# THE MICRO-GRID AS A STOCHASTIC HYBRID SYSTEM
## *Two Formal Frameworks for Advanced Computing*

Karel Macek[1,2] and Martin Střelec[1]
[1]*Honeywell Laboratories, Honeywell, V Parku 2326/18, Prague, Czech Republic*
[2]*Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University, Prague, Czech Republic*

Keywords: Micro-Grid Modeling, Stochastic Hybrid Systems.

Abstract: A Micro-Grid (MG) is an autonomous local energy network that involves various energy generation, consumption, storage, distribution and transfer devices. A MG energy management has to ensure satisfaction of energy demands through the coordination of generation and storage devices. Especially in complex MGs, significant savings can be achieved if the operation is optimized. For proper optimization, the system has to be described in sufficient detail to be used as the input for the optimization procedure. The major contribution of this paper is showing that a MG can be modeled as a Stochastic Hybrid Systems (SHS). Therefore, the usual tools for SHS can be applied here and solve practical problem. This is sketched out at the end of this work.

# 1 INTRODUCTION

MG has been defined more or less informally at many places (Chowdhury et al., 2009). This paper offers two formal frameworks for its modeling. Section 2 introduces the graph representation of a MG, its energy flows and balances. Afterwards, MG representation as a SHS is discussed in Section 3. In the Section 5, application of both frameworks will be illustrated on concrete examples that are motivated by several interesting potential applications. Some of them are being solved, implemented and tested within the MoVeS project[1] where the SHS are addressed in a multidisciplinary way.

# 2 GRAPH REPRESENTATION

**MG Structure.** Let $G = (\mathcal{V}, \mathcal{E})$ be an oriented graph. For each unit $v \in \mathcal{V}$ we introduce set of incoming edges $N^+(v) = \{(x, v) \in \mathcal{E}\}$ and out-coming edges $N^-(v) = \{(v, x) \in \mathcal{E}\}$. With respect to these sets, we can split the units into three disjoint groups $\mathcal{V} = \mathcal{V}_{in} \cup \mathcal{V}_m \cup \mathcal{V}_{out}$ where $\mathcal{V}_{in} = \{v \in \mathcal{V} | N^+(v) = \emptyset\}$, $V_{out} = \{v \in \mathcal{V} | N^-(v) = \emptyset\}$ and $\mathcal{V}_m = \mathcal{V} \setminus (\mathcal{V}_{in} \cup \mathcal{V}_{out})$. These sets represent the MG structure: $\mathcal{V}_{in}$ to the source units, $\mathcal{V}_m$ to transformation devices and transfer units, and $\mathcal{V}_{out}$ to terminal units.

[1]http://www.movesproject.eu

We distinguish transfer units $\mathcal{V}_{m,f} \subset \mathcal{V}_m$ and transformation or conversion units $\mathcal{V}_{m,c} = \mathcal{V}_m \setminus \mathcal{V}_{m,f}$ by the energy types. Let $\Phi$ be a finite set of energy types . Each edge $e \in \mathcal{E}$ is labelled by function $\phi : \mathcal{E} \to \Phi$, i.e. by type of energy flowing through it. Then, we can say that the transfer units are those where energies of the same type meet, i.e. $\mathcal{V}_{m,f} = \{v \in \mathcal{V}_m | \forall e_1, e_2 \in N^+(v) \cup N^-(v) : \phi(e_1) = \phi(e_2)\}$.

**Energy Flows and Internal States.** After the basic structure was introduced, we will list some quantities, related with this structure. We distinguish power flows on the edges and internal states in units $\mathcal{V}$.

Hence, we can consider the power flow $P_f : \mathcal{E} \times \mathcal{T} \to \mathbb{R}_+$ where $\mathcal{T}$ is an ordered set of time instants which can be either continuous or discrete. Hence $P_f(e, t)$ is the actual power flow in the edge $e \in \mathcal{E}$ at time instant $t \in \mathcal{T}$.

Further, we consider mapping $H : \mathcal{S} \times \mathcal{T} \to \mathbb{R}$ that represents internal states $\mathcal{S}$ (e.g. energy stored in a storage). Each internal state is considered to be associated with a unit $\mathcal{S}(v)$.

