Efficient Distributed Fusion Filtering Algorithms for Multiple Time Delayed Systems

Il Young Song and Moongu Jeon
School of Information & Communications, Gwangju Institute of Science and Technology
Oryong-Dong, Buk-Gu, 500-712, Gwangju, South Korea

Keywords: Distribute Fusion, Multi Sensor, Kalman Filter, Time-delayed System, Receding Horizon.

Abstract: In this paper, we provide two computational effective multi sensor fusion filtering algorithms for discrete-time linear uncertain systems with state and observation time delays. The first algorithm is shaped by algebraic forms for multi rate sensor systems, and then we propose a matrix form of filtering equations using block matrices. The second algorithm is based on exact cross-covariance equations. These equations are useful to compute matrix weights for fusion estimation in a multidimensional-multisensor environment. Also, our proposed filtering algorithm is based on the receding horizon strategy in order to achieve high estimation accuracy and stability under parametric uncertainties. We demonstrate the low computational complexities of the proposed fusion filtering algorithm and how the proposed algorithm robust against dynamic model uncertainties comparing with Kalman filter with time delays.

1 INTRODUCTION

In the past decades, state estimation problem for dynamic systems with time delays has received a great deal of research interest. The time delay phenomenon in state variables is unavoidable in many real systems (Anderson and Moore, 1979), such as low earth orbit (LEO) satellite communication systems (Glistic et al., 1996). Ignorance of the computation of these delays could cause unpredictable and unsatisfactory system performance with traditional Kalman filters.

Using finite-memory estimation, we can obtain an estimate based on data from the recent past only (receding horizon). As a result, finite-memory filters such as receding horizon Kalman filters are more robust against model uncertainties and numerical errors than standard Kalman filters, which utilize all measurements (Kim et al., 2006 and Kim et al., 2007). Thus, a receding horizon filter was chosen in this study.

Based on aforementioned literature, and to the best of the authors' knowledge, there are no existing results for the receding horizon filtering for linear systems with time delays. Motivated by the above problems, we focus on estimating the state of a discrete-time linear system with time delays in both the state and observation matrices, using a receding horizon strategy. The main contribution of the paper is to propose a fusion filtering algorithm using fusion formulas for the systems with time-delays. Moreover, a matrix form of filtering equations using block matrices is also discussed, because this form is useful to simply the filtering equations and derivation of crucial Lyapunov-like equations for receding horizon mean and covariance of systems with an arbitrary number of time delays. Finally, the obtained results are valid for general linear systems having time delays in both dynamic and observation models.

The rest of this paper is organized as follows. In Section II, the problem statement and description of the Kalman filter with time delays (KFTD) are given. In Section III, we present the receding horizon filter for discrete-time linear systems with time delays. Here, the exact recursive equations for determining receding horizon initial conditions (mean and covariance) are derived and discussed. In Section IV, two computational effective multi sensor fusion receding horizon filtering algorithms are presented. To achieve the fusion filtering, local cross-covariances are required. Thus, the equations of the exact cross-covariance are derived using the proposed form. In Section V, the effectiveness and comparative analysis of the proposed filter with the KFTD are then presented. Finally, a brief conclusion is given in Section VI.

Young Song I. and Jeon M..
Efficient Distributed Fusion Filtering Algorithms for Multiple Time Delayed Systems.
DOI: 10.5220/0003970703510356
In *Proceedings of the 9th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2012), pages 351-356
ISBN: 978-989-8565-21-1

2 PROBLEM STATEMENT

The discrete-time linear uncertain systems with state and observation time-delays considered in this paper described by stochastic recursive equation with time-delays,

$$x(k+1) = \sum_{h=0}^{M} F(k-h)x(k-h) + w(k),\ M \geq 0,\ k = 0,1,..., \quad (1)$$

where $x(k) \in R^n$ is an unknown state and $F(k-h) \in R^{n \times n}$, $h = 0,1,\dots,M$ are time-varying system matrices. It is assumed that $x(-s) \sim N(\bar{x}_0, P_0)$, $s = 0,1,\dots,M$ are initial conditions and a systems noise $w(k) \in R^n$ is a zero-mean white Gaussian noise with covariance $\mathrm{cov}\{w(k)w(s)\} = Q(k)\delta_{ks}$, and δ_{ks} is the Kronecker function.

Suppose that the overall discrete measurement are composed N measurement sub-vectors (local sensors) $y^{(1)}(k),\dots,y^{(N)}(k)$, i.e.,

$$Y(k) = \left[\left(y^{(1)}(k)\right)^T \cdots \left(y^{(N)}(k)\right)^T\right]^T \in R^m,$$

$$y^{(i)}(k) = \sum_{d=0}^{L_i} H^{(i)}(k-d)x(k-d) + v^{(i)}(k),\quad y^{(i)}(k) \in R^{m_i},\ L_i \geq 0, \quad (2)$$

$$i = 1,\dots,N,\ m_1 + \cdots + m_N = m,$$

where $y^{(i)}(k) \in R^{m_i}$ represents the local i-th sensor measurement, $v^{(i)}(k) \in R^{m_i}$ is the i-th measurement matrix, and $v^{(i)}(k) \in R^{m_i}$ is a zero-mean white Gaussian noise with covariance $\mathrm{cov}\{v^{(i)}(k)v^{(i)}(s)\} = R^{(i)}(k)\delta_{ks}$.

