

An Integrated Approach for Efficient Mobile Robot Trajectory Tracking and Obstacle Avoidance

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Abstract: An approach for nonholonomic two-wheeled mobile robot trajectory tracking and obstacle avoiding is presented in this paper. If the desired trajectory is provided by high level planner, trajectory tracking problem can be solved in various ways. In this paper, tracking is provided using proportional-integral (PI) or fuzzy logic controller (FLC). Unfortunately, tracking is never perfect, due to uncertainties and obstacles can change their positions in time. In order to overcome these difficulties, additional correction controller must be used. Here is proposed fuzzy controller, which slightly changes control action of the tracking controller in order to prevent collision with obstacles. This approach is proved to be efficient even in dynamic environments. Simulation results are presented as illustration of the proposed approach.

1 INTRODUCTION

In recent years, due to growing popularity and importance of wheeled mobile robots (WMRs) in many applications, motion control problems dedicated to WMRs attracted great attention. Trajectory tracking problem can be considered as a part of mobile robot navigation problem, which has been intensively researched, e.g. (Laumond, 1998; LaValle, 2006; Masehian and Sedighizadeh, 2007).

Considerable research efforts have been made on trajectory tracking control of two-wheeled differentially driven mobile robots. Despite the apparent simplicity of the WMR kinematic model, the design of stabilizing control law is challenging due to the existence of nonholonomic constraints.

Variou control strategies have been presented such as: sliding-mode control, e.g. (Bloch and Drakunov, 1994), backstepping procedure, e.g. (Taner and Kyriakopoulos, 2003), dynamic feedback linearization, e.g. (Oriolo et al., 2002), Lyapunov-type techniques, e.g. (Mastellone et al., 2008), adaptive control, e.g. (Fukao et al., 2000), model predictive control, e.g. (Kühne et al., 2005) and intelligent techniques, based on neural networks and fuzzy logic, e.g. (Jiang et al., 2005; Oh et al., 2005).

In general, closed-loop results obtained using classic control approaches may present undisable oscillatory motions. From the other hand, fuzzy logic may be good option for uncertain systems,

whose behaviour can be described linguistically. In this paper, two tracking controllers will be designed, nonlinear PI and fuzzy controller. Unfortunately, due to uncertainties and obstacles movements, collision with obstacles could happen even if the high level planner provided collision free path. It is the reason why additional fuzzy controller must be introduced, which will correct the control action of the tracking controller, when mobile robot comes close enough to the obstacle.

The rest of the paper is organized as follows: description of the WMR kinematic model is given in Section 2, design of the control structure in Section 3, simulation results in Section 4, while the conclusion is given in Section 5.

2 KINEMATIC MODEL OF THE TWO-WHEELED MOBILE ROBOT

Schematic model of WMR is shown on Figure 1. Derivation of the kinematic equations of the two-wheeled mobile robot is given in (Susic et al., 2011). World coordinate frame is denoted by $\{X, O, Y\}$, while $\{x_b, COM, y_l\}$ denotes local coordinate frame, attached to the robot, whose origin is placed at the robot's centre of mass (COM). State variables are position and orientation of the robot, i.e. COM

position (x, y) and angle ϕ between x axes of the world and local coordinate frame, while ω_L and ω_D denote angular velocities of the left and right side wheels of the robot, respectively, and represent control inputs, while v_c denotes COM linear velocity and $\dot{\phi}$ denotes robot angular velocity around COM. If \dot{x} and \dot{y} denote projections of velocity vector v_c onto coordinate axis of global coordinate system, kinematic model of the mobile robot is given by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} \cos \phi & \frac{r}{2} \cos \phi \\ \frac{r}{2} \sin \phi & \frac{r}{2} \sin \phi \\ -\frac{r}{2b} & \frac{r}{2b} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_D \end{bmatrix} \quad (1)$$