**Random Factors and Control.** Here, we will mention the random factors affecting the operational conditions of the MG. We use notation of a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ for all considered random factors.

The uncertainty affects the MG's external conditions like ambient temperature, humidity etc. Those conditions are denoted $d \in \mathcal{D} \subset \mathbb{R}^{n_d}$ and we use link function $\delta : \mathcal{T} \times \Omega \to \mathcal{D}$ for them so $d(t) = \delta(t, \omega)$.

Macek K. and Střelec M..
THE MICRO-GRID AS A STOCHASTIC HYBRID SYSTEM - Two Formal Frameworks for Advanced Computing.
DOI: 10.5220/0003952101410144
In *Proceedings of the 1st International Conference on Smart Grids and Green IT Systems* (SMARTGREENS-2012), pages 141-144
ISBN: 978-989-8565-09-9

The uncertainty affects also the power supplies $P_s$ at $\mathcal{V}_{in}$ and power demands $P_d$ at $\mathcal{V}_{out}$. Another factors influences particular unit $v \in \mathcal{V}$ are its controlled inputs. The control is considered to involve all possible decision in the system where we can consider various types of control strategies. Analogously to $\omega \in \Omega$ we will use notation $\lambda \in \Lambda$ for control. We use a link function $\zeta : (\mathcal{V}_{in} \cup \mathcal{V}_{out}) \times \mathcal{T} \times \Lambda \times \Omega$. Thus, we write:

$$P_s(u,t) = \zeta(u,t,\lambda,\omega) \qquad u \in \mathcal{V}_{in} \qquad (1)$$
$$P_d(u,t) = \zeta(u,t,\lambda,\omega) \qquad u \in \mathcal{V}_{out} \qquad (2)$$

Notation $P_d$ covers case when the demand is partly adjusted (covered by $\lambda$). Typical example is performing of demand response action where loads are adjusted according to utility requirements. Power demands are still partly influenced by random factors (covered by $\omega$). Also notation $P_s$ enables the possibility to consider random influences in source units (e.g. power grid instability).

Finally we make a link to actual setting of the MG. This setting is time dependent and the actual values are given by the strategy $\lambda$, but also by random factors $\omega$. So we can write $r \in \mathcal{R}$ and consider the link function $\rho : \mathcal{T} \times \Lambda \times \Omega \to \mathcal{R}$ so $r(t) = \rho(t,\lambda,\omega)$

The setting $r$ involve the unit commitment[2], the dispatch at the transfer units and other factors that can be affected in the MG with respect to these units.

**Source Units.** Let us assume the source units $\mathcal{V}_{in}$ have only one output, i.e. $N^-(v) = 1 \quad \forall v \in \mathcal{V}_{in}$. The power flow on the only edge is equal to the supply at the source unit:$P_f((u,v),t) = P_s(u,t)$.

**Transfer and Transformation Units.** For the units from $\mathcal{V}_m$ , a very general model is assumed in form of mappings, namely for each unit $u \in \mathcal{V}$ there exists a function $\xi_u : \mathbb{R}^{|N^+(u)|} \times \mathbb{R}^{|\mathcal{S}(u)|} \times \mathcal{D} \times \mathcal{R} \times (N^-(u) \cup \mathcal{S}(u)) \to \mathbb{R}$. This relationship has to be satisfied in the whole system, i.e. for output edges $(u,v)$:

$$P_f((u,v),t) = \xi_u\left( (P_f((w,u),t))_{w \in N^+(u)}, (H(s,t))_{s \in \mathcal{S}(u)}, d, r, v \right), \qquad (3)$$

and for internal states of each unit $u$, i.e. $s \in \mathcal{S}(u)$:

$$H(s,t) = \xi_u\left( (P_f((w,u),t))_{w \in N^+(u)}, (H(s,t))_{s \in \mathcal{S}(u)}, d, r, s \right). \qquad (4)$$

Mapping $\xi_u$ represents a kind of dispatch in the transfer units and some energy conversion in the transformation units. For generality sake, we do not provide any model details of particular units here.

Note that one shall consider the co-domain of the mapping $\xi_u$, that is $\mathrm{dom}(\xi_u)$, explicitly. E.g. the allowed chiller's input power is within an interval that depends on the ambient temperature.

