Multivariable Discrete Time Repetitive Control System

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Abstract: This paper deals with an iterative learning control law for multivariable systems. The desired inputs are supposed to be known and periodic. The principle of the control is to make outputs as close as possible to desired inputs at each new period. After the design of multivariable repetitive controller, we give the stability condition of the algorithm and some simulation results.

1 INTRODUCTION

The theory of the modern control was successfully used in the control of several industrial processes. There are at the moment several analytical methods for the choice of the controller that permit to obtain an asymptotic stability and an acceptable static error, but few of them specify the transient response of the system. This limitation motivated the researchers to develop a new concept of control for the systems that repeat the same operation, known under the name of iterative learning control either repetitive control. The objective of such control is to improve the performances to every new period (Figure 1).

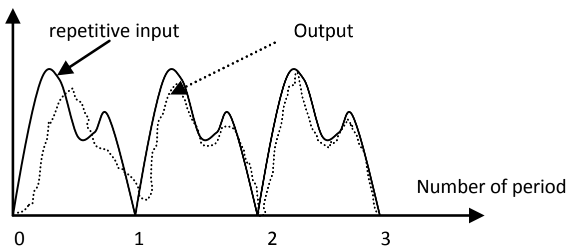

Figure 1: Example of periodic output.

Typical examples are industrial robots, which most of their tasks are of this kind; e.g. pick and place, painting, etc. Other examples are control of numerical control machines, hard-disc drive or many mechanical systems having revolving mechanisms inside.

Several researchers were interested in this type of control law (Arimoto et al. (1984), Sugie and Ono (1991), Moore et al. (1992), Xu and Tan (2003), Ahn et al. (2007) and Saari et al. (2010)). Most of their works were focused on the problem of the control in the multivariable case. They approached this problem by an analysis in the state space. The criticism made to this analysis is that it did not take into account the dynamics of the process to be controlled in the convergence condition of this algorithm (Curtelin et al. (1993)). The problem was resolved in the case of Single-Input Single-Output (SISO) systems by making an analysis by transfer function (Saari et al. (2010)). By respecting the convergence condition, the error goes to zero after an infinite number of periods. This induces the inversion of the process. The problem of non minimum phase process appears. This kind of problem was resolved by introducing the approached inverse of the process in the repetitive filter (Tomizuka et al. (1988) and Saari et al. (1994a, 1994b, 1996, 2010)).

In this paper, we are going to generalize the solution found for SISO systems to a Multi-Inputs Multi-Outputs (MIMO) system by using the notion of transfer matrix.

2 PROBLEM FORMULATION

The principle of repetitive control is presented by Figure 2, where, G is the transfer matrix of the process supposed to be stable and where $\dim(G) = n \times n$. H is the transfer matrix of the repetitive filter where $\dim(H) = n \times n$. Y_d is the vector of periodic reference signal of dimension *n*. Y^i

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and U^i are respectively the vectors composed by output and control signals of the period *i*. Both vectors are of dimension *n*. The memory blocks are introduced to indicate that the used signals are memorized in order to be used in the next period.

From Figure 2, one has:

$$Y^{i+1} = G \cdot U^{i+1} \; . \quad (1)$$

The control algorithm is then given by the following equation:

$$U^{i+1} = U^i + H \cdot E^i \; , \quad (2)$$

where E^i is the error vector of dimension *n* given by:

$$E^i = Y_d - Y^i \; . \quad (3)$$

By replacing (2) in (1) and taking into account (3), one obtains:

$$E^{i+1} = \left(I - G \cdot H\right) \cdot E^i \; . \quad (4)$$

From (4), one can deduce the following theorem that gives the convergence condition of the repetitive algorithm.

Theorem 1

The repetitive control algorithm (2) converges and the error decreases under certain norm;

$$\left\| E^{i+1} \right\| < \left\| E^i \right\| \quad (5)$$

if and only if:

$$\left\| I - G \cdot H \right\|_\infty < 1 . \quad (6)$$

■

The proof is obvious from (4).

