Predictive Control of Unmanned Formations

Martin Saska and Libor Přeučil
Department of Cybernetics, Czech Technical University in Prague, Technická 2, Prague, Czech Republic

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Abstract: A *receding horizon control* based approach for guiding of autonomous formations of nonholonomic robots in a leader-follower constellation is proposed in this paper. The presented method ensures dynamic obstacle avoidance, formation coordination as well as failure tolerance. The robustness of the algorithm is verified by numerical multi-robot experiments. Besides, effects of system's parameters on the algorithm performance are investigated.

1 INTRODUCTION

Formation driving of nonholonomic robots in an environment with dynamic obstacles are required in many robotic applications. As an example, one can mention airport snow shovelling by formations of snow plows (Saska et al., 2008) being our target application. In these tasks, the trajectory planning for the whole formation and the formation maintenance and stabilization have to be encapsulated into a complex system being able of responding to dynamic environment and unforeseen events.

Plenty of studies investigating formation control and planning have been published recently, e.g. (Zhang, 2010; Liu and Jia, 2012). These algorithms are mostly focused on tasks of formation following a predefined path and formation stabilization in desired positions. However, there is a lack of adequate methods in the literature for providing flexible control inputs for members of the formation responding to the dynamic environment and handling together optimality and stability of the *leader-to-goal* and *followers-in-formation* tasks. We propose a novel method solving these challenges.

In the proposed system, the response to unforeseen events such as appearing dynamic obstacles or failures of team members is ensured by Receding Horizon Control (RHC), which is also known as the model predictive control. RHC is an optimization based technique often used for stabilizing linear and nonlinear dynamic systems. For a detailed survey of RHC methods we refer to (Mayne et al., 2000) and references reported therein. The works applying RHC for the formation driving are presented in (Dunbar and Murray, 2006; Chen et al., 2010). Algorithms in these papers have utilized RHC for the formation stabilization and/or following predefined trajectory in a workspace without obstacles.

Our contribution is a new concept of RHC that combines both, the trajectory planning to a goal region for the entire group and the maintenance and stabilization of the formation, into one optimization process via an additional planning horizon. This approach enables to navigate the formation in such a way that the local image as well as the overall structure of the environment are appropriately incorporated. On top of that, the contribution of this paper lies in the study of RHC's parameters on the algorithm performance.

2 METHOD DESCRIPTION

Leader Trajectory Planning and Control. The main idea of the receding horizon control is to solve a moving finite horizon optimal control problem for a system starting from current state or configuration $\psi(t_0)$ over the time interval $[t_0, t_N]$ under a set of constraints on the system states and control inputs. In this framework, the length $t_N - t_0$ of the time interval $[t_0, t_N]$ is known as the control horizon. As a result of the optimal control loop, a sequence of N states of the system (transition points) is found. Between these points, the control inputs, which navigate the robot from one transition point to the following one, are constant. After a solution from the optimization problem is obtained on a control horizon, a portion of the computed control actions is ap-

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plied on the interval $[t_0, \Delta tn + t_0]$, known as the receding step. This process is then repeated on the interval $[t_0 + \Delta tn, t_N + \Delta tn]$ as the finite horizon moves by *time steps* defined by the sampling time Δtn, yielding a state feedback control scheme strategy. Advantages of the RHC scheme become evident in terms of adaptation to unknown events and change of strategy depending on new goals.

In the presented approach, we propose to solve the collision free trajectory planning and the optimal control together in one optimization step. We extend the standard RHC method with one control horizon into an approach utilizing two finite time intervals T_N and T_M. The first time interval T_N should provide immediate control inputs for the formation regarding the local environment. The difference $\Delta t(k+1) = t_{k+1} - t_k$ between transition points is kept constant in this time interval. The second interval T_M takes into account information about the global characteristics of the environment to navigate the formation to the goal. The transition points in this part can be distributed irregularly to effectively cover the environment. During the optimization process, more points are automatically allocated in the regions where a complicated maneuver of the formation is needed. This is enabled due to the varying values of time $\Delta t(k+1) = t_{k+1} - t_k$ between the transition points. Both these control intervals, T_N and T_M together form a trajectory Ω from an actual position of the robot into a desired target through $N + M$ transition points.

