliminary design of MS. However, optimizing a mechatronic system based on DAE is computationally expensive, due to the considerable number of simulation evaluations.

For this reason, substituting DAE system with surrogate models, using statistical methods, is one way

of alleviating this burden. The polynomial Response Surface Method (RSM) (Box and Wilson, 1951) is commonly considered as the first surrogate modelling technique. It uses a polynomial formulation to approximate an exact function. Other techniques such as Kriging (Kriging, 1951) and Artificial Neural Networks (ANNs) of Radial Basis Functions (RBF) (Hardy, 1971), (Hopfield, 1982) and (Powel, 1985) are also used to model complex relationships between inputs and outputs. Surrogate models are also known as metamodels (Blanning, 1975).

In this paper, both RMS and ANNs of RBF surrogate models have been generated from a Modelica model of an Electric Vehicle (EV). After their validation and comparison of their accuracy, the ANN of RBF surrogate model has been chosen to optimize the electric motor and the gear ratio of the EV. The optimization based on the surrogate model has been compared with an optimization based on the Modelica model. Results found are compared with a real case of an electric vehicle developed by general motors (GM EV1). Results show an interesting reduction in computing time of optimization without significantly affecting the accuracy.

2 SURROGATE MODELLING

RSM and ANNs of RBF have been used in this study due to their high accuracy and ease of use. The prin-

cial features of these two surrogate modelling techniques are described in the following section.

2.1 Response Surface Method

As it has been mentioned, RSM uses a polynomial formulation to approximate an exact function or a simulation process $F(X)$; where X is the input design vector that can be represented by a matrix as:

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \quad (1)$$

n and m are the number of design variables and the number of sample points, respectively. The approximation function $F_a(X)$ is therefore expressed as:

$$F_a(\mathbf{X}) = a + \sum_{i=1}^n b_i x_i + \sum_{i=1}^n c_i x_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^n d_{ij} x_i x_j + \dots \quad (2)$$

Polynomial degree in RSM models is frequently less than or equal to 2. Coefficients $a, b_i, c_i, d_{ij}, \dots$ are estimated by means of the least squares method.

2.2 Artificial Neural Network of Radial Basis Functions Method

In an ANN of RBF, a response $F(X)$ is approximated with $F_a(X)$ as a linear combination of radial basis functions Φ_j .

$$F_a(\mathbf{X}) = \sum_{j=1}^m \omega_j \cdot \Phi_j(\|\mathbf{X} - \mu_j\|) \quad (3)$$

μ_j are the centres of the radial functions and ω_j are the weighting coefficients that can be determined by the least squares method.

2.3 Validating a Surrogate Model

The surrogate model $F_a(X)$ and the approximated function $F(X)$ are related as follows:

$$F_a(\mathbf{X}) = F(\mathbf{X}) + \mathbf{E} \quad (4)$$

The validation of the surrogate model is performed through the evaluation of the residual vector \mathbf{E} . This evaluation can be made by calculating several quantities such as the mean error (Em), the root mean square error ($Erms$) and the maximum error ($Emax$), which are respectively expressed as:

$$Em_i = \frac{\left| \sum_{j=1}^m (F(x_{ji}) - F_a(x_{ji})) \right|}{m}; 1 \leq i \leq n \quad (5)$$

$$Erms_i = \frac{\sqrt{\sum_{j=1}^m [F(x_{ji}) - F_a(x_{ji})]^2}}{m}; 1 \leq i \leq n \quad (6)$$

$$Emax_i = \max_j (|F(x_{ji}) - F_a(x_{ji})|); 1 \leq i \leq n; 1 \leq j \leq m; \quad (7)$$

3 APPLICATION: ELECTRIC VEHICLE OPTIMIZATION

Figure 1 shows EV model which has been performed using Modelica language. The EV is composed of a battery (312 V), a power converter, a controller, an electric motor and a transmission. The objective of

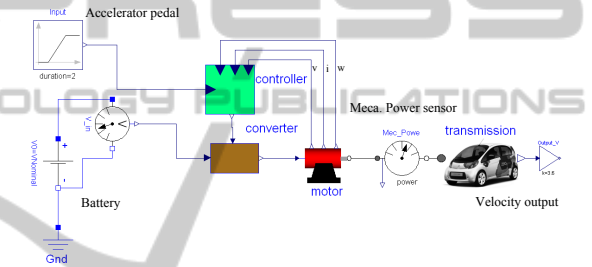


Figure 1: Electric vehicle model (Modelica).