If the convergence condition (6) is verified the error vector tends towards a null value after an infinite number of periods ($\lim_{i \to \infty} \left\| E^i \right\| \to 0$), this is equivalent to $Y^\infty \to Y_d$ and then, the control signal vector after an infinite number of periods inverts the process dynamic ($U^\infty = G^{-1} \cdot Y_d$) which seems to be impossible when the plant to be controlled is non invertible.

However, since Y_d is an a priori known signal, it is possible to generate the off-line control signal vector even if the plant is non invertible (saari et al. (2010)).

Moore et al. (1992) show that to satisfy the repetitive control convergence condition, the repetitive controller *H* will contain the inverse of the process. The question is then, what can we do when the process is non invertible?

In the following, we will examine several situations of the process to be controlled and consequently we will give the best choice of repetitive filter.

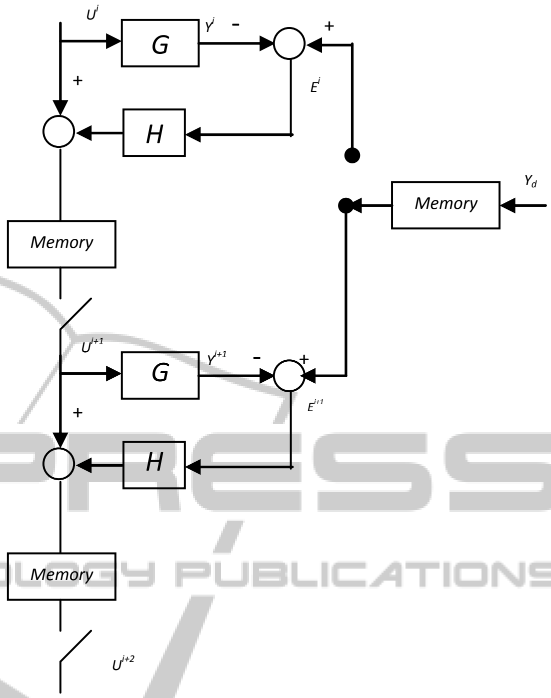

Figure 2: Scheme of the repetitive control.

3 CASE OF STABLE PROCESS

Let *G* the discrete stable transfer matrix of the process given under the form:

$$G(z^{-1}) = \frac{z^{-d}}{D(z^{-1})} \cdot N(z^{-1}) \, , \quad (7)$$

where *d* denotes the delay. $D(z^{-1})$ is the polynomial denominator of the transfer matrix *G*, containing the poles of the process. $N(z^{-1})$ is a matrix which elements are polynomials.

In this section, we approach the problem of the choice of repetitive filter in the cases of invertible and non invertible processes.

3.1 Case of an Invertible Process

A first idea consists in choosing the repetitive filter *H* such that it compensates only the delay while verifying the convergence condition (6).

The simplest expression of *H* is:

$$H(z^{-1}) = z^d \cdot H_0 \, , \quad (8)$$

with H_0 a constant matrix with an appropriate dimension.

The drawback of this method is that there is no method which guides us in the choice of H_0.

A second idea with the choice of the repetitive filter H consists in setting the inverse of the transfer matrix $G(z^{-1})$ multiplied by a gain kr such as:

$$H(z^{-1}) = kr \cdot z^{d} \cdot D(z^{-1}) \cdot \left[N(z^{-1})\right]^{-1}. \tag{9}$$

Theorem 2

The repetitive control algorithm described by Figure 2, with the repetitive filter (9) for invertible system (7), converges to zero error vector if and only if: $0 < kr < 2$.

■

Proof:

By examining the convergence condition (6) and by taking into account (7) and (9), one obtains:

$$\left\| I - kr \cdot I \right\|_{\infty} < 1, \tag{10}$$

that allows us to write:

$$\forall \omega \in [0, \pi], \quad |1 - kr| < 1,$$

and gives us finally:

$$0 < kr < 2.$$

3.2 Case of Non Invertible Process

The idea in this case, is to put in the repetitive filter an estimate of the inverse of the process transfer matrix.