3 DESIGN OF THE CONTROL SYSTEM

Proposed controller consists of three parts: trajectory tracking controller (TTC), obstacle avoiding controller (OAC) and combined controller (CC). It is assumed that the collision free trajectory is already provided by high level planner, i.e. virtual vehicle trajectory is known. TTC provides tracking of the desired trajectory. For this purpose two controllers will be presented, nonlinear PI and FLC. PI controllers are simple and widely used in industrial practice, while FLCs are intelligent control strategies which are proved to be efficient in control of complex systems. Main drawback of FLCs is large number of parameters which has to be adjusted. Tracking is never perfect, so, at this point, it is not ensured that robot will pass from starting to destination point safely. For this purpose fuzzy OAC

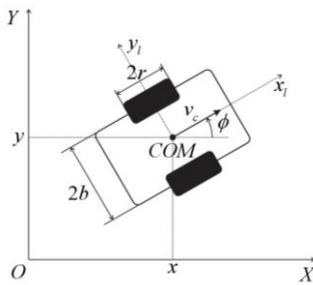


Figure 1: Kinematic model of mobile robot.

is proposed, which generates correction control signal which moves robot away from the obstacle. The last part of the control structure is combined controller. Its role is to combine the control signals obtained from TTC and OAC into control inputs of

the mobile robot, i.e. to make compromise between “tracking” and “avoiding” action of the controller.

3.1 Design of the Trajectory Tracking Controllers

3.1.1 Nonlinear Pi Controller

Tracking controller generates control action which tries to direct robot to the desired trajectory. Let (x, y, ϕ) denote robot position and orientation, (x^*, y^*, ϕ^*) desired position and orientation at the same time instant, and \mathbf{v}^* desired velocity. Velocity generated by controller is denoted by \mathbf{v}_z and can be obtained as:

$$\mathbf{v}_z = \mathbf{v}^* + \Delta \mathbf{v}_z, \quad \Delta \mathbf{v}_z = \begin{cases} \frac{K_p (\|\mathbf{e}_z\| - d)}{\|\mathbf{e}_z\|} \left(\mathbf{e}_z + \frac{1}{T_i} \mathbf{e}_{zi} \right), & \|\mathbf{e}_z\| > d \\ 0, & \|\mathbf{e}_z\| \leq d \end{cases} \quad (2)$$

$$d = d_0 - \min \{ d_0, \|\mathbf{v}^*\| / k \}$$

where $\Delta \mathbf{v}_z$ is velocity correction, k is positive gain and d is the dead-zone size, dependent on \mathbf{v}^* . Tracking error and integral of tracking error are denoted by $\mathbf{e}_z = [x^* - x \quad y^* - y]^T$ and $\mathbf{e}_{zi} = \int \mathbf{e}_z dt$, respectively. Proportional gain is denoted by K_p , while T_i stands for integral constant.

Velocity correction is nonlinear function of errors sum, i.e. dead-zone around desired point is introduced. Introducing nonlinearity is necessary, because it does not allow oscillations of robot's position near the desired point. Dead-zone size decreases when desired velocity increases. Parameter d_0 determines the maximal value of tracking error near the destination point.

Angular velocities of the motors (ω_L, ω_D) are weighted sums of the linear and angular velocities of the robot ($v_c, \dot{\phi}$). So, control generated by controller (ω_{Lt}, ω_{Dt}) is given by:

$$\omega_{Lt} = a_v \|\mathbf{v}_z\| - a_\phi \Delta \phi_z, \quad \omega_{Dt} = a_v \|\mathbf{v}_z\| + a_\phi \Delta \phi_z \quad (3)$$

where $\|\mathbf{v}_z\|$ and ϕ_z denote magnitude and angle of the velocity vector \mathbf{v}_z given by (2), $\Delta \phi_z$ approximates derivative of the ϕ_z , while weights a_v and a_ϕ are control parameters. These parameters weight straight-line and turning capabilities. Derivative approximation $\Delta \phi_z$ is given by:

$$\Delta \phi_z = \begin{cases} \frac{\phi_z - \phi}{T_s}, & \|\mathbf{e}_z\| > d \\ \frac{\phi^* - \phi}{T_s}, & \|\mathbf{e}_z\| \leq d \end{cases} \quad (4)$$

where T_s denotes the sampling time. It can be seen from (4) that controller tries to align orientations of the real and virtual robot when they are close enough, i.e. if their distance is less or equal d .