**Terminal Units.** Analogically to the source units, we assume the terminal units will have only one input, i.e. $N^+(v) = 1$. At this point, however we do not assume the equality between flows and demands. Instead of this, we consider:

$$\Delta P(v,t) = P_d(v,t) - P_f((u,v),t) \quad \forall v \in \mathcal{V}_{out}, t \in \mathcal{T} \qquad (5)$$

The $|\Delta P(v,t)|$ shall be possibly minimal. However, the concept of demand satisfaction is still subject of investigation.

**Cost Model and Switching.** The basic cost model is given by costs for consumed resources for given price $c_r : \mathcal{T} \times \mathcal{V}_{in} \to \mathbb{R}_+$. Thus:

$$C_r = \sum_{v \in \mathcal{V}_{in}} \int_{\tau \in \mathcal{T}} c_r(\tau,v) P_s(v,\tau) d\tau. \qquad (6)$$

However, there are additional costs that have to be also consider. Significant additional costs represent costs related to switching the units on. The switching is given by the $r \in \mathcal{R}$. One can introduce simply a mapping $o : \mathcal{T} \times \mathcal{R} \times \mathcal{V}_m \to \{0,1\}$ telling whether the unit is on or off. Considering the unit $v \in \mathcal{V}_m$ and decisions $r \in \mathcal{R}$, we introduce the set of switching times $S(v,r)$ as follows:

$$S(v,r) = \{t \in \mathcal{T} | o(t,r,v) = 1 \wedge \lim_{\tau \to t^-} = 0\}. \qquad (7)$$

Consequently, the the costs related to the switching units can be represented as $c_s : \mathcal{V}_m \to \mathbb{R}$ and involved as $C_s = \sum_{v \in \mathcal{V}_m} c_s(v)|S(v,r)|$. Finally, the overall costs will be $C = C_s + C_r$.

**Problem Formulation.** At the very general level, the optimal control of a MG can be formulated in terms of stochastic objective $f_0$ and stochastic constraints $f_1, f_2, \dots f_{n_c}$ where $f_i : \Lambda \times \Omega \to \mathbb{R}$.

## 3 GENERAL STOCHASTIC HYBRID SYSTEM

General Stochastic Hybrid Systems (GSHS) are a class of stochastic continuous time hybrid dynamical systems which are characterized by hybrid state defined by two components: continuous and discrete state. GSHS captures not only dynamics, but mainly the interaction between both states (Bujorianu and Lygeros, 2006).

[2]Unit commitment determines whether given unit is switched on or off.

**Definition 1.** (Bujorianu and Lygeros, 2006) A General Stochastic Hybrid System (GSHS) is a collection $H = ((Q, n, \chi), b, \sigma, Init, \psi, R)$ where

- $Q$ is a countable set of discrete variables;
- $n : Q \to \mathbb{N}$ is a map giving the dimensions of the continuous state spaces;
- $\chi : Q \to \mathbb{R}^{n(\cdot)}$ maps each $q \in Q$ into an open subset $X^q$ of $\mathbb{R}^{n(q)}$;
- $b : X(Q, n, \chi) \to \mathbb{R}^{n(\cdot)}$ is a vector field;
- $\sigma : X(Q, n, \chi) \to \mathbb{R}^{n(\cdot)\times m}$ is a $X^{(\cdot)}$-valued matrix, $m \in \mathbb{N}$;
- $Init : \mathcal{B}(X) \to [0, 1]$ is an initial probability measure on $(X, \mathcal{B}(S))$;
- $\psi : \hat{X}(Q, n, \chi) \to \mathbb{R}_+$ is a transition rate function;
- $R : \hat{X} \times \mathcal{B}(\hat{X}) \to [0, 1]$ is a transmission measure.

**Definition 2.** A stochastic process $x_t = (q(t), x(t))$ is called a GSHS execution if there exists a sequence of stopping times $T_0 = 0 < T_1 < T_2 \leq \ldots$ such that for each $k \in \mathbb{N}$,

- $x_0 = (q_0, x_0^{q0})$ is a $Q \times X$-valued random variable extracted according to the probability measure $Init$;
- For $t \in [T_k, T_{k+1}), q_t = q_{T_k}$ is constant and $x(t)$ is a (continuous) solution of the SDE:

$$dx(t) = b(q_{T_k}, x(t))dt + \sigma(q_{T_k}, x(t))dW_t \ , \tag{8}$$

  where $W_t$ is a the $m$-dimensional standard Wiener process;
- $T_{k+1} = T_k + S^{i_k}$ is chosen according to a so called survivor function;
- The probability distribution of $x(T_{k+1})$ is governed by the law $R((q_{T_K}, x(T_{k+1}^-)), .)$.