We also assume that the initial states $x(-s)$, $s = 0,1,\dots,M$, system noise $w(k)$, and measurement errors $v^{(i)}(k)$, $i = 1,\dots,N$ are mutually uncorrelated, i.e.,

$$\mathrm{cov}\{x(-s), w(k)\} = 0,\ \mathrm{cov}\{x(-s), v^{(i)}(k)\} = 0,$$
$$\mathrm{cov}\{w(k), v^{(i)}(k)\} = 0, \mathrm{cov}\{v^{(i)}(k), v^{(j)}(k)\} = 0, \quad (3)$$
$$s = 0,1,...,M;\ i,j = 1,...,N;\ i \neq j.$$

The main problem associated with such systems (1) and (2) is to find the optimal (in mean square sense) estimate of the unknown state based on the overall receding horizon sensor measurements $Y_{k-\Delta}^{k}$ with receding horizon time intervals Δ_i, $i = 1,\dots,N$, i.e.,

$$Y_{k-\Delta}^{k} = \left\{Y_{[k-\Delta_1:k]}^{(1)},\ ...,\ Y_{[k-\Delta_N:k]}^{(N)}\right\},$$
$$Y_{[k-\Delta_i:k]}^{(i)} = \left\{y^{(i)}(k-\Delta_i),\ y^{(i)}(k-\Delta_i+1),\ ...,\ y^{(i)}(k)\right\}, \quad (4)$$
$$i = 1,...,N.$$

There are two multi sensor fusion filtering algorithms. The first algorithm represents an optimal filtering (OF) algorithm, i.e., a mean-square estimate of a state vector using the overall measurement vector $Y(k)$ (2) is calculated by the optimal filtering equations presented in Priemer and Vacroux (1969) and Mishra and Rajamani (1975). However the OF algorithm is computationally expensive and it requires big memory sources, especially when the number of sensors N >> 1.

On the other hand, the second multi sensor algorithm is referred as fusion filtering (FF) which is achieved by combining N local estimates based on individual (local) sensor measurements $y^{(i)}(k)$, $i = 1,\dots,N$. The FF is suboptimal, but since the FF has parallel structure, it can be effectively adoptable for multisensory environment with the following advantages such as increase data input rates, simple fault detection, low computational complexity, and so on.

Therefore, since the FF can be adoptable in a multisensory environment, in this paper, the FF is considered for the system (1) and (2). To derive the FF, the local filtering estimates of a state vector based on individual sensor measurements $y^{(i)}(k)$ are required.

The KFTD's equations for the system (1) and (2) presented by Priemer and Vacroux (1969) and Mishra and Rajamani (1975). Using KFTD's equations, we propose their receding horizon version for estimation of state $x(k)$ using overall receding horizon measurements $Y_{k-\Delta}^{k}$ in (4). The details of the new receding horizon Kalman filter with time-delays are given in the next section.

3 LOCAL RECEDING HORIZON KALMAN FILTER WITH TIME-DELAYS

To find $\hat{x}^{(i)}(k|k)$ based on receding horizon measurements $Y_{[k-\Delta_i:k]}^{(i)}$ we propose to use KFTD equations on the receding horizon interval $s \in [k-\Delta_i,\ k]$. We obtain

$$\hat{x}^{(i)}(s-m|s) = \hat{x}^{(i)}(s-m|s-1) + G_m^{(i)}(s)\left[y^{(i)}(s) - \sum_{d=0}^{L_i} H^{(i)}(s-d)\hat{x}^{(i)}(s-d|s-1)\right], \quad (5)$$

$$s = k-\Delta_i, k-\Delta_i+1, ..., k;\ m = 1,2,...,\bar{M}_i,\ \bar{M}_i = \max\{M, L_i\}.$$

$$\hat{x}^{(i)}(s|s-1) = \sum_{h=0}^{M} F^{(i)}(s-h-1)\hat{x}^{(i)}(s-h-1|s-1). \quad (6)$$

$$\hat{x}^{(i)}(s|s) = \hat{x}^{(i)}(s|s-1) + G_0^{(i)}(s)\left[y^{(i)}(s) - \sum_{d=0}^{L_i} H^{(i)}(s-d)\hat{x}^{(i)}(s-d|s-1)\right], \quad (7)$$

where the receding horizon filter gains $G_m^{(i)}(s)$, $m=0,1,\ldots,\bar{M}_i$ and error auto-covariances