The inverse of the transfer matrix $G(z^{-1})$ appears under the form:

$$\left[G(z^{-1})\right]^{-1} = z^{d} \cdot D(z^{-1}) \cdot \left[N(z^{-1})\right]^{-1}. \tag{11}$$

One has:

$$N(z^{-1})^{-1} = \frac{adj\left[N(z^{-1})\right]}{\det\left[N(z^{-1})\right]}, \tag{12}$$

where $adj\left[N(z^{-1})\right]$ is the adjoint of matrix $N(z^{-1})$ and $\det\left[N(z^{-1})\right]$ is the determinant polynomial of $N(z^{-1})$. $\left[G(z^{-1})\right]^{-1}$ is stable if the roots of $\det\left[N(z^{-1})\right]$ are located inside the unit circle.

Let us decompose $\det\left[N(z^{-1})\right]$ into a polynomial containing the roots situated inside the unit circle $N^{+}(z^{-1})$ and another one containing both roots situated outside of the unit circle and possible delay $N^{-}(z^{-1})$:

$$\det\left[N(z^{-1})\right] = N^{+}(z^{-1}) \cdot N^{-}(z^{-1}). \tag{13}$$

We suggest to take the repetitive filter $H(z^{-1})$ under the form:

$$H(z^{-1}) = kr \cdot \frac{z^{d} \cdot D(z^{-1}) \cdot N^{-}(z)}{n \cdot N^{+}(z^{-1})} \cdot adj\left[N(z^{-1})\right]. \tag{14}$$

with $n \geq \max_{\omega \in [0,\pi]} \left| N^{-}(e^{-j\omega}) \right|^{2}$.

kr is called the repetitive filter gain and $N^{-}(z)$ is obtained by replacing every z^{-1} in $N^{-}(z^{-1})$ by z.

It is necessary to note that this filter has a strong similarity with the zero phase tracking controller (Tomizuka (1987)).

Theorem 3

The repetitive algorithm described by Figure 2, with the repetitive filter (14) for non invertible system (7), converges to zero error vector if and only if: $0 < kr < 2$.

■

Proof:

Let us examine the convergence condition (6). By taking into account (7) and (14) as well as (12), one obtains:

$$\left\| I - kr \cdot \frac{N^{-}(z^{-1}) \cdot N^{-}(z)}{n} \cdot I \right\|_{\infty} < 1. \tag{15}$$

One can then write for (15):

$$\max_{\omega \in [0,\pi]} \left(\left| 1 - \frac{kr}{n} \cdot \left[N^{-}(e^{-j\omega}) \cdot N^{-}(e^{j\omega}) \right] \right| \right) < 1,$$

that gives us:

$$0 < kr < \min_{\omega \in [0,\pi]} \left[2 \cdot \frac{n}{N^{-}(e^{-j\omega}) \cdot N^{-}(e^{j\omega})} \right],$$

and finally:

$$0 < kr < 2.$$

4 CASE OF AN UNSTABLE PROCESS

In the case of an unstable process, the scheme of the repetitive control (Figure 2) is modified and becomes:

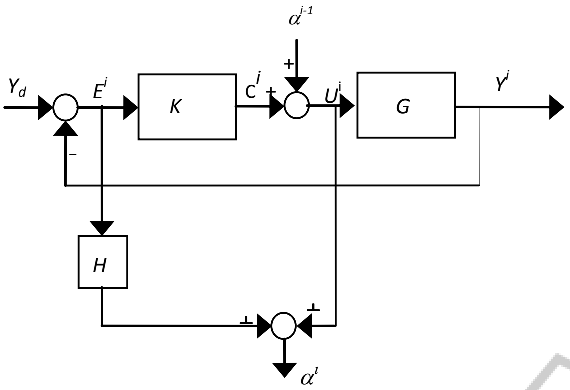

Figure 3: Repetitive control in closed loop configuration.

where $K(z^{-1})$ is a the controller transfer matrix that stabilizes the loop where $\dim(K) = n \times n$. C^i is the vector of the output controller's of dimension n, α^i of dimension n, is an anticipate vector function of the past error vector and the past control vector and i indicate the number of period.

