

ONLINE MATHEMATICS EDUCATION

E-Math for First Year Engineering Students

Steen Markvorsen and Karsten Schmidt

DTU Mathematics, Technical University of Denmark, Copenhagen, Denmark

Keywords: E-learning, Multifaceted E-approach, First year University Mathematics.

Abstract: We consider the technology enhanced learning of first year engineering mathematics and in particular the application of E-learning objects and principles in the course Mathematics 1 which has a yearly intake of 750 students at the technical University of Denmark. We show that with non-linear multimedia technology and e-learning principles it is possible to strengthen and enhance the students' desire and ability to prepare for the teaching and to read and enjoy the textual representations of the course materials.

1 INTRODUCTION

With an enrollment of 750 students once a year the first year course Mathematics 1 at the Technical University of Denmark is one of the largest courses at university level in Denmark. Since its first re-formation 10 years ago a number of interesting and valuable assets concerning undergraduate math education in general have emerged from that course. These assets have now been transplanted into the next 2010 re-formation, which is our main concern in this report.

We are convinced that physical attendance to lectures as well as to exercise sessions are still of utmost importance for most students, but we are also convinced that the overall outcome of the teaching can be improved considerably with the aid of e-learning techniques.

The purpose of this paper is to show that it is possible to strengthen the students' preparation for the lectures and it is possible to strengthen their ability to learn from the textual representations of the mathematics in question. This can be done by introducing a wide spectrum of web-based learning objects and activities which build directly upon the students' own up-to-date knowledge and use of modern multimedia technology.

The strategy has two parts: First of all the individual students' preparation can be freed from the classical linear string of text and progression, which has been the classical way to learn mathematics. Secondly the e-learning technologies have the potential to create new feed back to the mathematical text in a way which eventually will facilitate the final syl-

labus or curriculum as something which is actually owned by the individual student, based upon his or her own experiences, work, and progression through the course. This specific and explicit ownership of the resulting text makes it readily available also for later courses.

Specifically we show that it is possible to design web based learning which satisfies these goals via the following three-step strategy:

- Present the main topics, concepts, and points in many very different ways
- Provide help through links and video examples
- Replace traditional textbooks by easily accessible and easy-to-search eNotes, which contain more layers of understanding, and offer more and different ways of reading and study.

We will give a few glimpses from the present architecture of the course and discuss some results and challenges concerning such a modern large scale and multifaceted teaching of first year undergraduate calculus and linear algebra. The curriculum material in question is worldwide usually considered as standard service material and thus taught in standard courses as a much needed background for almost all engineering disciplines. Correspondingly these courses are also usually taught from standard textbooks and in a very standard way.

Our main goal with this paper is to point out that even with a fairly well established essentially invariant curriculum we have by now a unique chance to apply the computer driven technologies and internet facilities to enhance the teaching of undergraduate

mathematics and to inspire the young students even more into the learning of basic linear algebra and calculus.

2 BACKGROUND

In 2007-2008 DTU Mathematics conducted a thorough investigation of the study habits among the students in the course Mathematics 1, see (Ratleff et al., 2009). The focus of the evaluation was partly on the students' participation in the various learning activities (preparations for the lessons, participation in the lectures and in the exercises, and concerning their strategies for homework) and partly on their use of learning resources (text books, demo worksheets in Maple, and the students' own notes, respectively). The primary goal of the investigation was to find out about the role and importance of the traditional textbook for the learning process. The result was meant to support a decision concerning the pertinent question whether the textbooks (which were from 1992-1993) should be reprinted, new texts should be written, or we should simply use different learning resources.

The assessment showed for example that the students to a large degree change their study habits during the first year of study. They typically find their own strategy for survival as a way to get through and pass the course. One of these strategies is expressed in the title of the ensuing report: "I read less, but understand more" – which is a direct citation from one of the students who were interviewed.

For the present paper the following three specific conclusions from the 2007-2008 assessment are of particular importance:

1. The students' preparation for the teaching was generally decreasing throughout the year. After the 3rd week of the course 65% of the students reported that they had generally prepared for the classes; this number had decreased to 30 % after week 19. The time used for the preparation had likewise declined in the same period by more than 40 %.
2. The importance of the textbooks for and in the learning process was also strongly declining during the year of study. The students were asked how much of their learning outcome could be ascribed to the individual activities and resources. The average was 12 % for the text books after week 3 in the course and then only 6% after week 19. Moreover it was clear from the responses that the textbooks were mostly used for skimming the text and for looking up formularies and examples.
3. The importance of Maple as a facilitator for doing e.g. the homework and exercises was, on the other hand, increasing. In the above question concerning the learning outcome the corresponding number for the effect of using the Maple demo work sheets increased from 6 % in after week 3 to 12 % after week 19.