$$P^{(ii)}(s_1,s_2\,|\,s)=\mathrm{cov}\left\{e^{(i)}(s_1\,|\,s),e^{(i)}(s_2\,|\,s)\right\},\qquad(8)$$
$$e^{(i)}(s_1\,|\,s)=x(s)-\hat{x}^{(i)}(s_1\,|\,s),\quad s_1,s_2\le s.$$

are described by

$$G_m^{(i)}(s)=\sum_{d=0}^{L_i}P^{(ii)}(s-m,s-d\,|\,s-1)\left(H^{(i)}(s-d)\right)^T\qquad(9)$$
$$\times\left[R^{(i)}(s)+\sum_{d_1,d_2=0}^{L_i}H^{(i)}(s-d_1)P^{(ii)}(s-d_1,s-d_2\,|\,s-1)\left(H^{(i)}(s-d_2)\right)^T\right]^{-1}.$$

$$P^{(ii)}(s-h_1,s-h_2\,|\,s)=P^{(ii)}(s-h_1,s-h_2\,|\,s-1)\qquad(10)$$
$$-G_{h_1}^{(i)}(s)\sum_{d=0}^{L_i}H^{(i)}(s-d)P^{(ii)}(s-d,s-h_2\,|\,s-1),$$

$$P^{(ii)}(s+1,s+1|s)=\sum_{h_1,h_2=0}^{M}F(s-h_1)P^{(ii)}(s-h_1,s-h_2|s)F^T(s-h_2)+Q(s).\qquad(11)$$

In contrast to KFTD filtering, the local receding horizon Kalman filtering with time delay (LRHKFTD) (5)-(11) needs to initialize (M+1) receding horizon initial conditions at $s=k-\Delta_i$ which represent an unconditional means and covariances, i.e.,

$$\hat{x}^{(i)}(k-\Delta_i-\bar{M}_i+1|k-\Delta_i)=E\{x(k-\Delta_i-\bar{M}_i+1)\}\overset{def}{=}m(k-\Delta_i-\bar{M}_i+1),$$
$$\hat{x}(k-\Delta_i-\bar{M}_i+2|k-\Delta_i)=E\{x(k-\Delta_i-\bar{M}_i+2)\}\overset{def}{=}m(k-\Delta_i-\bar{M}_i+2),\qquad(12)$$
$$\cdots\cdots\cdots\cdots$$
$$\hat{x}(k-\Delta_i+1|k-\Delta_i)=E\{x(k-\Delta_i+1)\}\overset{def}{=}m(k-\Delta_i+1).$$

and

$$P^{(ii)}(h_1,h_2\,|k-\Delta_i)=\mathrm{cov}\{x(h_1),x(h_2)\}\overset{def}{=}P(h_1,h_2),\qquad(13)$$
$$h_1,h_2=k-\Delta_i-\bar{M}_i+1,\ldots,k-\Delta_i+1.$$

Remark 1. The horizon initial means (12) are described by

$$m(t+1)=\sum_{h=0}^{M}F(t-h)m(t-h),\quad t=0,1,2,\ldots,k-\Delta_i+1\qquad(14)$$

with initial conditions

$$m(0)=m(-1)=m(-2)=\ldots=m(-M)=\bar{x}_0.\qquad(15)$$

Remark 2. The receding horizon initial covariances (13) satisfy Lyapunov-like recursive equations

$$P^{(ii)}(t+1,t+1)=\sum_{h_1,h_2=0}^{M}F(t-h_1)P^{(ii)}(t-h_1,t-h_2)F^T(t-h_2)+Q(t),\qquad(16)$$
$$t=0,1,2,\ldots,k-\Delta_i+1,$$

$$P^{(ii)}(t-h_1+1,t-h_2+1)=\sum_{l_1=0}^{M}F(t-h_1-l_1)P^{(ii)}(t-h_1-l_1,t-h_2+1)$$
$$+Q(t-h_1)\delta_{t-h_1,t-h_2},\quad k-h_1<k-h_2,\qquad(17)$$
$$P^{(ii)}(t-h_1,t-h_2)=P^{(ii)}(t-h_2,t-h_1)^T,\quad t-h_1<t-h_2$$

with initial conditions

$$P^{(ii)}(-s_1,-s_2)=P_0^{(ii)},\quad s_1,s_2=0,1,\ldots,M.\qquad(18)$$

Derivation of Lyapunov-like equations for mean and covariance (14)-(18) is given in Appendix.

4 TWO COMPUTATIONALLY EFFICIENT MULTI SENSOR FUSION ALGORITHMS

To apply the receding horizon Kalman filtering with time delay (5)-(11) to the real computation with MATLAB, using the repetition of (5)-(11) is less effective than direct matrix multiplications because the matrix operations, i.e., multiplications, divisions, and inversions take many optimized computational algorithms in MATLAB. Therefore, we change the filtering equation (5)-(11) into a matrix form for the computational benefits on MATLAB.