The trajectory planning and the static as well as dynamic obstacle avoidance problem can be then transformed to the minimization of a single cost function $J(\Omega)$ subject to sets of constraints. During the optimization, both control vectors and transition points act as variables and can be optimized to get the desired solution. The proposed cost function consists of three parts as $J(\Omega) = J_{total\_time}(\Omega) + \alpha J_{obst\_dist}(\Omega) + \beta J_{deviation}(\Omega)$.

The endeavor of the trajectory planning to reach a desired goal as soon as possible is expressed in the first part, which represents the total time required for reaching the goal if using the trajectory Ω. It is a sum of time differences $\Delta t(\cdot)$ between all transition points of Ω. The second part $J_{obst\_dist}(\Omega)$ is an avoidance function, which contributes to the final cost if an obstacle is closer to the trajectory than a certain detection radius and it approaches infinity if distance to the closest obstacle is equal to an avoidance radius. The part $J_{deviation}(\Omega)$ is employed only if it is required to follow a preferred path during reaching the target (as the following of runway axes shown in Fig. 2-5). This part represents the biggest deviation of a transition point from the desired path to follow. If the aim of the planning is to reach the target independently on a desired path, this part is neglected (as shown in Fig. 1). The influence of all parts of the cost function is adjusted by constants α and β.

The minimization of the cost function is subject to a set of equality constraints representing a kinematic model of the utilized vehicles. This satisfies that the obtained trajectory stays feasible with respect to kinematics of nonholonomic robots. Besides, it is subject to a set of inequality constraints that characterizes bounds on the velocity and curvature of the virtual leader. These bounds are determined by the shape of the formation and motion constraints of each of the follower. Finally, a stability constraint guaranteeing that the obtained trajectory will enter the target has to be employed. This inequality constraint represents distance between the target and the last transition point of Ω. The constraint is satisfied if this distance is bellow a given threshold.

Trajectory Tracking for the Followers. The presented approach relies on the well known leader-follower method (Barfoot and Clark, 2004), where the followers track the leader's trajectory, which is distributed within the group. The followers are maintained in relative distance to the leader in curvilinear coordinates. Employing this concept, the trajectory computed as the result of the previous section will be used as an input of the trajectory tracking for the followers. We apply the classical RHC based method with one control interval T_N for a discrete-time trajectory tracking. Such a scheme enables to respond to events in the environment behind the actual position of the leader and to incorrect movement of a neighbour in the formation. One can find implementation details on this approach in (Saska et al., 2011), where such a trajectory planning has been used for a spline path following.

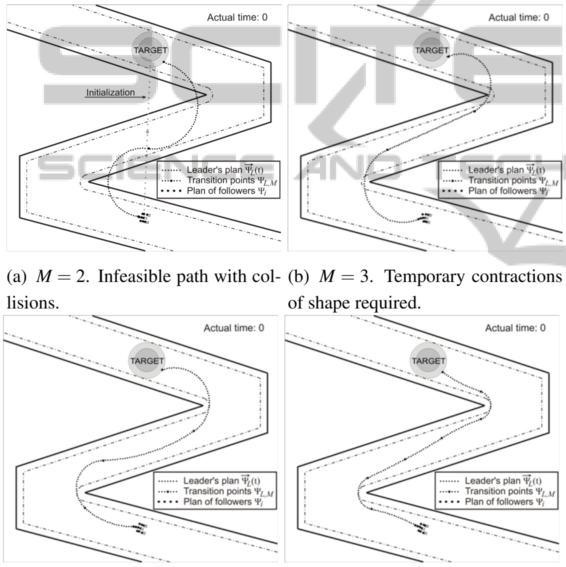
3 EXPERIMENTAL RESULTS AND PARAMETERS SETTING

Let us now discuss the influence of parameters n, N and M and show performance of the method via numerical experiments and simulations. In Table 1, where the situation from Fig. 1 was solved, the quality of results (values of the cost function) and computational time[1] are presented. As expected, the quality of the results increases with growing parameter M, as is also shown in Fig. 1, but the necessary computational

[1] In the 2nd and higher steps of the planning loop, the computational time is notably decreased due to possible re-initialization using the result from the previous step.