this study is to optimize the maximum electric power Pe_{Max} of the motor and the overall transmission gear ratio G_r . This optimization should respect a performance constraint of acceleration to reach a velocity $V_{10} = 100 \text{ km/h}$ in 10 seconds. The electric motor is modelled by the following equations (parameters and variables are described in Tables 1 and 2).

$$k \cdot \omega_m = V_{emf} \quad (8)$$

$$T_m = k \cdot i \quad (9)$$

$$V_e = L \cdot \frac{di}{dt} + R \cdot i + V_{emf} \quad (10)$$

$$P_e = V_e \cdot i \quad (11)$$

The transmission model is defined by:

$$F_t = M \cdot g \cdot f_r + \frac{1}{2} \cdot d_{air} \cdot C_D \cdot A_f \cdot v^2 \quad (12)$$

$$+ (M + I \cdot \frac{G_r^2}{\eta_g \cdot r^2}) \cdot a + M \cdot g \cdot \sin(\alpha)$$

$$P_m = F_t \cdot v \quad (13)$$

For the purpose of comparing the simulation results with a real case, we chose values that match the electric vehicle EV1 of General Motors (GM), which are

Table 1: Electric vehicle parameters.

parameter	value
Vehicle mass $M(kg)$	1540
Gravity acceleration $g(m/s)$	9.81
Coefficient of rolling resistance f_r	0.0048
Density of the air $d_{air}(kg/m^3)$	1.205
Drag coefficient C_D	0.19
Frontal area $A_f(m^2)$	1.8
Moment of inertia of the motor $I(kg.m^2)$	0.03
Radius of the tyre $r(m)$	0.3
Slope angle $\alpha(rad)$	0
Motor characteristics $k(N.m/A)$	0.2
Motor internal resistance $R(\Omega)$	0.02
Motor inductance $L(H)$	0.01
Gear system efficiency η_g	0.95
Gear ratio G_r	opt.
Motor maximum power $Pe_{Max}(W)$	opt.
Motor maximum speed $\omega_M(rad/s)$	const.
Motor critical speed $\omega_C(rad/s)$	const.

published in (Larminie and Lowry, 2003). These parameters are given in Table 1. As indicated in Table 1, the maximum electric power Pe_{Max} and the gear ratio of the system connecting the motor to the axle G_r are to be optimized. The maximum motor speed ω_M and the critical speed ω_C are defined as constraints. ω_C is the minimum speed above which the electric motor powers the transmission efficiently. Table 2 gives the variables which are dependant of time during simulation. We introduce an optimizing objective variable

Table 2: Electric vehicle variables.

Motor rotational speed $\omega(rad/s)$	
Electromotive forces $V_{emf}(V)$	
Motor torque $T_m(N.m)$	
Electric current in motor $i(A)$	
Voltage input $V_e(V)$	
Tractive effort $F_t(N)$	
Vehicle velocity $v(m/s)$	
Vehicle acceleration $a(m/s^2)$	
Electric motor instant power $P_e(W)$	
Mechanical power $P_m(W)$	
Velocity at 10 seconds $V_{10}(km/h)$	$\approx 100km/h$

$V_{10}(km/h)$ that represents the vehicle velocity at 10 seconds. The goal is to reach $100km/h$. In the acceleration test, the controller and the power converter act to power the electric motor with current in order to provide a torque T_m defined as:

$$\begin{cases} T_m = p_a \cdot Pe_{max} / \omega_C & (\text{if } \omega \leq \omega_C) \\ T_m = p_a \cdot Pe_{max} / \omega_M & (\text{if } \omega \geq \omega_M) \\ T_m = p_a \cdot Pe_{max} / \omega & \text{else} \end{cases} \quad (14)$$

The percentage of acceleration (p_a), during the acceleration test, reaches 100% in 2 seconds.

Based on the precedent mathematical formulation, a Modelica model of the EV has been elaborated to be simulated on an interval time of 30 seconds. The input design vector X for the surrogate models is defined as:

$$X = [G_r, Pe_{Max}, \omega_c, \omega_M] \quad (15)$$

The output variable is defined by V_{10} which is determined by the Modelica simulations at every input design point.

The design space limits for X are defined by : $2 \leq G_r \leq 13$; $90000 \leq Pe_{Max} \leq 110000$; $600 \leq \omega_C \leq 900$; $1000 \leq \omega_M \leq 1400$.

The sample points of the design space domain have been determined using the Design of Experiment (DoE) technique with a Latin Hypercube method (Mackay et al., 1979).

Automatic evaluation of the output variable has been performed using iSIGHT software¹, which has been also used to elaborate both RSM and ANN of RBF surrogate models.