3.1.2 Fuzzy Logic Controller

Controllers based on fuzzy logic are proved to be efficient in control of complex systems, where other control strategies do not provide satisfactory performance. FLCs try to mimic action of experienced operator.

Proposed controller is Takagi-Sugeno-Kang (TSK) type and has three inputs (“distance to virtual vehicle” - Euclidian distance between real and virtual COM position, “angle” - angle at which real robot sees the virtual and “orientation difference” - difference between virtual and real robot orientations) and two outputs (“linear velocity” - linear velocity of the WMR, normalized on [0,1] and “angular velocity” - angular velocity of the WMR, normalized on [-1,+1]). Membership functions of the linguistic values of inputs and outputs are given on the Figures 2 and 3.

According to (3), outputs of the whole FLC can be obtained as:

$$\omega_{Dt} = K_v \bar{v}_t + K_\omega \bar{\omega}_t, \omega_{Lt} = K_v \bar{v}_t - K_\omega \bar{\omega}_t \quad (5)$$

where \bar{v}_t and $\bar{\omega}_t$ are normalized linear and angular velocities produced by the fuzzy inference mechanism, while K_v and K_ω are weights which give the relative importance to straight forward and turning capabilities.

Fuzzy rule base is given by Table 1. Membership functions are represented by abbreviations, defined on Figure 3. Also, every cell is represented by two linguistic values. The first one corresponds to the “linear velocity”, while the second one corresponds to the “angular velocity”. As can be seen from Table 1, there are two sets of fuzzy rules. The first one takes “distance to virtual vehicle” and “angle” as inputs. This set of rules is active when robot is not close enough to the desired point, and controller tries

Table 1: Fuzzy rule base of the TTC.

		“angle”				
		BL	FL	F	FR	BR
“distance to virtual vehicle”	N	Z/HP	S/Z	S/Z	S/Z	Z/HN
	M	Z/FP	M/HP	M/Z	M/HN	Z/FN
	F	Z/FP	L/HP	L/Z	L/HN	Z/FN
		“orientation difference”				
		Z/HN	Z/HN	Z/Z	Z/HP	Z/HP
	VC	LN	MN	S	MP	LP

to bring the robot close to this point. Second set of rules takes “distance to virtual vehicle” and “orientation difference” as inputs. This set of rules is active when robot comes close enough to the desired point, trying to align robot’s and desired orientation. Thus, the idea is to introduce set of rules which keeps orientation of the real and virtual robot aligned when they are close enough.

3.2 Design of the Obstacle Avoiding Controller

Path planning algorithm in complex scenarios with large number of obstacles might generate path that guides robot very close to the obstacles. Tracking is not perfect, so obstacle avoidance is not ensured yet.

Obstacle avoiding fuzzy controller is two-input (“distance to obstacle” and “obstacle viewing angle”) and one-output (“angular velocity

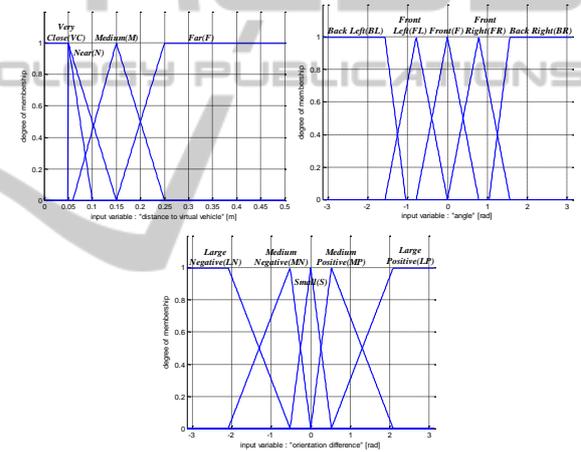


Figure 2: Membership functions of the TTC’s inputs.

