IMAGE RESTORATION VIA HUMAN PERCEPTION AND LIE GROUPS*

Vittoria Bruni[1], Elisa Rossi[2] and Domenico Vitulano[2]
[1]*Dept. SBAI University of Roma Sapienza, Via A. Scarpa 16, 00161 Rome, Italy*
[2]*Istituto per le Applicazioni del Calcolo, C.N.R., Via dei Taurini 19, 00185 Rome, Italy*

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Abstract: This paper presents a novel and user's independent model for image restoration based on Lie group transformations. They allow to define a redundant set of transformations from which automatically select the ones that better invert the physical formation of the defect. Hence restoration consists of gradually reducing the visual perception of the defect. Extensive experimental results on original photographs, from Alinari Archive, affected by semi-transparent blotches show the potential of the proposed approach in removing degradation in different contexts without altering the original content of artworks.

1 INTRODUCTION

The growing demand for archived material as well as the improvement of computer technologies has led to the need of reliable and useful tools for digital and (semi)-automatic removal of degradation, such as noise, blotches, line-scratches, tear, moire, shake and flicker (Kokaram, 1998).

The model proposed in this paper focuses on a generalization of the plethora of existing restoration methods for the aforementioned kinds of degradation — see next section for a short review. In order to make more concrete the content of the paper, from now on we will focus on the class of semitransparent degradations whose main peculiarity is that part of the original information often survives in the degraded area (Crawford et al., 2007). The main goal is the definition of a general framework that is as much as possible independent of a priori specific assumptions on the degradation under exam. The framework is required to select suitable restoration transformations from a redundant set only accounting for the semi-transparency of the degraded region. To this aim, the visual perception of degradation cannot be neglected (Winkler, 2005). Image defects are detected by human eye 'at first glance' even in complicated contexts. This means that degradation represents an 'anomaly' in any natural image. Hence, the reduction of the visual contrast of the degraded region (visual anomaly) should decrease the visual contribution of the degraded area without creating new artifacts. A local contrast-based restoration process that embeds transformations in a Lie group gives us the opportunity of defining a redundant set of transformations that also contains the inversion of the unknown degradation process. In addition, it allows to develop a restoration algorithm that automatically selects the more suitable transformations for points having the same visual contrast.

This approach is two-fold innovative:

i) The combination of Human Visual System (HVS) and Lie algebra allows the proposed model to have not a precise target to converge — as it usually happens in Lie algebra based approaches (see, for instance, (Drummond and Cipolla, 2000; Porikli et al., 2006; Mansouri and Mukherjee, 2004; Mukherjee and Acton, 2007)). The exact final solution is not known in advance and the model is only required to force the contrast of the final solution to be in a suitable range of values according to 'typical' contrasts of the surrounding clean image — blotch has to be invisible. To this aim, the modified successive mean quantization transform (SMQT) (Nilsson et al., 2005) is employed for contrast-based classification of degraded pixels;

ii) The rich set of Lie groups transformations overcomes the search of the solution of the restoration

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problem through simple translation and shrinking operations, that are commonly used by existing competitors.

The reminder of the paper is the following. Next section contains the motivations of the work and a short state of the art of image restoration methods. Section 3 briefly introduces Lie group transformations. Section 4 presents the proposed restoration model. Finally, Section 5 contains results and concluding remarks.

2 MOTIVATION OF THE WORK

Although semitransparent degradation includes defects like blotches, scratches and so on, for the sake of clarity, we will focus on the former.

Blotches are usually caused by dirt or moisture on archived material that partially or completely obscures some image regions. They appear as irregularly shaped regions with a darker average intensity and a slight variation in color, as shown in Fig.1. Hence, the lack of distinctive features, like shape and color, makes their detection and restoration not trivial tasks. Blotches must be restored using clean information from the immediate vicinity of the blotch, although this is not essential. However, part of the original information still survives even after the degradation process. This is due to its physical formation that can be modeled by the spreading and penetration of water droplets into material. During the spreading process, the radius of the wet region grows to an equilibrium point. From this point on, the liquid is absorbed depending on the porosity of the considered medium. In ideal conditions, the central pores absorb more than the external pores, since they come in contact with the liquid earlier. For complete spreading and absorption processes, one can expect a smooth transition to the unaffected area; on the contrary a spurious edge is evident.

Hence, the degraded image J at the point $\mathbf{x} = (x,y)^T$ can be modeled as

$$J(\mathbf{x}) = \mathcal{T}(I(\mathbf{x})),$$

where $\mathcal{T}$ is a proper composition of transformations and I is the original image. The goal should be to find the inverse of $\mathcal{T}$ in order to reconstruct the original image I. Unfortunately, the evolution of a drop involves different parameters, such as drop geometry and the regularity of paper surface, that are unknown in real applications. Hence $\mathcal{T}$ is unknown as well as its inverse $\mathcal{T}^{-1}$. The proposed model employs the projective Lie group as a redundant set of transformations where automatically select the best $\mathcal{T}^{-1}$. The selected transformations are not global but they are adapted to the local properties of the damaged area, according to a contrast-based classification. External information is involved just in the definition of the admissible range values for degraded pixels and for comparing global measures like the inner contrast between the damaged area and its surrounding regions, making the model quite independent as of external features.