Table 1: Computational times required for planning the first step of the algorithm, the maximal computational times required for the second and higher steps of planning and values of the cost function $J(\Omega)$ after the first step of the algorithm. The algorithm was used with different values of parameter M and fixed constants $N = 4$, $n = 2$. The results have been obtained with Pentium 4 CPU 3.2GHz using function *fmincon* of Matlab.

M	t [s] (1.st)	$t_{\max}$ [s] ($\geq$ 2.st)	cost [-] (1.st)
2	34.7	0.28	∞
3	34.9	0.29	38.8
4	43.4	0.36	27.8
5	54.7	0.49	21.1
6	68.4	0.61	15.7
7	82.5	0.75	13.1
8	99.3	0.93	11.6



(a) $M = 2$. Infeasible path with collisions. (b) $M = 3$. Temporary contractions of shape required.

(c) $M = 4$. Feasible solution. (d) $M = 7$. Shorter path than in (c).

Figure 1: Solution of the *formation to target zone problem* with different setting of parameter M.

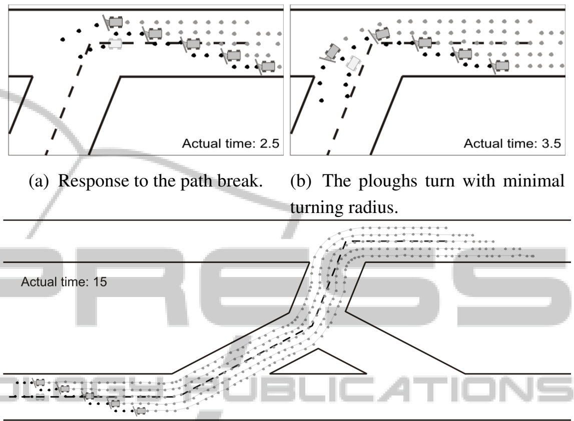
time of optimization is increased.

The influence of parameters n and N will be demonstrated using the results of the RHC method applied for driving of formations of autonomous ploughs at airport with two parallel runways connected via several auxiliary roads (see Fig. 2). The task of the ploughs is to follow the axes of roads that need to be cleaned. The crucial problem of the method is to overcome unsmooth connections of axes and still to maintain the optimal coverage of the runways. We purposely did not smooth the desired path in a post-processing to highlight the effect of the variables n and N.

Now, let us define an interval of stabilization $IS(Th)$ to be able to numerically characterise the influence of different setting of variables n and N. The interval of stabilization $IS(Th)$ will be the length of the desired path between the connection of line segments of the desired path and the point from which the difference between the virtual leader position and the desired position on the followed path stays less than or equal to a threshold Th as long as the planning interval N does not reach the next connection of line segments.

Table 2: Values of $IS(d_r/4)$ of solutions obtained by the algorithm with a different setting of parameters N and n.

$n \backslash\backslash N$	2	3	4	6	8
1	∞	2.44	2.19	1.50	1.50
2	∞	2.35	1.99	1.48	1.44
3	x	∞	2.44	2.05	1.50
4	x	x	∞	2.16	1.49
5	x	x	x	2.72	2.69



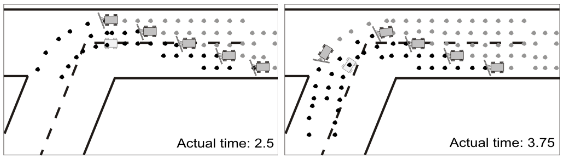
(a) Response to the path break. (b) The ploughs turn with minimal turning radius.

(c) The formation after the mission accomplishment with depicted passed trajectories of the ploughs.

Figure 2: Formation driving algorithm with parameters $N = 3$ and $n = 2$.

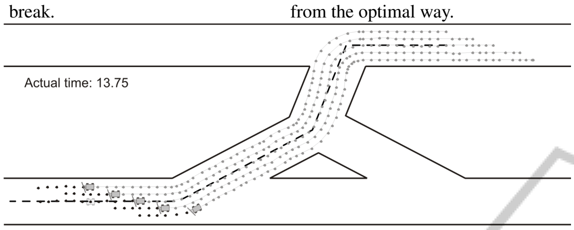
The values of $IS(Th)$ for $Th = d_r/4$, were d_r is width of the robot, obtained from simulations of the task in the Fig. 2 using the RHC method with different settings of parameters N and n are presented in Table 2. The obvious result is the correlation of solutions' quality with the value of N. Longer time horizon can response to the sudden changes of the formation heading better. For the lowest values of N the process can be even unstable (denoted as ∞ in the table). Contrariwise a too big value of the parameter n causes a long period of the formation driving without any possibility to respond to obstacles or to a breakage of the desired path.