Based on this scheme, the control law is then:

$$U^{i+1} = U^i + H \cdot E^i + K \cdot E^{i+1} . \tag{16}$$

By multiplying the left side of both terms of (16) by G, one obtains:

$$Y^{i+1} = Y^i + G \cdot H \cdot E^i + G \cdot K \cdot E^{i+1} , \tag{17}$$

and then:

$$E^{i+1} = E^i - G \cdot H \cdot E^i - G \cdot K \cdot E^{i+1} , \tag{18}$$

that gives us finally:

$$E^{i+1} = (I + G \cdot K)^{-1} \cdot (I - G \cdot H) \cdot E^i . \tag{19}$$

From (19), the repetitive algorithm will converge to zero error (under certain norm) if:

$$\left\| (I + G \cdot K)^{-1} \cdot (I - G \cdot H) \right\|_\infty < 1 , \tag{20}$$

and knowing that:

$$\| A \cdot B \|_\infty \le \| A \|_\infty \cdot \| B \|_\infty ,$$

where A and B are two complex matrices.

If one notes:

$$\left\| (I + G \cdot K)^{-1} \right\|_\infty = \gamma , \tag{21}$$

then the condition (20) becomes:

$$\left\| (I - G \cdot H) \right\|_\infty < 1/\gamma . \tag{22}$$

In the case of an invertible process, the repetitive filter $H(z^{-1})$ was taken like (9) and the convergence condition of the repetitive algorithm is given by the following theorem:

Theorem 4

The repetitive algorithm described by Figure 3 with the repetitive filter given by (9) and for invertible system (7), converges to zero error vector, if and only if:

$$1 - 1/\gamma < kr < 1 + 1/\gamma . \tag{23}$$

■

Proof:

By replacing in the convergence condition (22) G and H by their expressions given respectively by (7) and (9), one obtains:

$$\| I - kr \cdot I \|_\infty < 1/\gamma , \tag{24}$$

that leads to:

$$\forall \omega \in [0, \pi] \quad |1 - kr| < 1/\gamma ,$$

and gives us finally:

$$1 - 1/\gamma < kr < 1 + 1/\gamma .$$

In the case of a non invertible process, the repetitive filter $H(z^{-1})$ is taken by the expression (14). In that case, the convergence condition of the repetitive algorithm is given by the following theorem:

Theorem 5

The repetitive algorithm described by Figure 3 for non invertible system (7) and using the repetitive filter (14), converges to zero error vector if and only if:

$$\delta < kr < \beta , \tag{25}$$

with:

$$\delta = \max_{\omega \in [0,\pi]} \left[(1 - 1/\gamma) \cdot \frac{n}{N^-(e^{-j\omega}) \cdot N^-(e^{j\omega})} \right] .$$

$$\beta = \min_{\omega \in [0,\pi]} \left[(1 + 1/\gamma) \cdot \frac{n}{N^-(e^{-j\omega}) \cdot N^-(e^{j\omega})} \right] .$$

■

Proof:

From the convergence condition (22) and by taking into account (7), (14) and (12), one obtains:

$$\left\| I - kr \cdot \frac{N^-(z^{-1}) \cdot N^-(z)}{n} \cdot I \right\|_\infty < 1/\gamma . \tag{26}$$

With the same approach which was made for (15), one has:

$$\max_{\omega\in[0,\pi]}\left(\left|1-\frac{kr}{n}\cdot\left[N^{-}(e^{-j\omega})\cdot N^{-}(e^{j\omega})\right]\right|\right)<1/\gamma\ ,$$

and gives us finally: $\delta < kr < \beta$.

5 SIMULATION RESULTS

To highlight the theoretical developments made previously, let us consider the process described by the following stable transfer matrix:

$$G(z^{-1})=\frac{z^{-1}}{1+0.3z^{-1}}\begin{bmatrix}1 & 1+1.5z^{-1}\\ 1+0.2z^{-1} & 0\end{bmatrix}.$$

We are in the case of non invertible stable system (section 3.2).