2.1 Blotch Restoration: A Short State of the Art

A good restoration method is required to preserve all the original content of the artwork for historical and artistic purposes. Inpainting methods or texture synthesis approaches (Bertalmio et al., 2000; Bertalmio et al., 2003; Criminisi et al., 2004; Efros and Freeman, 2001; Kokaram, 2002) are not appropriate for the restoration of partially missing data regions, since they completely discard the original information that is still contained in those regions.

On the contrary, existing approaches that exploit the semi-transparency property, like (Crawford et al., 2007; Stanco et al., 2005; Greenblatt et al., 2008; Bruni et al., 2011), make implicit or explicit assumptions on the physical model that causes the degradation so that their restoration consists of 'ad hoc' operations. For example, in (Stanco et al., 2005) semi-transparency is modeled as a linear dependence between the intensity values of the degraded and original region, assuming similar statistic features inside and outside the degraded one. A non linear model closer to the visual appearance of degradation is used in (Greenblatt et al., 2008): it uses *flattening* to emphasize blotches darkness and *enhancement* to exploit the local image statistics. Affine point-wise transformations are employed in (Crawford et al., 2007) and (Bruni et al., 2011). While the former involves a minimization algorithm that emphasizes the propagation of information from the outside-in, the latter relies on a precise model of both physical and visual characteristics of the specific degradation kind.

3 A SHORT REVIEW ABOUT LIE ALGEBRA

A finite *Lie group* G is a set of elements equipped with a group multiplication $* : G \times G \rightarrow G$ such that:

i) $\forall g_1, g_2, g_3 \in G\ (g_1 * g_2) * g_3 = g_1 * (g_2 * g_3)$;

ii) $\exists\, e \in G$, identity element, such that $\forall\, g \in G\ e * g = g * e = g$;

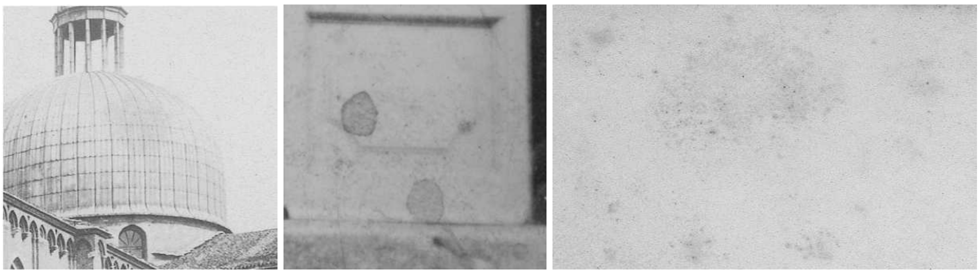

Figure 1: Examples of semi-transparent blotches in real photographs.

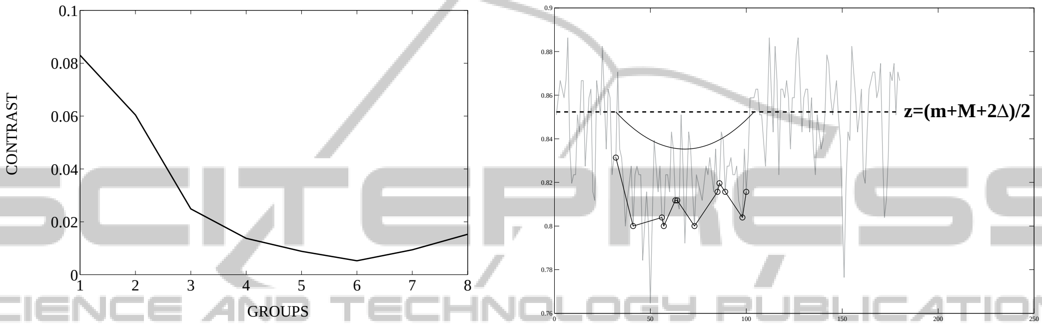

Figure 2: Left: Contrast curve γ of B. Right: Plane section of a blotch (*light gray*), the surface of the $i-th$ group (piecewise curve) and the corresponding paraboloid (dark and solid line).

iii) $\forall\, g \in G \quad \exists\, g^{-1} \in G : g * g^{-1} = g^{-1} * g = e$

and a differentiable manifold of finite dimension, i.e. a space locally diffeomorphic to R^n, if n is the dimension of G. Moreover the group operation $*$ and the inverse map ($G \to G$, $g \mapsto g^{-1}$) are C^∞ with respect to the differentiable structure of the manifold. So a Lie group G has algebraic properties coming from the group structure and geometric properties coming from the differentiable structure and they are related. Finally, every finite Lie group can always be viewed as a matrix group.
A *Lie algebra* $\mathfrak{g}$ is a vector space endowed with a bilinear operation,

$$[\ ,\ ] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}, \quad (X,Y) \mapsto [X,Y],$$

called Lie bracket, antisymmetric and satisfying Jacobi identity.