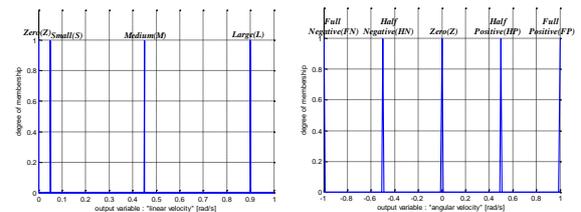


Figure 3: Membership functions of the TTC’s outputs.

correction”, normalized on [0,1] system. Meaning of the fuzzy inputs and output is similar as in TTC design. Correction should be generated such that mobile robot moves away from the obstacle, but only when it comes close enough to it.

Membership functions of the fuzzy inputs and output are given on Figures 4 and 5. Output of the whole OAC is:

$$\Delta\omega = K_{\Delta}\Delta\bar{\omega} \quad (6)$$

where $\Delta\bar{\omega}$ is output of the fuzzy inference system, and K_{Δ} is output gain.

Fuzzy rules are given by Table 2. Abbreviations are also used for membership function representation. Avoiding action is the strongest when the obstacle is straight ahead of the robot.

Table 2: Fuzzy rule base of the OAC.

		"obstacle viewing angle"					
		BR	R	FR	FL	L	BL
"distance to obstacle"	N	SP	HP	FP	FN	HN	SN

3.3 Design of the Combined Controller

The task of the CC is to combine outputs from the TTC and OAC in order to obtain control signals of the mobile robot. It is basically a weighed sum of TTC and OAC outputs, whose weights depend on distance between robot and obstacle, i.e. it gives relative importance to "tracking" and "avoiding" action. If the robot is close enough to the obstacle, OAC output becomes dominant, otherwise TTC output is dominant. The outputs of the CC are given by:

$$\omega_L = K_1\omega_{Lt} - K_2\Delta\omega, \quad \omega_D = K_1\omega_{Dt} + K_2\Delta\omega \quad (7)$$

where K_1 and K_2 denote the weights, given by (8), ω_{Lt} and ω_{Dt} are the outputs of the TTC and $\Delta\omega$ is the output of the OAC.

$$K_1 = \min\left(1, \frac{1 - K_1^{\min}}{d_c}d + K_1^{\min}\right) \quad (8)$$

$$K_2 = \max\left(0, -\frac{K_2^{\max}}{d_c}d + K_2^{\max}\right)$$

where d denotes distance between robot and obstacle, d_c denotes minimum distance to obstacle when OAC becomes active, K_1^{\min} is the minimum contribution of the tracking signal in the overall control, while K_2^{\max} is the maximum contribution of the avoiding in the overall control.

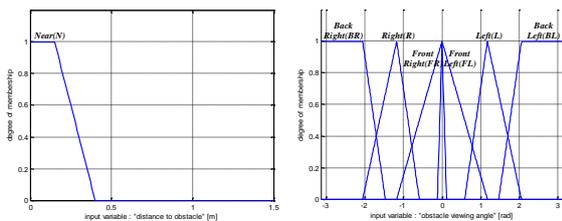


Figure 4: Membership functions of the OAC's inputs.

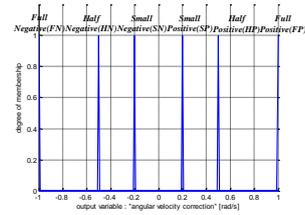


Figure 5: Membership functions of the OAC's output.

4 SIMULATION RESULTS

Proposed algorithm for mobile robot trajectory tracking is implemented in MATLAB package.

Environment with seven circular obstacles is adopted. Tracking performance will be presented in two different scenarios. In the first scenario, planner knows exact position of the obstacles, so generated desired trajectory is guaranteed to be collision free. In further text, this scenario will be denoted by Scenario I. In the second scenario, some obstacles slightly changed their positions, while desired trajectory remained the same (in further text denoted by Scenario II). Mean-square error (MSE) is adopted as a measure of tracking quality, and MSE values obtained in these scenarios are given by Table 3.