4.1 Matrix Form of Filtering Equations

The equations (5)-(11) can be represented by block matrices. Let us assume the following block matrices

$$F(s-h)=\begin{cases}F(s-h), & 0\le h\le M,\\ 0_{n\times n}, & M<h\le\bar{M}_i,\ \bar{M}_i=\max\{M,L_i\},\end{cases}\qquad(19)$$
$$H^{(i)}(s-j)=\begin{cases}H^{(i)}(s-j), & 0\le j\le L_i,\\ 0_{n\times n}, & L_i<j\le\bar{M}_i.\end{cases}$$

$$\hat{X}^{(i)}(s\,|\,s)=\left[\left(\hat{x}^{(i)}(s\,|\,s)\right)^T\ \left(\hat{x}^{(i)}(s-1\,|\,s)\right)^T\ \cdots\ \left(\hat{x}^{(i)}(s-\bar{M}_i\,|\,s)\right)^T\right]^T,$$
$$\bar{F}(s)=\left[F(s)\ \ F(s-1)\ \cdots\ F(s-\bar{M}_i)\right],$$
$$\bar{H}^{(i)}(s)=\left[H^{(i)}(s)\ \ H^{(i)}(s-1)\ \cdots\ H^{(i)}(s-\bar{M}_i)\right],$$
$$\bar{G}^{(i)}(s)=\left[\left(G_0^{(i)}(s)\right)^T\ \left(G_1^{(i)}(s)\right)^T\ \cdots\ \left(G_{\bar{M}_i}^{(i)}(s)\right)^T\right]^T,\qquad(20)$$
$$\Omega^{(ii)}(s\,|\,s)=\begin{bmatrix}P^{(ii)}(s,s\,|\,s) & \cdots & P^{(ii)}(s,s-\bar{M}_i\,|\,s)\\ \vdots & \ddots & \vdots\\ P^{(ii)}(s-\bar{M}_i,s\,|\,s) & \cdots & P^{(ii)}(s-\bar{M}_i,s-\bar{M}_i\,|\,s)\end{bmatrix},$$
$$A=\left[I_n\ \ 0_{n\times n\bar{M}_i}\right]\in R^{n\times n(\bar{M}_i+1)},\ B=\left[I_{n(\bar{M}_i+1)}\ \ 0_{n(\bar{M}_i+1)\times n}\right]\in R^{n(\bar{M}_i+1)\times n(\bar{M}_i+2)},$$
$$\Omega_0=\left[P_{jh}\right],\quad P_{jh}=P_0,\ j,h=1,\ldots,(\bar{M}_i+1),$$

where I_n is an $n\times n$ indent matrix, $0_{n\times n}$ is an $n\times n$ zero matrix, and $\Omega^{(ii)}(0|0)=\Omega_0$.

Then, based on (5)-(11), the filtering equations are rewritten using (19), (20):

$$
\begin{aligned}
&\hat{X}^{(i)}(s|s-1) = B\begin{bmatrix} \bar{F}(s-1)\hat{X}^{(i)}(s-1|s-1) \\ \hat{X}^{(i)}(s-1|s-1) \end{bmatrix}, \\
&\Omega^{(ii)}(s|s-1) \\
&= B\begin{bmatrix} \bar{F}(s-1)\Omega^{(ii)}(s-1|s-1)\bar{F}^T(s-1)+Q(s-1) & \bar{F}(s-1)\left(\Omega^{(ii)}(s-1|s-1)\right)^T \\ \Omega^{(ii)}(s-1|s-1)\bar{F}^T(s-1) & \Omega^{(ii)}(s-1|s-1) \end{bmatrix} B^T, \\
&\hat{X}^{(i)}(s|s) = \hat{X}^{(i)}(s|s-1) + \bar{G}^{(i)}(s)\left[y^{(i)}(s) - \bar{H}^{(i)}(s)\hat{X}^{(i)}(s|s-1)\right], \\
&\bar{G}^{(i)}(s) = \Omega^{(ii)}(s|s-1)\left(\bar{H}^{(i)}(s)\right)^T\left[\bar{H}^{(i)}(s)\Omega^{(ii)}(s|s-1)\left(\bar{H}^{(i)}(s)\right)^T + R^{(i)}(s)\right]^{-1}, \\
&\Omega^{(ii)}(s|s) = \left(I_{n(\bar{M}_i+1)} - \bar{G}^{(i)}(s)\bar{H}^{(i)}(s)\right)\Omega^{(ii)}(s|s-1).
\end{aligned} \tag{21}
$$

Finally, the local estimate $\hat{x}^{(i)}(k|k)$ and error-covariance $P^{(ii)}(k,k|k)$ at current time k are described as

$$\hat{x}^{(i)}(k|k) = A\hat{X}^{(i)}(k|k),\ P^{(ii)}(k,k|k) = A\Omega^{(ii)}(k|k)A^T. \tag{22}$$

Differently from (5)-(11), the local estimate $\hat{x}^{(i)}(k|k)$ is directly calculated using (22). Moreover, (21) is shaped like the Kalman filter as well as more simple (5)-(11) on MATLAB.