If G is a Lie group, its tangent space at identity, $\mathfrak{g}$, which is a vector space, has a Lie algebra structure. Hence, $\mathfrak{g}$ is a vector space of the same size of G endowed with a Lie bracket. If G is a matrix group, the Lie bracket is the matrix commutator, i.e. $[X,Y] = XY - YX$. Since G is a differentiable manifold, there exists a correspondence between its tangent space at identity (the Lie algebra $\mathfrak{g}$) and G itself, that is the exponential map, $\exp : \mathfrak{g} \to G$. Let $X \in \mathfrak{g}$ be a tangent vector at e in G; locally there exists the integral curve of X, a smooth curve starting from the identity with tangent vector X, i.e. $\gamma : [0,T] \to G$ such that $\gamma(0) = e$ and $\dot{\gamma}(0) = X$; we define $\exp(X) = \gamma(1)$. The exponential map is a local diffeomorphism around the origin of $\mathfrak{g}$ and it gives a natural way to move from $\mathfrak{g}$ (vector space) to G (manifold). For the matrix group, it corresponds to matrix exponential: $\exp(X) = \sum_{n=0}^{\infty} \frac{X^n}{n!}$. For a complete treatment of Lie groups and Lie algebras see (Varadarajan, 1974) and (Helgason, 1962).

3.1 Lie Group Transformations

Most of the matrix Lie groups can be used to describe transformations in the plane or in the space. For instance, rotations in the plane are represented by the group

$$SO_2R = \left\{ \left(\begin{smallmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{smallmatrix} \right), \theta \in R \right\}.$$

The dimension of the group can be thought as the number of free parameters needed to describe the transformations. In case of plane rotation we just need one parameter, θ, so the dimension of SO_2R is 1. We can think about its Lie algebra elements, which are tangent vectors at the identity, as the infinitesimal transformations (rotation of an 'infinitesimal angle') of the points.

In this paper we are interested in using projective transformations in the space and they can be described as a group matrix, P_3, acting on the space points expressed in homogeneous coordinates, with the convention that the fourth value in the coordinates is always scaled back to 1. Projective transformations are characterized by 15 parameters, that is the dimension of P_3, described by the following basis of its Lie algebra representing translations, scaling, shear and projections:

$$G_1 = \begin{bmatrix} 0&0&0&1\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix}, G_2 = \begin{bmatrix} 0&0&0&0\\0&0&0&1\\0&0&0&0\\0&0&0&0 \end{bmatrix}, G_3 = \begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0 \end{bmatrix},$$

$$G_4 = \begin{bmatrix} 0&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&0 \end{bmatrix}, G_5 = \begin{bmatrix} 0&0&-1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0 \end{bmatrix},$$

$$G_6 = \begin{bmatrix} 0&1&0&0\\-1&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix}, G_7 = \begin{bmatrix} 1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix},$$

$$G_8 = \begin{bmatrix} 0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix}, G_9 = \begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0 \end{bmatrix}, G_{10} = \begin{bmatrix} 0&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix},$$

$$G_{11} = \begin{bmatrix} 0&0&1&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix}, G_{12} = \begin{bmatrix} 0&0&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&0 \end{bmatrix},$$

$$G_{13} = G_1^T, G_{14} = G_2^T, G_{15} = G_3^T,$$

with T the transpose operator.

Hence, every real linear combination of $G_1,\ldots,G_{15}$ is an infinitesimal projective transformation in the space that corresponds to a transformation of the group P_3 thanks to the exponential map. The infinitesimal transformation of a generic point $p = \begin{bmatrix} x\\y\\z \end{bmatrix}$ is $\tilde{L}_j = G_j \begin{bmatrix} x\\y\\z\\1 \end{bmatrix}$ $j = 1,\ldots,15$ whose affine coordinates L_j respectively are

$$L_1 = \begin{bmatrix} 1\\0\\0 \end{bmatrix} \; L_2 = \begin{bmatrix} 0\\1\\0 \end{bmatrix} \; L_3 = \begin{bmatrix} 0\\0\\1 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} 0\\z\\-y \end{bmatrix} \; L_5 = \begin{bmatrix} -z\\0\\x \end{bmatrix} \; L_6 = \begin{bmatrix} y\\-x\\0 \end{bmatrix}$$

$$L_7 = \begin{bmatrix} x\\0\\0 \end{bmatrix} \; L_8 = \begin{bmatrix} 0\\y\\0 \end{bmatrix} \; L_9 = \begin{bmatrix} 0\\0\\z \end{bmatrix}$$

$$L_{10} = \begin{bmatrix} y\\0\\0 \end{bmatrix} \; L_{11} = \begin{bmatrix} z\\0\\0 \end{bmatrix} \; L_{12} = \begin{bmatrix} 0\\z\\0 \end{bmatrix}$$

$$L_{13} = \begin{bmatrix} x^2\\xy\\xz \end{bmatrix} \; L_{14} = \begin{bmatrix} xy\\y^2\\yz \end{bmatrix} \; L_{15} = \begin{bmatrix} xz\\yz\\z^2 \end{bmatrix}.$$

3.2 Surfaces Distance Minimization by Projective Transformation

The relation between Lie algebras and Lie groups allows us to define an iterative procedure able to map a given surface S_1 to another one, S_2, in $\mathbb{R}^3$. $\forall p \in S_1$, let n_p be the unit normal at S_1 in the point p and d_p the distance between p and S_2 along n_p. Hence, $\sum_{p\in S_1} d_p$ is the distance between S_1 and S_2.