The 2007-2008 assessment showed that the extensive use of Maple – in particular the Maple demo work sheets – to some extent also substituted the classical use of the textbooks. There were both positive and negative effects of this. Some students reported that the more experimental work with Maple implied a better understanding of the concepts and topics, whereas others admitted that they often used Maple as a black box solver without actually understanding the mathematics behind. We refer to (Schmidt et al., 2010) which contains a deeper analysis of the pitfalls and potentials that are associated with the extensive use of Maple in a first year mathematics course like Mathematics 1.

The decrease in the time for and quality of preparation for the teaching and the decline in the actual classical intended use of the textbooks seems to be quite correlated. One of the most important starting points for the current development and re-thinking of the course is our conviction that a carefully paced and possibly individually enhanced preparation is a much needed prerequisite for the students' outcome from lectures and exercises. Moreover we are convinced that the ability and the desire to read and to cope with the mathematical concepts in a textual representation is of utmost importance for transfer – i.e the students' ability to carry over and actually use the mathematics in other disciplines and technology applications.

3 WEB-BASED LEARNING OBJECTS

Based on the findings reported in the above section, DTU Mathematics decided in 2010 to replace the traditional textbooks and the corresponding printed weekly menus by a Mathematics 1 portal.

As mentioned above we believe that physical attendance, lectures and group exercises are crucial hallmarks of effective teaching and learning, but we also face the facts and challenges that the course in question is mandatory for more than 750 first year

students, that it must give fundamental mathematical training and services to 11 very different study lines at DTU, and that each one of our students at the same time is struggling to find the working methods and styles which are best suited for him or her at DTU.

Therefore we have introduced a wide range of web based learning objects and activities into the teaching agenda - to support and drive the good old classical curriculum but now also to open up and offer plenty of windows and pathways into the core material. Traditional teaching materials such as textbooks and printed exercises and weekly sheets are replaced by the internet portal where YouTube-based video instructions, e-Notes and electronic exercises suggest and offer a much less linear form of learning than before. In order to create both new solid challenges for the strongest students and at the same time keep the weaker students well motivated and engaged we have found that the assets mentioned in the introduction (the three bullets) are of invaluable importance for this enterprise.

In particular there are then obvious new challenges that the authors of the e-Notes (replacing the traditional textbooks) are facing. The e-Notes must contain several layers corresponding to different purposes: They must display the concepts and the theory, they must serve as the target for links from the online exercises, they must provide inspirations for videos, they must take advantage of and unfold parallel visually enhanced exercises, and finally they must provide several choices for printing "your own book" - depending on which parts of the content the reader wants to emphasize. Similarly the e-Notes should offer flexible and different ways of reading: studying contiguous text, skimming, finding theorems and formulas, checking examples etc.

Originally our working title for the e-Notes was *transfer notes*. We consider them as an integral – yet compactly distilled – part of the whole enterprise. They function as a constant reference for all the other activities so that there is in principle a link to the relevant section, example, or theorem from every other activity in the course. The idea is thus twofold – to keep the e-Notes as a *text* and to actively help lifting this text into every corner of the course. Technologically the text is constantly only one click away – through the portal of the course. We have the hope – to be seen and reported – that the students in this way will be able to 'carry' and 'transfer' what they have learnt much more readily into their next courses in the engineering disciplines.



Figure 1: All lectures are available on video.

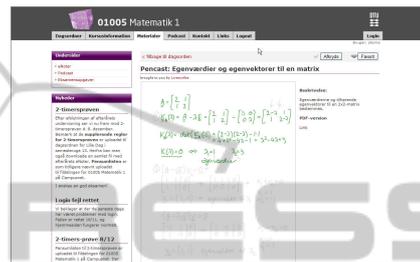


Figure 2: Look and listen to a careful pencast solution.

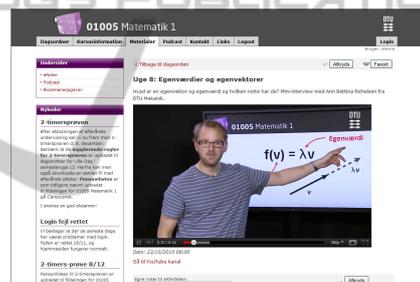


Figure 3: Prepare for the lecture – view the appetizer video!.

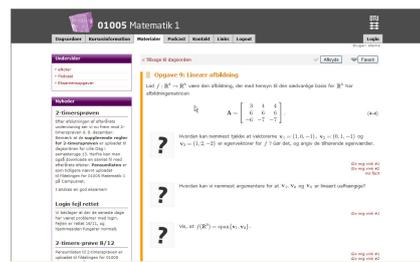


Figure 4: An exercise with clearly marked questions.