# 4 ILLUSTRATIVE EXAMPLE

In this section, an example of the definition of a MG in abstract frameworks described in sections 2 and 3 is introduced. Figure 1 shows small scale MG that consists of several devices and includes two energy types - electrical and thermal energy. The MG is connected to the main distribution grid and includes the local power source (photovoltaic panels). Two chillers transform the electricity onto cooling energy that supplies cooling load or can be store in water tank. The cooling energy is distributed by water distribution circuits. Valves divide the cooling energy from chillers between the water tank and the cooling load.

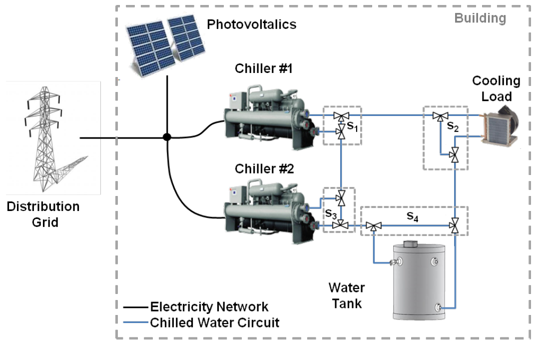


Figure 1: Example of a MG.

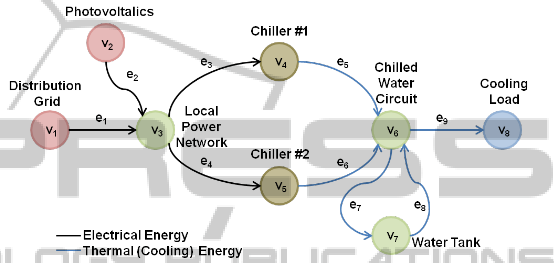


Figure 2: Graph Representation of the Microgrid.

**Microgrid in Graph Representation.** Figure 2 shows the graph representation of the considered MG according to the notation introduced in Section 2. Distribution grid and photovoltalics are two source units $\mathcal{V}_{in} = \{v_1, v_2\}$ that supply the MG with electrical energy. Local power network $v_3 \in \mathcal{V}_{m,f}$ constitutes the transfer unit that interconnects power generation and consumption side represented by two chillers $v_4, v_5 \in \mathcal{V}_{m,c}$. Chillers are transformation units which consume electricity and produce cooling (thermal) energy. Chilled water circuit represented by transfer unit $v_6 \in \mathcal{V}_{M,f}$ transfer cooling energy from chillers to terminal units $v_8 \in \mathcal{V}_{out}$. Water tank is an energy storage where thermal energy can be stored and can be later utilized. Because water tank $v_7 \in \mathcal{V}_m$ operates with same type of energy is considered as special case of transfer unit $\mathcal{V}_{m,f}$. Valves $s_1, s_2, s_3, s_4$ are part of the setting $r$ affecting energy flows $P_f$ on the graph's edges $\mathcal{E}$. The $r$ involves also (i) the chillers' commitment $o$, (ii) chillers' input power control $l$. Control of the storage tank is already cover by the valves $s_4$. Therefore setting $r$ can be written as $r \equiv (s_1, s_2, s_3, s_4, o(v_4), o(v_5), l)$. Energy management systems has to ensure satisfaction of cooling load via modulating of bypassing valves and via suitable chillers' and commitment. From the internal states, we will consider only the state of the storage tank $S(v_7) = \{p\}$ that is considered as cooling energy of the tank. Let its index be $p$, then we write the actual value of the water tank state (stored energy) as $P_{ch}(t) \equiv H(v_7, p, t)$. As the disturbances we consider the ambient temperature, solar radiation, and building's occupancy, i.e. $d = (T_{oa}, h, o)$. All of them can be considered as realization of random processes over $(\Omega, \mathcal{F}, \mathcal{P})$ and foretasted, potentially by external providers (weather). Based on these forecasts, expected cooling loads $P_d(v_8)$ and photovoltaics generation $P_s(v_2, t)$ can be estimated and consequently used for optimization. The grid power supply $P_s(v_1, t)$ is considered as subject of control. In case of transitions units, i.e. $v_3$ and $v_6$, the outputs depend on $l(t)$, respectively on the valves $s_1, s_2, s_3, s_4$. Finally the discharging power flow $P_f(e_8, t)$ from the water tank is determined by the state of charging $S(v_7, t)$ and the discharging control is given by valve $s_4$. The cost model $C$ is the same as introduced in general in the previous section. The link functions $\xi$ are in case chillers are based on the COP curves , depending on the input power flows ($P_f(e_3, t)$ and $P_f(e_4, t)$) and the ambient temperature $d_1 \equiv T_{oa}$. In case of transitions units, i.e. $v_3$ and $v_6$, the outputs depend on $l(t)$, respectively on the valves $s_1, s_2, s_3, s_4$. Finally the discharging power flow $P_f(e_8, t)$ is determined by the state of charging $S(v_7, t)$ and the discharging control $k(t)$. The cost model $C$ is the same as introduced in general in the previous section.