It is assumed that robot width is $2b = 30$ cm and wheel radius is $r = 6$ cm. Maximum angular velocities of the wheels are $\omega_L^{\max} = \omega_D^{\max} = 15$ rad/s. It is assumed that the robot position and orientation measurements are corrupted with white Gaussian noise, which standard deviations are 1cm and 1° , respectively. Starting point is (2,4.5)m, while the destination point is (4.5,0.5)m for the virtual robot. Starting point and orientation of the real robot is $(x_0, y_0, \phi_0) = (1.85\text{m}, 4.3\text{m} - \frac{3\pi}{4}\text{rad})$.

Results obtained in the Scenario I have been presented on Figures 6, 7 and 8. Error on the Figures 6, 7, 9 and 10 is defined as distance between desired (x^*, y^*) and robot's position (x, y) at the same time instant. Snapshots of the vehicles on Figure 8 have been taken at the following time instants: (0,3,6,10,15,35)s. Virtual robot is presented by blue dotted line, real robot with PI TTC controller with red solid line, while the one with FLC TTC with green solid line. Parameters of all controllers have been adjusted experimentally. PI controller parameters are as follows: $K_p = 6, T_i = 0.5, k = 10, d_0 = 0.05\text{m}, a_v = 66.68, a_\phi = 9.335$. PI outputs have been filtered with simple first-order continuous filter whose transfer function is $G_F(s) = \frac{1}{0.2s+1}$. Choice of the controller parameters is critical.

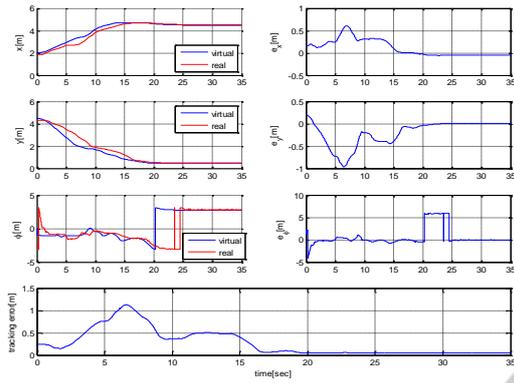


Figure 6: Tracking errors with PI TTC and OAC in Scenario I.

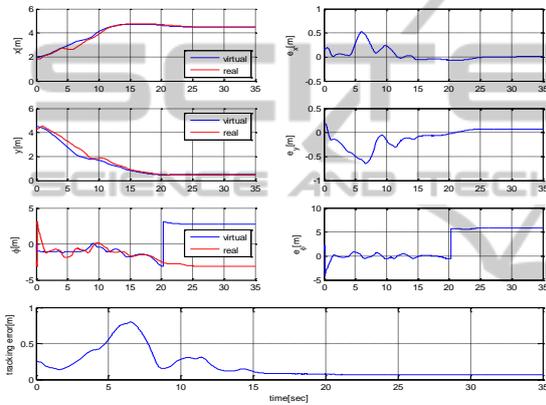


Figure 7: Tracking errors with FLC TTC and OAC in Scenario I

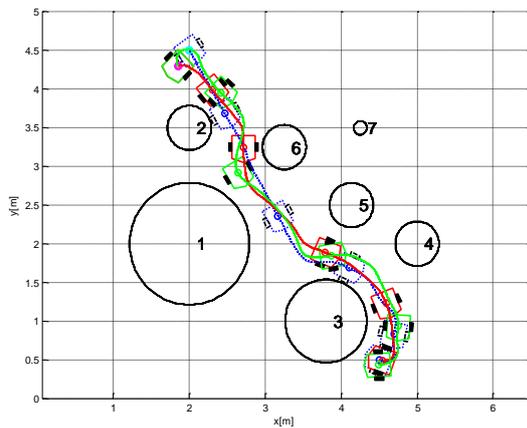


Figure 8: Comparative 2D view of robots motion in Scenario I.

Increase of proportional gain K_p enhances the tracking performance, but increases the presence of noise in control signals. Decrease of T_i decreases the tracking error, but may lead to instability.

Parameters a_v and a_ϕ weight straight-line and turning capabilities, so larger value of a_ϕ is recommended. Gains of the FLC are also chosen as: $K_v = K_\omega = 15$. These parameters weight straight motion and turning capabilities, respectively, so larger values of K_ω are advisable, in order to ensure good tracking in sharp curves.