4.2 Distributed Fusion Form of Filtering Equations

Through (20)-(22) for $i = 1,\ldots,N$, we obtain N LRHKFTDs $\hat{x}^{(1)}(k|k),\ldots,\hat{x}^{(N)}(k|k)$ with the corresponding local error-covariance $P^{(11)}(k,k|k)$ $,\ldots,P^{(NN)}(k,k|k)$. Then, the distributed fusion estimate $\hat{x}^{FF}(k|k)$ is determined using the following fusion formula presented by Shin et al. (2006).

$$\hat{x}^{FF}(k|k) = \sum_{i=1}^{N} C^{(i)}(k)\hat{x}^{(i)}(k|k),\ \sum_{i=1}^{N} C^{(i)}(k) = I_n, \tag{23}$$

where $C^{(i)}(k),\ i,j = 1,\ldots,N$ is $n\times n$ matrix weights which are defined as

$$C(k) = \left(D^T P_e^{-1}(k) D\right)^{-1} D^T P_e^{-1}(k),\ D = \left[I_n \ldots I_n\right]^T, \tag{24}$$

where

$$
\begin{aligned}
&C(k) = \left[C^{(1)}(k),\ldots,C^{(N)}(k)\right] \in R^{n\times nN}, \\
&P_e(k) = \left[P^{(ij)}(k,k|k)\right]_{i,j=1}^{N} \in R^{nN\times nN}, \\
&P^{(ij)}(k,k|k) = \mathrm{cov}\left\{e^{(i)}(k|k), e^{(j)}(k|k)\right\}, \\
&e^{(i)}(k|k) = x(k) - \hat{x}^{(i)}(k|k),\ i,j = 1,\ldots,N,\ i \neq j.
\end{aligned} \tag{25}
$$

In order to compute the matrix weights $C^{(i)}(k),\ i,j = 1,\ldots,N$, the local cross-covariances $P^{(ij)}(k,k|k),\ i,j = 1,\ldots,N,\ i \neq j$ are required. Derivation of (23)-(25) is given in Shin et al. (2006). In the next section, the effectiveness of the fusion filtering is presented.

5 NUMERICAL EXAMPLE

In this section, an example for discrete-time dynamic systems with parametric model uncertainty is presented. We compare the accuracies and implementation time between two fusion algorithms: the first is OF algorithm (see section 4.1) and the second is the FF algorithm (see section 4.2). The example demonstrates the robustness and effectiveness of our proposed LRHKFTD (5)-(11) in terms of mean square errors (MSEs).

We now consider the following LEO satellite communication system with multiple time delay and uncertainty (Glistic et al., 1996). LEO satellite channels impart severe spreading in delay and Doppler on the transmitted signal. The state vector represents the received signal level [dB].

$$\begin{cases} x(k+1) = \left(0.995+\delta_1(k)\right)x(k) + \left(0.190+\delta_2(k)\right)x(k-1) + \left(0.107+\delta_3(k)\right)x(k-2) + w(k), \\ y(k) = 0.4x(k) + 0.1x(k-1) + 0.4x(k-2) + v(k), \end{cases} \tag{26}$$

where $w(k) \sim N(0, Q(k))$ and $v(k) \sim N(0, R(k))$ are uncorrelated white Gaussian system and measurement noises, respectively, $Q(k) = 0.02^2$, $R(k) = 0.5.$ The initial values are $x(0) \sim N(\bar{x}(0), P(0))$, $\bar{x}(0) = 1$ [dB] and $P(0) = 1;$ $\delta(k) = \{\delta_1(k),\ \delta_2(k),\ \delta_3(k)\}$ are uncertain model parameters which is assumed to satisfy

$$\delta(k) = \begin{cases} |\delta_1(k)| \le 0.05, |\delta_2(k)| \le 0.1, |\delta_3(k)| \le 0.01, & k \in T_{UI}, \\ 0, & \text{otherwise}, \end{cases} \tag{27}$$

where $T_{UI} = [40;\ 60]$ is the uncertainty interval (UI). The common receding horizon length Δ of the LRHKFTDs is taken as $\Delta_{com} = 5$. Finally, two fusion receding horizon filters: OF and FF with the LRHKFTDs (5)-(11) and two fusion non-receding horizon filters: OF and FF with KFTD for the system model (26) with the uncertainty $\delta(k)$ which takes the form (27) are compared.