Let L_j^p, for $j = 1,\ldots,15$, be the infinitesimal projective transformation L_j applied to the point p. The goal is to estimate 15 real parameters, $\alpha_1,\ldots,\alpha_{15}$, such that the infinitesimal projective transformation $\sum_{j=1}^{15} \alpha_j L_j^p$, projected onto the normal direction n_p, minimizes the distance between S_1 and S_2, i.e.

$$(\alpha_1,\ldots,\alpha_{15}) = \min_{\alpha_j} \sum_{p\in S_1} \left[d_p - \sum_{j=1}^{15} \alpha_j \left(L_j^p \cdot n_p \right) \right]^2.$$

Therefore, $\vec{\alpha} = (\alpha_1 \ldots \alpha_{15})^T$ is such that $\vec{\alpha} = \vec{A}^{-1}\vec{b}$, where the matrix $\vec{A}$ and the column vector $\vec{b}$ are

$$A_{jk} = \sum_{p\in S_1} \left(L_j^p \cdot n_p \right) \left(L_k^p \cdot n_p \right)$$

and

$$b_k = \sum_{p\in S_1} d^p \left(L_k^p \cdot n_p \right).$$

The exponential map transforms the infinitesimal transformation $G = \sum_{j=1}^{15} \alpha_j G_j$ into a projective transformation T of the group i.e.,

$$T = \exp(G) = \sum_{n=0}^{\infty} \frac{G^n}{n!}.$$

Finally, S_1 is updated applying T to its points: $S_1^{(1)} = T(S_1)$. The minimization process can be then iterated using the couples of curves $(S_1^{(1)}, S_2)$, and so on.

For the numerical computation of $\exp(G)$ applied to generic point $\mathbf{x}$, a 4th order Runge Kutta algorithm can be used — see (Drummond and Cipolla, 2000) for details. It is equivalent to cut the 4th order series expansion of the matrix exponential and apply it to the point $\mathbf{x}$, that is

$$T(\mathbf{x}) \approx \left(I + G + \frac{1}{2}G^2 + \frac{1}{6}G^3 + \frac{1}{24}G^4 \right)(\mathbf{x}),$$

but it directly manages affine coordinates.

4 THE PROPOSED MODEL

Lie group transformations offer a redundant set of transformations as well as a direct and fast method for

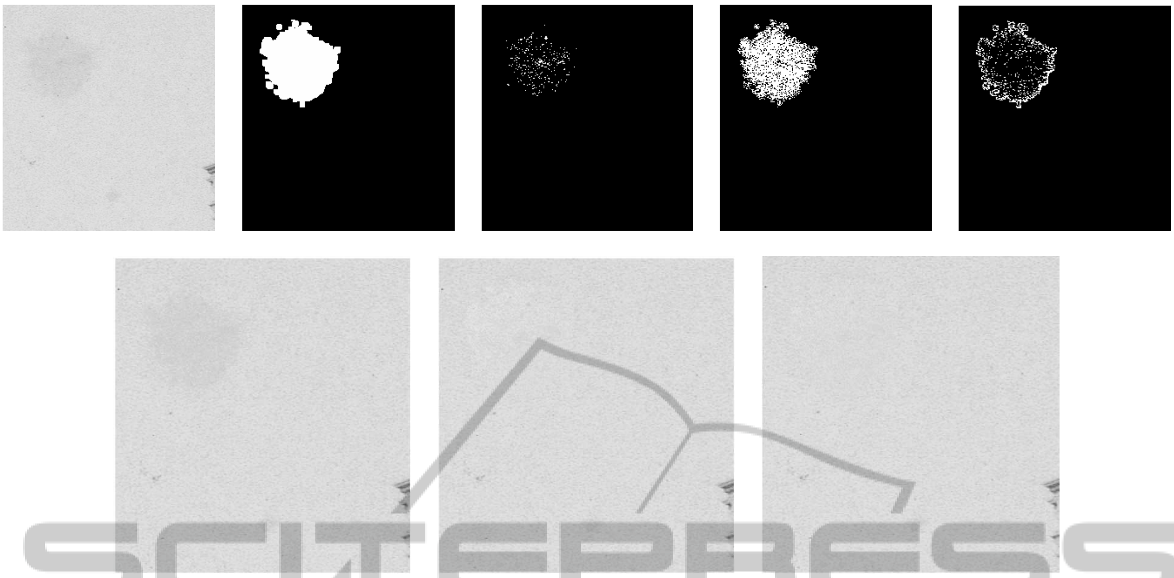

Figure 3: Top: Degraded image, its detection mask, D_1, D_2 and D_3 masks. Bottom: Restored images after: processing D_1 (left), processing D_2 (middle) and final masking (right).