The formulation of the optimization problem is defined as follows:

$$\begin{cases} \min Pe_{Max}, \text{ for } 90000 \leq Pe_{Max} \leq 110000 \\ \min G_r, \text{ for } 2 \leq G_r \leq 13 \\ \max V_{10}, \text{ for } 98 \leq V_{10} \leq 101 \\ 1100 \leq \omega_M \leq 1400 \\ 600 \leq \omega_C \leq 900 \end{cases} \quad (16)$$

To solve the non-linear multi-objective optimizing problem, a sequential quadratic programming algorithm called Non-Linear Programming by Quadratic Lagrangian (NLPQL) (Schittkowski, 1985) has been used. NLPQL is well suited for non-linear problems with few objectives and continuous design space.

4 RESULTS AND DISCUSSION

In this paper, we have combined Modelica modelling technique with surrogate model method for modelling and optimization of an electric vehicle. Both RSM and ANN of RBF surrogate models have been elaborated.

Table 3 gives the evaluation of residual vector \mathbf{E} for both RSM and ANN of RBF models.

Results show that ANN of RBF surrogate model presents a better accuracy than RSM model. Figure 2 shows the distribution of residuals for the both surrogate models. The results of validation prove the ca-

¹<http://www.simulia.com/products/isight.html>

Table 3: Residuals in RMS and RBF models.

	Em (%)	Erms (%)	E _{max} (%)
RSM	4.1	5.4	11.9
RBF	2.1	2.8	7.1

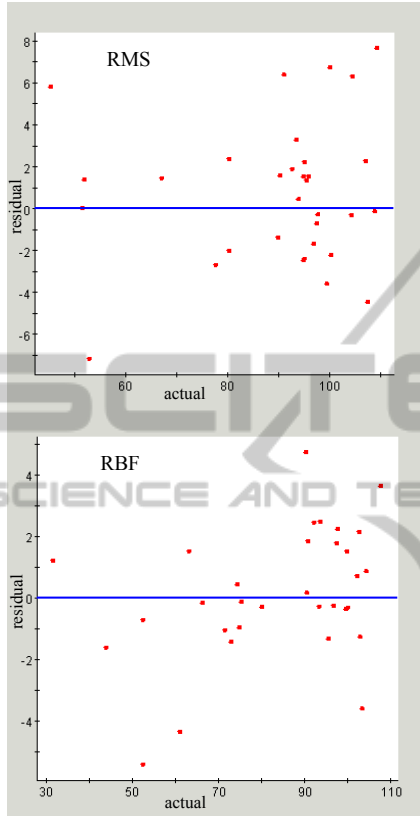


Figure 2: Residual distribution of surrogate models (RSM and RBF).

capacity of RBF models to approximate non linear responses. However, RSM model is simpler and easier to exchange between modelling platforms. In our case, the algebraic model elaborated for the RSM surrogate model is:

$$\begin{aligned}
 V_{10} = & -81.29 + 19.G_r + 0.002.Pe_{max} - 0.0847.\omega_c \\
 & + 0.0383.\omega_M - 1.1942.G_r^2 + 0.007.G_r.\omega_c \\
 & + 0.0003.G_r.\omega_M
 \end{aligned}
 \tag{17}$$

Such simple mathematical model is useful for system engineers at high abstraction level for system analysis and decision making.

A response surface ($V_{10} = F(G_r, Pe_{Max})$) for the RBF surrogate model is given by Figure 3. This response surface shows that V_{10} reaches a maximum, which justifies the need to optimize V_{10} .

Table 4 shows results of optimization based on

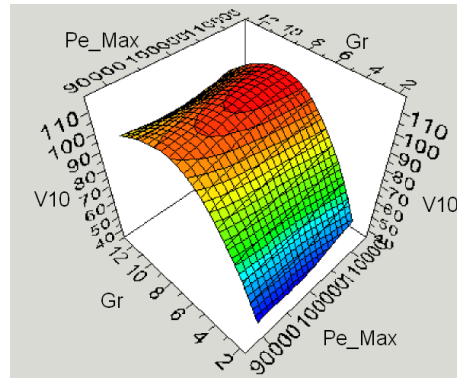


Figure 3: Response surface for ANN of RBF surrogate model: $V_{10} = F(G_r, Pe_{Max})$.

ANN of RBF surrogate model and a comparison with an optimization based on the Modelica model. Results show a good agreement between the two op-

Table 4: Optimization results.