We now present model (26) to show robustness of the proposed RHKFTD against the uncertainty. All simulations were evaluated in terms of MSEs of 1000 Monte Carlo runs. We compare the MSEs of OF with KFTD ("OFKF"), FF with KFTD ("FFKF"), OF with RHKFTD ("OFRHF") and FF with RHKFTD ("FFRHF") with common receding horizon length Δ_{com}, i.e.,

(A) OF with KFTD ("OFKF"):

$$P^{OFKF}(k,k|k) = E\left[x(k) - \hat{x}^{OFKF}(k|k)\right]^2,$$

(B) FF with KFTD ("FFKF"):

$$\mathbf{P}^{\mathrm{FFKF}}(\mathbf{k},\mathbf{k}|\mathbf{k}) = \mathbf{E}\left[\mathbf{x}(\mathbf{k}) - \hat{\mathbf{x}}^{\mathrm{FFKF}}(\mathbf{k}|\mathbf{k})\right]^2,$$

(C) OF with RHKFTD ("OFRHF"):

$$\mathbf{P}^{\mathrm{OFSW}}_{\Delta_{\mathrm{com}}}(\mathbf{k}|\mathbf{k}) = \mathbf{E}\left[\mathbf{x}(\mathbf{k}) - \hat{\mathbf{x}}^{\mathrm{OFSW}}_{\Delta_{\mathrm{com}}}(\mathbf{k}|\mathbf{k})\right]^2, \quad \Delta_{\mathrm{com}} = 5,$$

(D) FF with RHKFTD ("FFRHF"):

$$\mathbf{P}^{\mathrm{FFSW}}_{\Delta_{\mathrm{com}}}(\mathbf{k}|\mathbf{k}) = \mathbf{E}\left[\mathbf{x}(\mathbf{k}) - \hat{\mathbf{x}}^{\mathrm{FFSW}}_{\Delta_{\mathrm{com}}}(\mathbf{k}|\mathbf{k})\right]^2, \quad \Delta_{\mathrm{com}} = 5.$$

Our point of interest is the behaviour of the aforementioned filters, both inside and outside of the uncertainty interval $\mathbf{T}_{\mathrm{UI}} = [40;\ 60]$. In Fig.1 we observe that inside of the UI, the first pair of the receding horizon filters (OFRHF and FFRHF) demonstrate good performance compared to the non-receding horizon filters (OFKF and FFKF). Also in each pair, the OF filters give more accurate estimates than their FF versions. However, these differences are not big.

The estimation accuracy of the filters can be more clearly compared through MSEs in Fig. 2. In the figure we observe that within the $\mathbf{T}_{\mathrm{UI}} = [40;\ 60]$, the MSEs of the non-receding horizon filters (OFKF and FFKF) are remarkably larger than receding horizon versions (OFRHF and FFRHF). However, the differences between all OF and FF filters are negligible outside of the $\mathbf{T}_{\mathrm{UI}} = [40;\ 60]$. For this reason, our proposed algorithm is suitable for real implementations in a multisensory system with time delays.

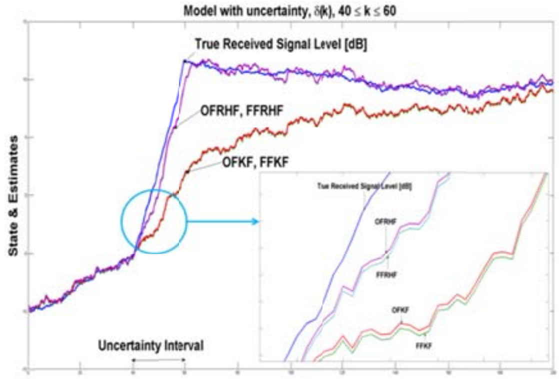

Figure 1: True received signal level and its estimates using OFRHF, FFRHF, OFKF, and FFKF.

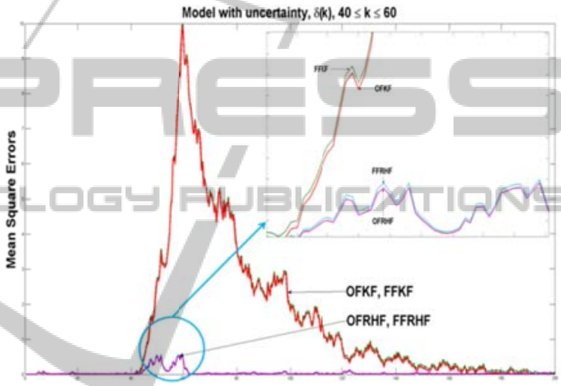

Figure 2: MSEs comparison between receding horizon filters (OFRHF and FFRHF) and non-receding horizon filters (OFKF and FFKF) with uncertainty.

6 CONCLUSIONS

In this paper we propose a new receding horizon filter for discrete-time linear system with time delays in both the state and observation matrices. Filtering equations shaped by classical forms for time-delayed systems are defined, and then we change the filtering equations into the matrix form because the matrix form has computational benefits on MATLAB. Also, the Lyapunov-like recursive equations for receding horizon initial mean and covariance of a system state with an arbitrary number of time delays are derived.