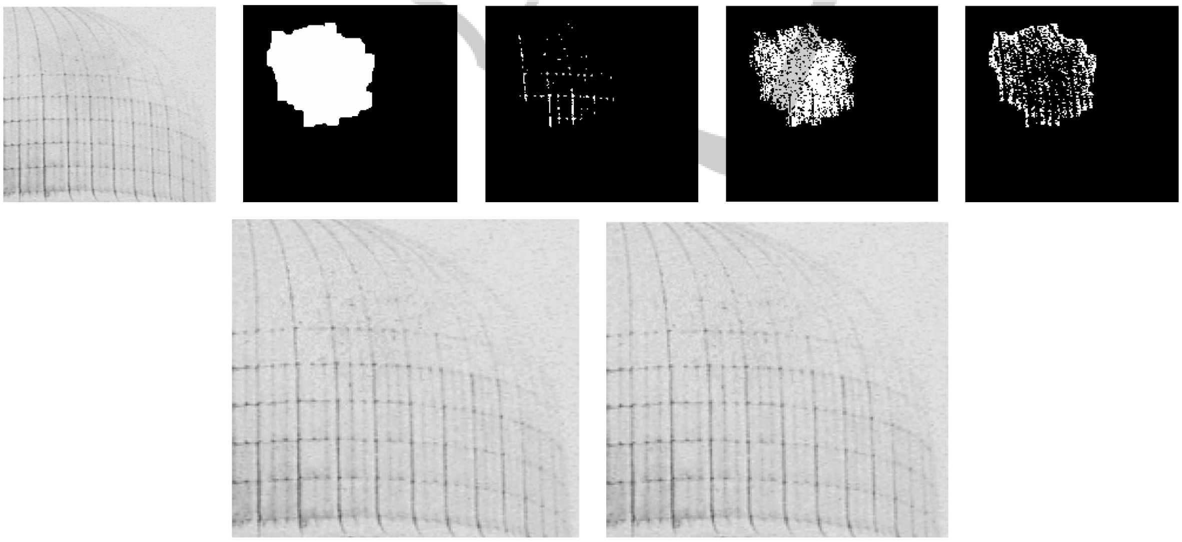

Figure 4: Top: Original degraded image, its detection mask, D_1, D_2 and D_3 masks. Bottom: Restored images after: processing D_2 (left) and final masking (right).

selecting them once the final target curve is known. Unfortunately, in case of digital restoration the final clean image is unknown. That is why some basic rules of HVS are used for defining suitable ranges of intensity values for the damaged area to be not visible with respect to its neighborhood. Despite the wide flexibility of Lie transformations, the minimization process is global. In other words, at each step the parameters $\{\alpha_j\}_{j=1,\dots,15}$ are the same for each point. Hence, if on one hand global transformations preserve the original information contained in the degraded region, on the other they forget that pixels may have been subjected to a different amount of degradation.

In order to find a tradeoff between preservation of original information and model flexibility,it is necessary to classify damaged pixels accounting for their visual importance and restore them accordingly.

4.1 Preprocessing of the Damaged Area

Let I be the analyzed image, B its damaged area and let E be a sufficiently small neighborhood of B such

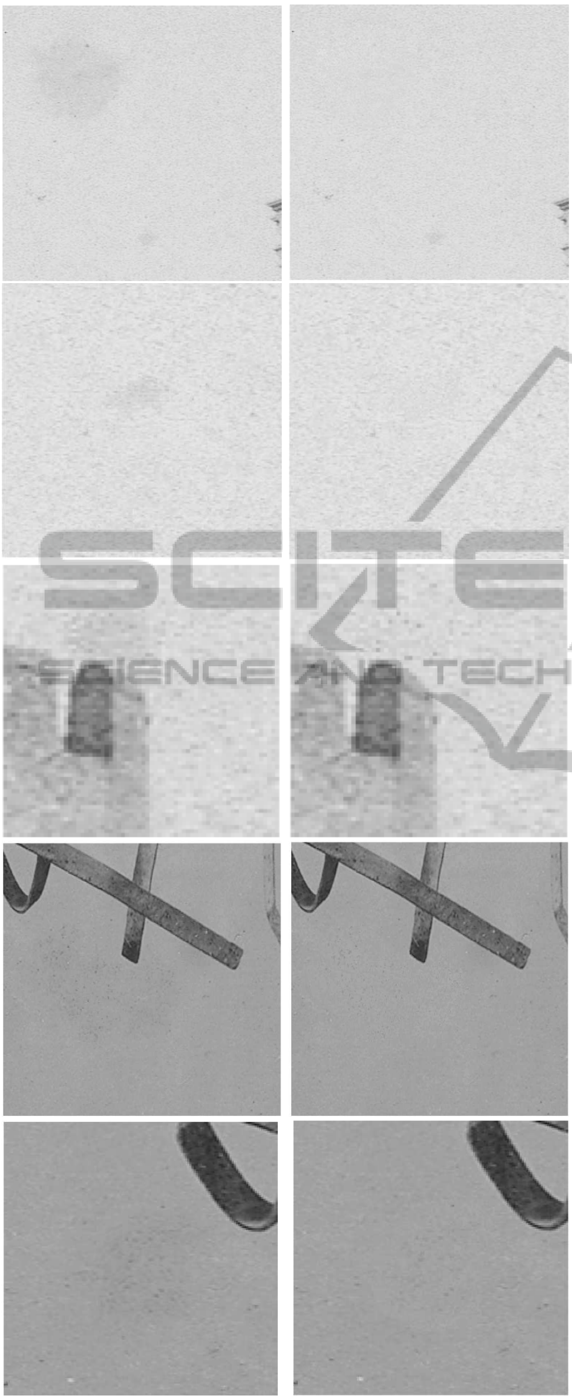

Figure 5: Original (left) and restored (right) images.