Let us calculate $det(N)$ that we put under the shape given by (13):

$$N^{-}(z^{-1})=z^{-1}(1+1.5z^{-1})$$
$$N^{+}(z^{-1})=-(1+0.2z^{-1})$$

that allows us to give the following repetitive filter:

$$H(z^{-1})=\frac{kr}{-1-0.2z^{-1}}\cdot\begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix},$$

with:

$$H_{11}=0$$
$$H_{12}=-0.072z^{-1}-0.396-0.592z-0.24z^{2}$$
$$H_{21}=-0.096z^{-2}-0.944-0.28z-0.24z^{2}$$
$$H_{22}=0.048+0.232z+0.24z^{2}$$

From theorem 3, kr must be included between 0 and 2 so that there is convergence of the repetitive algorithm. We choose then $kr=1$. Figure 4 represents the convergence condition. We can see that it is respected, seen that $\|I-GH\|_{\infty}$ is lower than 1 like imposed by (6).

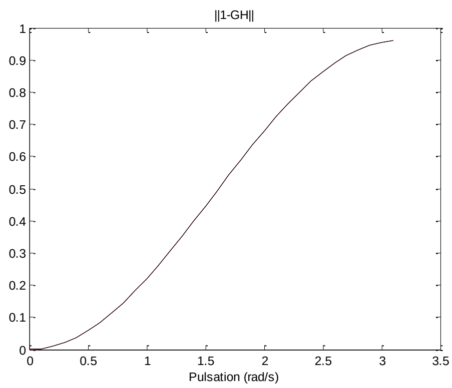

Figure 4: Convergence condition.

The purpose of this control is to track perfectly (with zero error) the periodic reference signals given by Figure 5 and which are represented over a period.

Figure 6 shows the behavior of the tracking error signals (e_1 and e_2) at the 30th period. One can see that there are practically zero. We obtain these results without inverting the process and consequently without divergence of the control signals u_1 and u_2, see Figure 7.

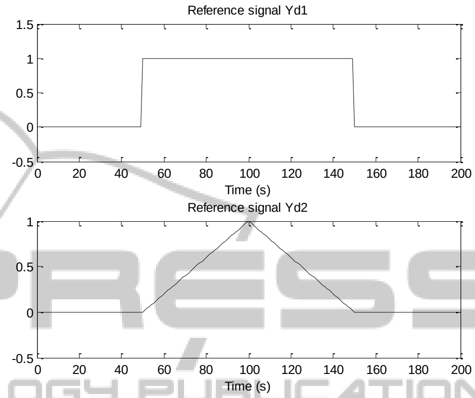

Figure 5: Reference signals.

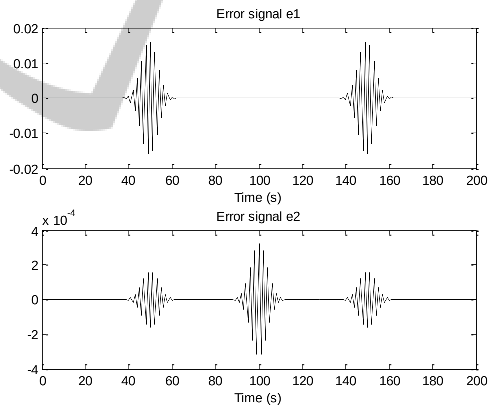

Figure 6: Control signal behavior at the 30th period.

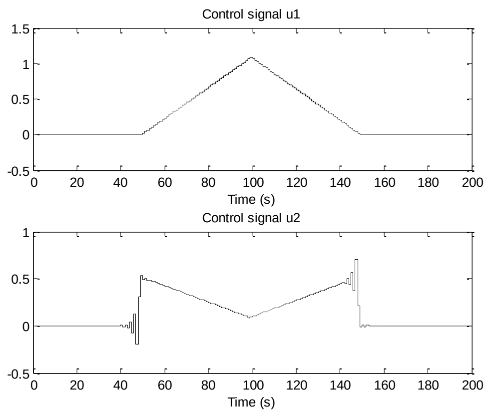

Figure 7: Control signal behavior at the 30th period.

6 CONCLUSIONS

In this paper, we have considered the problem of the repetitive control in the multivariable case by using the formalism of the transfer matrix. This formalism allowed us to consider the processes with stable and unstable inverse transfer matrix. Moreover, the case of a closed loop configuration was considered when the system to be controlled is unstable. This paper allowed us to generalize the solutions found for SISO systems. With this algorithm, we obtained good results (zero error vector) while avoiding the inversion of the process in order to have no divergence of the control signals.

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