To verify the effectiveness of the proposed RHKFTD, an example for discrete-time dynamic systems with parametric model uncertainty $\delta(k)$ is implemented. Through the implementations, it is demonstrated that the robustness of the proposed filter in terms of MSEs and the proposed filter can produce good results in real-time processing requirements.

ACKNOWLEDGEMENTS

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2011-0004889).

REFERENCES

Anderson, B. D. O., & Moore, J. B. (1979). *Optimal Filtering*, Prentice-Hall.

Glistic, S., Talvite, J., Kumpumaki, T., & Latva-aho, M. (1996). Design study for a CDMA-based LEO satellite network: downlink system level parameters, *IEEE Journal of Selected Topics in Signal Processing*, 14(9), 1796–1808.

Kim, D. Y., & Shin, V. (2006). An optimal receding horizon FIR filter for continuous-time linear systems. In *Proceedings Inter. Conf. SICE-ICCAS* (pp. 263–

265), Busan, Korea, Oct 2006.

Kim, D. Y., & Shin, V. (2007). Optimal receding horizon filter for continuous time nonlinear stochastic systems. In *Proceedings of the 6th WSEAS Inter. Conf. on Signal Processing* (pp. 112–116), Dallas, Texas, USA, Mar 2007.

Mishra, J., & Rajamani, V. S. (1975). Least-squares state estimation in time-delay systems with colored observation noise: an innovation approach, *IEEE Transactions on Automatic Control*, 20(1), 140–142.

Priemer, R., & Vacroux, A. G. (1969). Estimation in linear discrete systems with multiple time delays, *IEEE Transactions on Automatic Control*, 14(4), 384–387.

Shin, V., Lee, Y., & Choi, T. (2006). Generalized millman's formula and its applications for estimation problems, *Signal Processing*, 86(2), 257–266.

APPENDIX

Derivation of Equation for Receding Horizon Initial Mean (14). Taking expectation on both sides of (1) and using $\mathbf{E}\left[\mathbf{w}(\mathbf{t})\right]=\mathbf{0}$ we immediately obtain recursive equation (14) for mean $\mathbf{m}(\mathbf{t})=\mathbf{E}\left[\mathbf{x}(\mathbf{t})\right]$.

Derivation of Equation for Receding Horizon Initial Covariance (16). Subtracting (14) from (1) we obtain time propagation of the centered state,

$$\tilde{\mathbf{x}}(\mathbf{t}+1)=\sum_{\mathbf{h}=0}^{\mathbf{M}}\mathbf{F}_{\mathbf{h}}(\mathbf{t}-\mathbf{h})\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h})+\mathbf{w}(\mathbf{t}),\quad \mathbf{t}=0,1,2,\ldots, \tag{A.1}$$

Next we have

$$\begin{aligned}\tilde{\mathbf{x}}(\mathbf{t}+1)\tilde{\mathbf{x}}(\mathbf{t}+1)^{\mathbf{T}}&=\sum_{\mathbf{h}_1,\mathbf{h}_2=0}^{\mathbf{M}}\mathbf{F}_{\mathbf{h}_1}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2)^{\mathbf{T}}\mathbf{F}_{\mathbf{h}_2}^{\mathbf{T}}(\mathbf{t}-\mathbf{h}_2)+\mathbf{w}(\mathbf{t})\mathbf{w}(\mathbf{t})^{\mathbf{T}}\\&+\sum_{\mathbf{h}_1=0}^{\mathbf{M}}\mathbf{F}_{\mathbf{h}_1}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_1)\mathbf{w}(\mathbf{t})^{\mathbf{T}}+\sum_{\mathbf{h}_2=0}^{\mathbf{M}}\mathbf{w}(\mathbf{t})\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2)^{\mathbf{T}}\mathbf{F}_{\mathbf{h}_2}^{\mathbf{T}}(\mathbf{t}-\mathbf{h}_2).\end{aligned} \tag{A.2}$$

Taking expectation on both sides of (A.2) and using the fact that current noise $\mathbf{w}(\mathbf{t})$ does not depend on current and past states $\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_1)$, $\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2)$ we obtain recursive equation for covariance (16),

$$\mathbf{P}(\mathbf{t}+1,\mathbf{t}+1)=\sum_{\mathbf{h}_1,\mathbf{h}_2=0}^{\mathbf{M}}\mathbf{F}_{\mathbf{h}_1}(\mathbf{t}-\mathbf{h}_1)\mathbf{P}(\mathbf{t}-\mathbf{h}_1,\mathbf{t}-\mathbf{h}_2)\mathbf{F}_{\mathbf{h}_2}^{\mathbf{T}}(\mathbf{t}-\mathbf{h}_2)+\mathbf{Q}(\mathbf{t}). \tag{A.3}$$