that we can assume that E and B share the same information. In order to classify the points in B according to their contrast properties, the SMQT algorithm (Nilsson et al., 2005) is used. It groups pixels having comparable visual contrasts. More precisely, SMQT builds a binary tree using the following rule: given a set of data D and a real parameter L (number of levels), split D into two subsets,

$$D_0 = \left\{x \in D | D(x) \leq \overline{D}\right\}$$

and

$$D_1 = \left\{x \in D | D(x) > \overline{D}\right\},$$

where $\overline{D}$ is the mean value of D. D_0 and D_1 are the first level of the SMQT. The same procedure is recursively applied to D_0 and D_1 and so on until the L^{th} level, that is composed of 2^L subsets. We set $L = 3$ in order to obtain a $\log_2$ quantization of the damaged pixels (i.e. $2^3 = \log_2(256)$ groups, $B_1, \ldots, B_8$, where $[0, 255]$ is the gray scale range) and will assume $B_1, \cdots, B_8$ ordered from the darkest to the brightest pixel. We compute the inner contrast $C_j = \frac{\sigma_j}{\mu_j}$ of each B_j, with σ_j the standard deviation and μ_j the mean of B_j. Let $\gamma = \{C_1, \cdots, C_8\}$ be the discrete contrast curve of B, as in Fig. 2 (*Left*). Darker pixels in B, say D_1, are those sets B_j whose inner contrast is greater than k (where k is the least detection threshold given by the Weber's law, see (Winkler, 2005)). On the contrary, brighter pixels in B, say D_3, are those B_j whose inner contrast curve has positive first derivative. Let D_2 be the union of the remaining groups.

Summing up, SMQT quantization of the degraded region B combined with its contrast properties allows to split B into three regions: the darkest, D_1, the brightest, D_3 and the central one, D_2, where most of information lives. The whole range of values of damaged pixels has been then split into these three intervals: $[b_0, b_1]$, $]b_1, b_2[$ and $[b_2, b_3]$. Let M and m be respectively the maximum and the minimum value for E (except for outliers). $[m, M]$ is the range of admissible values for the final solution. In fact, a natural scene component is required to not exceed the range of values of the surrounding information in terms of visibility bounds. $[m, M]$ is proportional to the global contrast of the whole image I, i.e. $\frac{M-m}{\mu_I}$ where μ_I is the mean value of I. Since we are dealing with a local degradation, we can think that any transformation of the degraded region does not influence too much μ_I, so that it can be considered constant. That is why from now on we will just deal with ranges instead of contrasts.

The second goal of preproccesing is to understand if each group of pixels must be moved. In fact, even though the damaged area B appears darker than the neighborhood E, it is important to check if the intersection between E and B values contains points that are darker than the blotch, as it is the case for the blotch on the dome in Fig. 1 (leftmost). To this aim, it is necessary to check if the darker region of E, corresponding to those pixels whose value is in the interval

$[b_0,b_1]$, masks D_1. Setting the just noticeable threshold $\varepsilon = .33$ (Pappas et al., 2005), if their contrast ratio is included in $[1-\varepsilon,1+\varepsilon]$, then the same information lives outside the blotch, and D_1 must be left unchanged; otherwise D_1 must be transformed and mapped into the range $[m,M-\delta]$, where δ is such that $[M-\delta,M]$ and is proportional to $[b_2,b_3]$. It is worth stressing that we choose to move D_1 values in $[m,M-\delta]$ and not in $[m,M]$ in order to preserve eventual original information.

With respect to D_3, i.e. the brightest region of B, we check if it is masked by the region in E corresponding to the values in the interval $[b_2,b_3]$. D_3 mostly corresponds to the transition area from E to B, according to the physical properties of the damaged area. Hence, the ratio between the inner contrasts is included in $[1-\varepsilon,1+\varepsilon]$ and with high probability we don't need to transform it.
Finally, D_2 contains most of degradation and it must be mapped into the interval $[m,M]$. However, because of some rigidity of the model, it will be better to shift the interval, i.e. $[m+\Delta,M+\Delta]$. Δ measures the darkness of the damaged area and it is well represented by the difference between the mean values of B and E.

4.2 Processing of the Damaged Area

The surfaces distance minimization described in section 3.2 requires the definition of suitable surfaces living in the transformation domains, $[m+\Delta,M+\Delta]$ and $[m,M-\delta]$, of D_2 and D_1. To preserve the original information, HVS perception mechanisms can be embedded in the restoration process. In particular, we aim at processing in the same way points that are equally perceived by human eye: points having the same context (same visual contrast), have to converge to the same target surface. To this aim, we apply the SMQT algorithm to D_2. Each group in D_2 corresponds to a surface (defined by interpolation). The target surface of the i-th group is defined as a paraboloid cut by the plane

$$z = \frac{m+M+2\Delta}{2}$$

and whose vertex is proportional to the mean value of the group, according to the values interval. Each surface converges to the corresponding paraboloid, as shown in Fig. 2 (*Right*).