In Fig. 2, the simulation with setting of algorithm $n = 2$ and $N = 3$ is presented to demonstrate the effect of the insufficiently long time interval N. In all pictures, the black points denote an actual plan for the



(a) The first response to the path break.

(b) The first plough turns deviated from the optimal way.



(c) The robots after the mission accomplishment with depicted passed trajectories.

Figure 3: Formation driving algorithm with parameters $N = 6$ and $n = 5$.

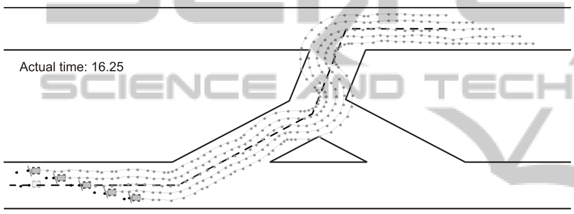


Figure 4: Formation driving algorithm with parameters $N = 2$ and $n = 1$.

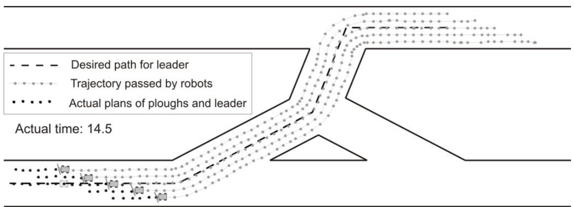


Figure 5: Formation driving algorithm with parameters $N = 6$ and $n = 2$.

ploughs as well as for the virtual leader and the grey points denote states visited during the previous movement. The dashed line represents the desired path that has to be followed by the virtual leader that is drawn by a contour in front of the formation.

The first snapshot in Fig. 2(a) was captured at time when the first response to the approaching change of heading is enabled. Nevertheless, it is too late to come through the curve optimally and so the deviation from the second path segment produces an uncleaned part of the runway (see Fig. 2(b)).

The second problem, the interval n being too long, is clarified in Fig. 3 where the algorithm with $n = 5$ and $N = 6$ was utilized. Again the first snapshot in Fig. 3(a) was captured at time of the first possible response to the path break. As one can see, the formation is already too close to the line break due to the long period where the robots just blindly executed preplanned control inputs. Therefore, the ploughs again overshoot the desired path (see Fig. 3(b)). Results of the simulation with the setting of parameters $n = 2$ and $N = 6$ are presented in Fig. 5 for comparison.

4 CONCLUSIONS

In this paper, we have described an approach of formation driving. The method is based on a model predictive control extended by an additional time horizon for navigation to a target region. The robustness of the algorithm and the influence of its parameters on the system performance were verified by simulations.

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REFERENCES

Barfoot, T. D. and Clark, C. M. (2004). Motion planning for formations of mobile robots. *Robotics and Autonom. Syst.*, 46:65–78.

Chen, J., Sun, D., Yang, J., and Chen, H. (2010). Leader-follower formation control of multiple non-holonomic mobile robots incorporating a receding-horizon scheme. *Int. J. Rob. Research*, 29:727–747.

Dunbar, W. and Murray, R. (2006). Distributed receding horizon control for multi-vehicle formation stabilization. *Automatica*, 42(4):549–558.

Liu, Y. and Jia, Y. (2012). An iterative learning approach to formation control of multi-agent systems. *Systems & Control Letters*, 61(1):148 – 154.

Mayne, D. Q., Rawlings, J. B., Rao, C. V., and Scokaert, P. O. M. (2000). Constrained model predictive control: Stability and optimality. *Automatica*, 36(6):789–814.

Saska, M., Hess, M., and Schilling, K. (2008). Efficient airport snow shoveling by applying autonomous multi-vehicle formations. In *Proc. of IEEE ICRA*.

Saska, M., Vonasek, V., and Preucil, L. (2011). Formation coordination with path planning in space of multinomials. In *Artificial Intelligence and Soft Computing*.

Zhang, F. (2010). Geometric cooperative control of particle formations. *IEEE Trans. on Autom. Control*, 55(3):800 –803.