Note that equation (A.3) contains auto-covariance,

$$\begin{aligned}&\mathbf{P}(\mathbf{t}-\mathbf{h}_1,\mathbf{t}-\mathbf{h}_2)=\mathbf{E}\left[\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2)^{\mathbf{T}}\right],\\&\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h})=\mathbf{x}(\mathbf{t}-\mathbf{h})-\mathbf{m}(\mathbf{t}-\mathbf{h}),\\&\mathbf{h}_1,\mathbf{h}_2=0,1,\ldots,\mathbf{M}.\end{aligned} \tag{A.4}$$

Derivation of Equation for Auto-covariance (17). Using "symmetric" property of auto-covariance $\mathbf{P}(\mathbf{t}-\mathbf{h}_2,\mathbf{t}-\mathbf{h}_1)=\mathbf{P}(\mathbf{t}-\mathbf{h}_1,\mathbf{t}-\mathbf{h}_2)^{\mathbf{T}}$ and without loss of generality we can assume that $\mathbf{k}-\mathbf{h}_1\geq\mathbf{k}-\mathbf{h}_2$. Substituting $\mathbf{t}\to\mathbf{t}-\mathbf{h}_1$ in (A.1) we obtain

$$\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_1+1)=\sum_{\mathbf{l}_1=0}^{\mathbf{M}}\mathbf{F}_{\mathbf{l}_1}(\mathbf{t}-\mathbf{h}_1-\mathbf{l}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_1-\mathbf{l}_1)+\mathbf{w}(\mathbf{t}-\mathbf{h}_1). \tag{A.5}$$

Multiplying both sides of (A.5) by $\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2+1)^{\mathbf{T}}$ and using (A.4) we obtain

$$\begin{aligned}\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_1+1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2+1)^{\mathbf{T}}&=\sum_{\mathbf{l}_1=0}^{\mathbf{M}}\mathbf{F}_{\mathbf{l}_1}(\mathbf{t}-\mathbf{h}_1-\mathbf{l}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_1-\mathbf{l}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2+1)^{\mathbf{T}}\\&+\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2+1)^{\mathbf{T}},\end{aligned} \tag{A.6}$$

and

$$\begin{aligned}\mathbf{P}(\mathbf{t}-\mathbf{h}_1+1,\mathbf{t}-\mathbf{h}_2+1)&=\sum_{\mathbf{l}_1=0}^{\mathbf{M}}\mathbf{F}_{\mathbf{l}_1}(\mathbf{t}-\mathbf{h}_1-\mathbf{l}_1)\mathbf{P}(\mathbf{t}-\mathbf{h}_1-\mathbf{l}_1,\mathbf{t}-\mathbf{h}_2+1)\\&+\mathbf{E}\left[\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2+1)^{\mathbf{T}}\right],\quad \mathbf{h}_1,\mathbf{h}_2=0,1,\ldots,\mathbf{M}.\end{aligned} \tag{A.7}$$

It's remain to calculate expectation in (A.7), i.e.,

$$\mathbf{E}\left[\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2+1)^{\mathbf{T}}\right]\quad\text{for}\quad \mathbf{t}-\mathbf{h}_1\geq\mathbf{t}-\mathbf{h}_2. \tag{A.8}$$

Calculating product $\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2+1)^{\mathbf{T}}$ using (A.5) and after that taking expectation we get

$$\begin{aligned}\mathbf{E}\left[\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2+1)^{\mathbf{T}}\right]&=\sum_{\mathbf{l}_2=0}^{\mathbf{M}}\mathbf{E}\left[\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2-\mathbf{l}_2)^{\mathbf{T}}\right]\mathbf{F}_{\mathbf{l}_2}^{\mathbf{T}}(\mathbf{t}-\mathbf{h}_2-\mathbf{l}_2)\\&+\mathbf{E}\left[\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\mathbf{w}(\mathbf{t}-\mathbf{h}_2)^{\mathbf{T}}\right].\end{aligned} \tag{A.9}$$

According to assumption $\mathbf{t}-\mathbf{h}_1\geq\mathbf{t}-\mathbf{h}_2$, "future" noise $\mathbf{w}(\mathbf{t}-\mathbf{h}_1)$ does not depend on current and past states $\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2-\mathbf{l}_2)$ therefore $\mathbf{E}\left[\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\tilde{\mathbf{x}}(\mathbf{t}-\mathbf{h}_2-\mathbf{l}_2)^{\mathbf{T}}\right]=\mathbf{0}.$ Next using property of white noise we obtain

$$\mathbf{E}\left[\mathbf{w}(\mathbf{t}-\mathbf{h}_1)\mathbf{w}(\mathbf{t}-\mathbf{h}_2)^{\mathbf{T}}\right]=\mathbf{Q}(\mathbf{t}-\mathbf{h}_1)\delta_{\mathbf{t}-\mathbf{h}_1,\mathbf{t}-\mathbf{h}_2}. \tag{A.10}$$

Finally using (A.7), (A.9) and (A.10) we get equation for auto-covariance (17).

This completes the derivation Lyapunov-like equations for receding horizon mean and covariances.