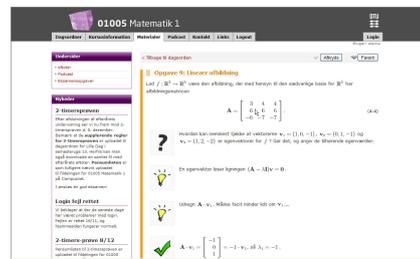


Figure 5: The exercise with 3 unfolded hints.

3.1 Balanced and Useful Sample Codes

The Maple system is a well functioning example of a merger between a calculator, pen and pencil, formularies, and editor tool which is used throughout the course for solving the math problems, for documenting homework, and for handling larger project exercises. All the figures in the eNotes are produced from simple Maple commands – so that the students can do their own modifications of those figures directly in their own work sheets and thereby enhance their study and reading of the eNotes even further, see Figure 6.

The screenshot shows a page from an eNote titled 'Sætning 27.18 Stokes' sætning i planen'. It contains mathematical derivations for Stokes' theorem in the plane. The equations are:

$$\int_C \text{Rot}(\mathbf{V}) \cdot n_r \, d\mu = \int_{\mathcal{D}} \mathbf{V} \cdot e_{\theta} \, d\mu$$

$$\int_C \text{Rot}(\mathbf{V}) \cdot (0, 0, 1) \, d\mu = \int_{\mathcal{D}} (V_1 V_2 - V_2 V_1) \cdot e_{\theta} \, d\mu$$

$$\int_C \text{Rot}(\mathbf{V}) \cdot (0, 0, 1) \, d\mu = \int_0^{\pi} (V_1 V_2 - V_2 V_1) \cdot b'(\theta) \, d\theta$$

$$\int_C \left(\frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} \right) d\mu = \int_0^{\pi} (V_1 b_1'(\theta) + V_2 b_2'(\theta)) \, d\theta$$
 and

$$\int_C \left(\frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} \right) \text{Jacobi}(u, v) \, d\mu = \int_0^{\pi} (V_1 b_1'(\theta) + V_2 b_2'(\theta)) \, d\theta$$
 The page also features three 3D plots: a vertical bar chart, a sphere with a grid, and a vector field plot. A large watermark 'SCITEPRESS AND TECHNOLOGY PUBLICATIONS' is visible across the page.

Figure 6: A glimpse of a page from the eNotes.

4 EVALUATION AND RESULTS

As a contribution to the evaluation of the 2010 reformation of the course Mathematics 1, DTU Mathematics has conducted a series of investigations in collaboration with LearningLab DTU, both before and after the 2010 reformation. We display here a few selected results, which contain information about the students' preparation for the teaching and about their outcome from the respective textual representation of the mathematical content and syllabus.

After week 8 in the study year 2009-2010 (i.e. before the reformation of 2010) and correspondingly after week 8 in 2010-2011 the students were asked to prioritize the various teaching activities and state from which activities they have learnt the most – in their own opinion. The total score concerning preparation, lectures, and exercises in classes displayed as in Table 1. The table shows that the active individual preparation before the teaching has been strengthened at the expense of the more passive listening to lectures.

Correspondingly the students were asked to pri-

Table 1: Increasing preparation.

Year	Preparation	Lectures	Exercises
2009	13.1 %	44.3 %	42.6 %
2010	21.6 %	37.0 %	41.5%

oritize the learning resources. The total scores concerning textbooks/eNotes, Maple Demos, and the students' own notes from lectures are shown in Table 2. The table shows that the students' appreciation of the new eNote representation of the mathematical content of the course has increased considerably at the expense of the Maple Demos and their own notes.

Table 2: From textbooks to eNotes.

Year	(e-)Text	Maple Demos	Own notes
2009	26.8%	38.0 %	35.2 %
2010	45.8 %	28.8 %	25.5 %

5 CONCLUSIONS

Taken as a whole the two tables 1 and 2 indicate that the combined effects of a wide spectrum of web-based learning objects has helped fulfil the overall aim: To strengthen the students' desire and ability to prepare for the teaching and to read and enjoy the textual representations of the course contents.

ACKNOWLEDGEMENTS

The investigation and the data that we mention in section 4 have been designed and collected by P. M. Hussmann (LearningLab DTU), P. Ratleff (Danish School of Education), H. Rotzén (DTU Informatics), and K. Schmidt (DTU Mathematics).

REFERENCES

Ratleff, P., Schmidt, K., and Hussmann, P. M. (2009). Reading less and understanding more. In *MONA, (2009) 3*, pp. 21-40.

Schmidt, K., Rattleff, P., and Hussmann, P. M. (2010). The impact of CAS use in introductory engineering mathematics. In *Progress in Industrial Mathematics at ECMI 2008. Mathematics in Industry, Volume 15, Part 2, (2010) 653-659*. Springer Verlag.