The iterative minimization process stops when the target surface has been reached in agreement with visibility bounds. More precisely, let S be the initial surface, S^n the solution at the n-th iteration and P the target paraboloid; S^n is the final solution if

$$\frac{\sum_{x\in S}|S^n(x)-P(x)|}{\sum_{x\in S}P(x)} \le k\,, \qquad (1)$$

where k is the least detection threshold given by Weber's law. The first member of previous equation corresponds to Weber's contrast evaluated at the points of the analysed surfaces. For D_1 we use the same rule with

$$z = \frac{m+M-\delta}{2}.$$

4.3 Masking Refinement

The iteration algorithm tends to preserve the original information at the expense of some rigidity (solutions tend to be dark). As a result, we need to stress the range values relative to D_2 in order to have sufficiently bright solutions and a sufficiently large range values to avoid oversmoothed solutions. This requires a final masking operation.

Specifically, let S_2 be the output of the minimization algorithm applied to D_2. Let V be the set of pixels in S_2 whose value is greater than $M-\varepsilon$, that is

$$V = \{p\in S_2 \mid v(p) > M-\varepsilon\},$$

where $v(p)$ is the value of the pixel p in S_2 and V^C its complementary set in S_2. The ratio between V and V^C is the inner contrast of S_2. If this ratio is included in $[1-\varepsilon,1+\varepsilon]$, we don't need masking; otherwise, we have to replace the values of some pixels in V with their original ones.

The main idea is to replace those pixels that originally were sufficiently bright and that became too bright in the minimization algorithm. More precisely, let

$$H = \{p\in V \mid v_{orig}(p) > \tau\}$$

and

$$V_{orig} = \{v_{orig}(p) \mid p\in V\}$$

where $v_{orig}(p)$ is the original value of the pixel p in D_2 and τ is a suitable constant related to the properties of V_{orig}, then $S_2(H) = V_{orig}(H)$. Note that $\tau = \overline{V}_{orig}+\varepsilon$, where $\overline{V}_{orig}$ is the mean value of V_{orig}.

4.4 Algorithm

Let I, B and E respectively be the initial image, the degraded area and its neighborhood.

Step 1. Split B into the groups D_1, D_3 and D_2 using SMQT;

Step 2. Estimate the range of the solution $[m,M]$ from the surrounding information;

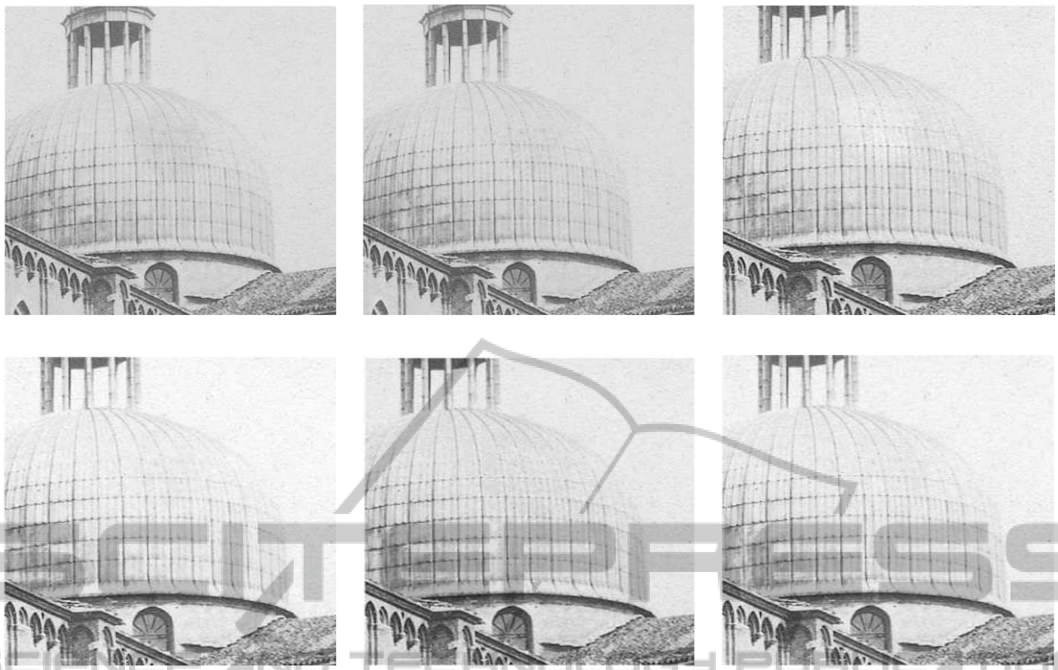

Figure 6: Blotch on a textured image. *Top* to *bottom* - *left* to *right*: degraded, restored image with the present method and restored using methods in (Greenblatt et al., 2008), (Stanco et al., 2005), (Crawford et al., 2007) and (Bruni et al., 2011).

Step 3. Check if D_1 has to be processed; if so, go to step 4, otherwise go to step 7;

Step 4. Apply SMQT to D_1;

Step 5. Compute paraboloids including the solution whose vertices are set according to the mean amplitudes of the groups computed in step 4 and the output of step 2;

Step 6. For each group in step 4, apply the iterative procedure in Section 3.2 where the target surfaces are the ones computed in step 5, until eq. (1) is satisfied;

Step 7. Apply steps 4-5-6 to D_2;

Step 8. Perform masking refinement.