Output gain of the OAC is adopted as $K_\Delta = 13.5$, while $K_1^{min} = 0.4$ and $K_2^{max} = 0.6$. This means that the minimal “tracking” contribution is 40%, while maximal “avoiding” contribution is 60% in overall control action. Critical distance to obstacle on which CC modifies “tracking” control action with “avoiding” contribution is $d_c = 45\text{cm}$. Figure 8 shows that the “avoiding” contribution degrades the quality of tracking near the obstacles, but robots move away from the obstacles, decreasing the risk of collision. In this case, FLC used as TTC is better solution.

Results obtained in Scenario II, when obstacles 1, 2, 3 and 6 changed their positions slightly are shown on Figures 9, 10 and 11. Snapshots of the vehicles have been taken at the following time instants: (0,3,7,13,35)s. FLC used as TTC provides better result again.

Table 3: Mean-square error in different scenarios.

Scenario	MSE value [cm]	
	PI	FLC
Scenario I	26.9	17.6
Scenario II	51.9	22.7

5 CONCLUSIONS

The solution of trajectory tracking with obstacle avoiding is presented in this paper. Although it is assumed that planner which provides collision free time-parameterized path is available, it is not necessary. It is enough that only “sketch” of the trajectory is provided, and OAC will correct control action and push mobile robot away from the obstacles. Proposed scheme can be used in different scenarios with obstacles of arbitrary shape. This approach can be applied even in dynamic environments in which exist moving obstacles. The proposed algorithms will be implemented in real time control of 4WD mobile robot platform.

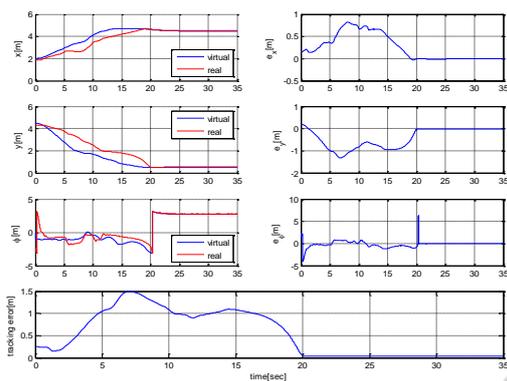


Figure 9: Tracking errors with PI TTC and OAC in Scenario II.

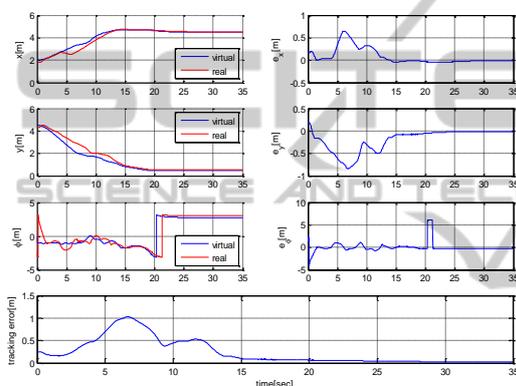


Figure 10: Tracking errors with FLC TTC and OAC in Scenario II.

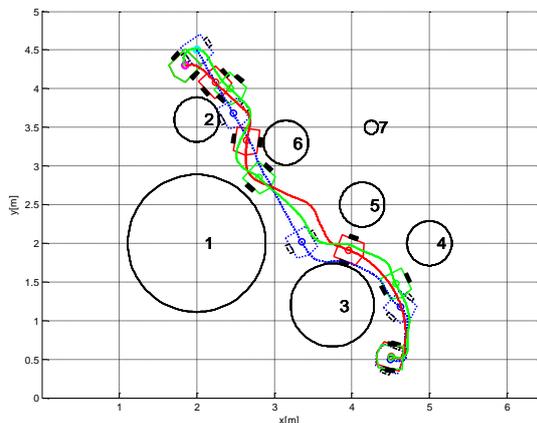


Figure 11: Comparative 2D view of robots motion in Scenario II.

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