**Microgrid as GSHS.** In this section, relation between GSHS defined in Section 3 and given example of the MG is described. Set of discrete states $Q$ is given by commitment $o$, i.e. $Q = \{0, 1\}^2$. Mapping $n : Q \to \mathbb{N}$ determines the dimension that remains always the same, since the continuous state $x$ involves $d, r, (P_f)_{e \in \mathcal{E}}, P_{ch}, P_d(v_9), C$ in all the cases. The discrete dynamics is represented as decisions about the commitment $o$. They determine the sequence of stopping times $T$ that change operation of the MG.

The continuous dynamic can constitute thermal properties of water, dynamical behavior of chillers etc. Diffusion term $\sigma$ constitutes stochastic influences affecting continuous dynamic $b$, both are related to the functions $\delta, \zeta, \rho$ and $\xi$. In given example, cooling load is considered as a stochastic process that can not be controlled and can be described by (8).

## 5 POTENTIAL APPLICATIONS

Microgrid modeled as a stochastic hybrid system can be used for investigation whether given system can reach undesired discrete modes. Model checking theory offers methods for investigation, not only whether given model can reach given state, but with what probability as well (Baier and Katoen, 2008).

Important role of MG energy management system is to minimize of operational costs of a MG with satisfaction of supplies to all loads. This can be achieved by suitable scheduling and utilization of internal and external energy sources. Adopting of a SHS framework for MG modeling, we can use methods related to SHS theory for solving of a scheduling problem, e.g. via the scenario approach (Campi et al., 2009).

## 6 CONCLUSIONS AND FUTURE WORK

Objective of this paper is not to model particular parts of a MG, but offers better insight into two formal abstract framework where a MG can be modeled. Graph representation is more less traditional point of view to a MG which is motivated by practical realization of MG's energy management solutions. This representation was formalized in Section 2. Stochastic hybrid systems were introduced in Section 3 and constitutes a framework for multidisciplinary modeling. Considering MGs as SHS, the definitions and models would have to be developed in more detail, especially regarding the continuous dynamics. A high level example of modeling a simple MG in both frameworks was showed in Section 4. Finally, Section 5 sketches out the benefit of considering MG as a SHS, namely model checking examining extreme situations in the system and stochastic dynamic programming for efficient scheduling.

## ACKNOWLEDGEMENTS

Research was supported by the European Commission under the project MoVeS, FP7-ICT-257005.

## REFERENCES

Baier, C. and Katoen, J.-P. (2008). *Principles of Model Checking (Representation and Mind Series)*. The MIT Press.

Bujorianu, M. and Lygeros, J. (2006). Toward a general theory of stochastic hybrid systems. *Lecture Notes in Control and Information Sciences (LNCIS)*, 337:3–30.

Campi, M. C., Garatti, S., and Prandini, M. (2009). The scenario approach for systems and control design. *Annual Reviews in Control*, pages 149–157.

Chowdhury, S., Chowdhury, S., and Crossley, P. (2009). *Microgrids and Active Distribution Networks*, volume 6 of *Renewable Energy Series*. The Institution of Engineering and Technology.