	RBF	Modelica	Err.(%)
G_r	9.15	10.1	9.4
$Pe_{Max}(kW)$	102	98	5.2
$V_{10}(km/h)$	99	100.7	1.7
$\omega_C(rad/s)$	828	780	6.1
$\omega_M(rad/s)$	1112	1154	3.6
Computing time(s)	4	332	

timizations with an important gain of computing time for the optimization based on the ANN of RBF surrogate model (4 seconds for RBF surrogate model and 332 seconds for Modelica model).

These results are close to those published for the electric vehicle GM EV1. Indeed, for EV1 the electric motor has a power $Pe_{Max} = 100 kW$, maximum speed of $12000 rpm (\omega_M = 1256 rad/s)$, $\omega_C = 600 rad/s$ and a transmission gear ratio $G_r = 11$ (Larminie and Lowry, 2003). The choice of G_r affects the performance of acceleration but also has an influence on other requirements such as the maximum vehicle velocity, which has not been considered in this optimization. This explains the little difference between values of G_r found by optimization and the value chosen by GM for EV1.

To verify the performance constraint of acceleration test with the case of EV1 vehicle, we have fixed the following values in the Modelica model: $Pe_{Max} = 100 kW$; $\omega_C = 600 rad/s$; $\omega_M = 1256 rad/s$ and $G_r = 11$.

Figure 4 shows the input signal of pedal acceleration and the output vehicle velocity. This figure confirms that the electric vehicle reaches a velocity of 100 km/h in 10 seconds.

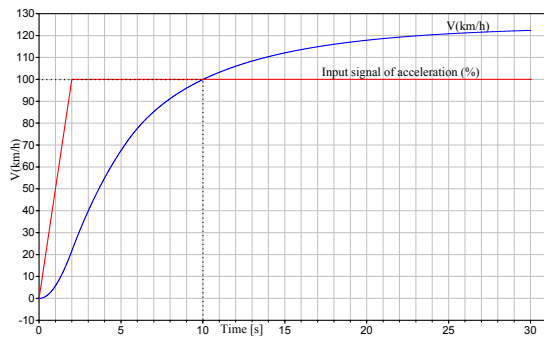


Figure 4: Results of Modelica simulation during an acceleration test of the EV (accelerator signal and vehicle velocity).

Figure 5 shows the variation of the electric current and power during the performance test of acceleration. The electric current reaches a maximum of 330 A and the electric power reaches a maximum of 98 kW. The value of the maximum current helps in the choice of the battery. The simulation near the opti-

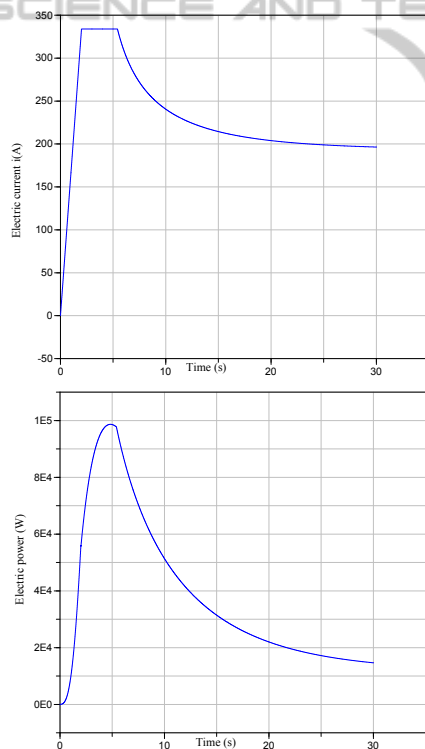


Figure 5: Electric current (top) and electric power (bottom) during acceleration test of the EV.

mal solution is performed to verify the output design variables. To improve the optimization accuracy, it is possible to perform a second optimization on the Modelica Model, but near the optimal solutions given by the optimization based on the surrogate models. In

this case, surrogate models play a role of assistant in optimization. This alternative has longer time of computing but with a better precision, and in most cases time will be shorter than direct optimization on Modelica model without knowing the neighbourhoods of the optimal solution.

5 CONCLUSIONS

An electric vehicle has been modelled using Modelica language. This model has been used as a support to develop ANN of RBF and RSM surrogate models. A comparison of accuracy of the two surrogate models has been made. It confirms that ANN of RBF method is more accurate than RSM in the case of electric vehicle modelling. ANN of RBF surrogate model has been used to optimize the electric motor and the gear ratio of the EV. Results show the important gain in computing time compared to direct optimization based on the Modelica model, without affecting a lot accuracy. Results of simulation in the case of acceleration test have been compared to a real case of an EV (GM EV1), and results show a good agreement with those published.

Thus, combining Modelica and surrogate modelling techniques is an effective method to reduce design time and minimize complexity of optimizing mechatronic systems.

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