Fig. 3 shows the results of some steps of the algorithm. In this case D_1 must be processed: the initial contrast ratio between D_1 and its corresponding external area is 2.5. On the contrary, the one between D_3 and its corresponding external region is 0.8; hence it is not necessary to process it.

For the blotch in Fig. 4, both D_1 and D_3 don't need to be processed — contrast ratios respectively are 1.1 and 0.9.

5 EXPERIMENTAL RESULTS AND CONCLUDING REMARKS

The proposed approach has been tested on selected images from the photographic Alinari Archive in Florence, affected by semi-transparent defects. Some results are shown in Figs. 5 and 6. As it can be observed, the visual appearance of the recovered images is very good: no artifacts appear, the texture of the background is well recovered as well as eventual details of the original image (see, for example, the edges of the dome).

The use of a selective algorithm avoids annoying halo effects at the border of the defect along with over-smoothing in the inner part of the restored region. As a result, the restored region is not still perceived as an anomaly on the image. The convergence process is different for each group of points so that it could happen that some groups converge after one or two iterations while others require longer convergence time. In that way the over-smoothing is avoided and the preservation of the inner information is guaranteed via the visibility based stopping criterion in eq. (1).

Moreover, the preprocessing of the damaged area and masking procedures allow the detection mask to be not precise (it can be larger than the degraded region) and to manage complicated cases where the degraded area intersects a darker region of the image as in Fig. 5, so that restoration does not create artifacts in correspondence to not degraded pixels. This is a great advantage, since the detection mask heavily influences restoration results of available restoration frameworks.

It is also worth highlighting that even though the proposed algorithm involves iterative procedures, it uses simple and fast operations and 4/5 iterations on average to converge.

Future research will be oriented to refine the proposed model to make it more flexible and adaptive to different amount of degradation while faithfully preserving original image information.

REFERENCES

Bertalmio, M., Sapiro, G., Caselles, V., and Bellester, B. (2000). Image inpainting. In *Proceedings of SIGGRAPH 2000*.

Bertalmio, M., Vese, L., Sapiro, G., and Osher, S. (2003). Simultaneous structure and texture image inpainting. In *IEEE Transactions on Image Processing, 12(8)*.

Bruni, V., Crawford, A., Kokaram, A., and Vitulano, D. (2011). Semi-transparent blotches removal from sepia images exploiting visibility laws. In *Signal Image and Video Processing*. Springer-Verlag.

Crawford, A., Bruni, V., Kokaram, A., and Vitulano, D. (2007). Multiscale semitransparent blotch removal on archived photographs using bayesian matting techniques and visibility laws. In *Proceedings of International Conference on Image Processing (ICIP 2007),S. Antonio, Florida*. IEEE Signal Processing Society.

Criminisi, A., Perez, P., and Toyama, K. (2004). Region filling and object removal by exemplar-based image inpainting. In *IEEE Transactions on Image Processing, 13(9)*. IEEE Signal Processing Society.

Drummond, T. and Cipolla, R. (2000). Application of lie algebras to visual servoing. In *International Journal of Computer Vision, 37(1)*.

Efros, A. and Freeman, W. (2001). Image quilting for texture synthesis and transfer. In *Proceedings of SIGGRAPH 2001*.

Greenblatt, A., Agaian, S., and Panetta, K. (2008). Restoration of images damaged by semi-transparent water blotches using localized image enhancement. In *Proceedings of SPIE 2008*.

Helgason, S. (1962). *Differential geometry and symmetric spaces*. Pure and Applied Mathematics, Vol. XII., Academic Press, New York-London.

Kokaram, A. (1998). *Motion Picture Restoration. Digital Algorithms for Artefact Suppression in Degraded Motion Picture Film and Video*. Springer Verlag.

Kokaram, A. (2002). Parametric texture synthesis for filling holes in pictures. In *Proceedings of International Conference on Image Processing (ICIP 2002)*. IEEE Signal Processing Society.

Mansouri, A. and Mukherjee, D. (2004). Constraining active contour evolution via lie groups of transformation. In *IEEE Transactions on Image Processing, 13(6)*. IEEE Signal Processing Society.

Mukherjee, D. and Acton, S. (2007). Affine and projective active contour models. In *Pattern Recognition, 40(3)*. Elsevier Science Inc.

Nilsson, M., Dahl, M., and Claesson, I. (2005). The successive mean quantization transform. In *Proceedings of ICASSP*. IEEE Signal Processing Society.

Pappas, T., Safranek, R., and Chen, J. (2005). Perceptual criteria for image quality evaluation. In *Handbook of Image and Video Processing*. Academic Press.

Porikli, F., Tuzel, O., and Meer, P. (2006). Covariance tracking using model update based on lie algebra. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR 2006)*.

Stanco, F., Tenze, L., and Ramponi, G. (2005). Virtual restoration of vintage photographic prints affected by foxing and water blotches. In *Journal of Electronic Imaging, 14(4)*.

Varadarajan, V. (1974). *Lie groups, Lie algebras and their representations*. Prentice-Hall Series in Modern Analysis. Prentice-Hall.

Winkler, S. (2005). *Digital Video Quality. Vision Models and Metrics*. Wiley.