INTRODUCTION TO MULTICRITERIA TECHNIQUES

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Abstract: The systematic analysis and decision making in companies, particularly in an environment of risk, are now a major challenge, namely the complexity of each problem. Multicriteria techniques are applied a long time ago and had an important development and expanded its application areas. The simple decisions, which are considered routine, given the frequency they repeat themselves tend now to be reviewed and reanalyzed in order to be efficient. Sort a decision as efficient as we rank a decision when compared with other decisions in which the chosen factors have worse performance, or deciding factors, for example, the ratio between consumption and production is less attractive.

1 THE COLLECTION OF INFORMATION AND FORMULATION OF THE PROBLEM

By analyzing a situation it is necessary to know the surrounding primarily internal and external environment. It is necessary to collect information about the company, its employees, its suppliers, its customers and to the legislation that regulates it, which is characterized by internal environment. Collect information about competitors, industry sector, European law in the case of application of quotas, which is characterized by the external environment.

Currently Operational Research offers a large set of theories, methods and models that allow the decision maker to reduce the degree of uncertainty in decision making as it can rely on models already tested and widely applied in different sectors.

The complexity of decisions are now often very large part of every decision and serves as a "lever" for the other decisions which influence and are influenced and, moreover, often increasing the complexity of using same resources, which involves choices regarding the allocation of human or material resources.

Reflecting the rapid evolution of markets but also the enormous dependence of each sector of the global economy, decisions are made based on deterministic models that do not increase the uncertainty of each decision, linked to other decisions that greatly increases the randomness.

The decision maker can minimize risk by collecting and "working" all available information concerning the system where it operates, the company he represents, to competitors, it aims to meet customers, regulators, among others.

Among the various paradigms presented by (Valadares Tavares et al. 1996) we emphasize the effectiveness of: while not ignoring the multiple sources of uncertainty and randomness, it is believed in the ability to establish effective systems, ie systems that achieve goals with predefined levels of safety or reliability very high.

One should emphasize the difference between decision making in nonprofit organizations, private enterprises and public enterprises which is justified by the difference in the Mission.

The set of steps are: comprehensive listing of all resources (human, financial and technical); listing of all feasible alternatives, identify the criteria that will influence and ultimately quantify each alternative / criterion.

2 PROBLEM FORMULATION

After the development of all the steps mentioned

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above the decision maker is faced with a set of alternatives which will select one based on clearly defined criteria.

A scale is assigned for each criterion which is defined by the amplitude, ie, a maximum value, minimum value and even the definition whether this criterion should be maximized or minimized.

If each criterion has a range quite different, which influences the results it does not raise any issue because they simply become more consistent. Set of Alternatives - A1; A2; A3;…; Ai

In order not to increase the complexity in solving the problem but to enter into account all factors relevant to the decision. Each criterion is to be analyzed individually or as a result of the analysis criteria of any other criteria, which implies a decrease in complexity.

According to the scale given to each criterion, each alternative is quantified for each criterion and Xij represents the quantification assigned to alternative i according to the criterion j.

Quantification Xij may correspond to a numerical scale or a qualitative scale (attributes).

The subordination of criteria is when the decision maker can define a relationship between the relative importance of these values (Keeney 1988 and 1992). Each structure of this type characterizes their own ethics (Valadares Tavares *et. al.*, 1996) not only the decision maker but the system where it is inserted. Even before choosing and applying the model one should identify all situations of subordination and eliminate all these dominated alternatives.

To evaluate a set of alternatives based on different criteria there are several methods in this paper we analyze:

- Compensating Methods;
- Non-Compensatory Models.

We will use software MacModel created in 2001 by José Coelho in IST (Instituto Superior Técnico) to multicriteria problem solving which will consist of the presentation of some outputs and especially the results and sensitivity analysis.

3 COMPENSATORY MODELS

Why use the term compensatory? It comes from the fact that an alternative may have certain criteria in a quantifier worst since the other criteria can restore the balance.

This is because this model aims at the integration of different criteria which is easily achieved by assigning each criterion a preference indicator, each indicator varies between 0 and 1 and the sum of all indicators is equal to unity. The indicator preferably assigned to each criterion represents the weight of each criterion in the final decision. Representing a tree in the same scheme previously presented:

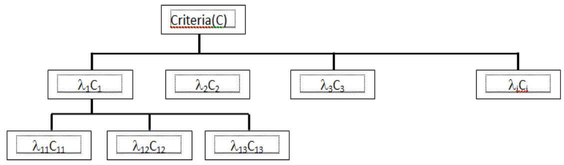


Figure 1: Assigning weights to each criterion and each sub-criterion.

$$\sum_{j=1}^{m} \lambda_j = 1$$

Trade-off between criteria a and b is calculated as follows: (Valadares Tavares *et. al*, 1996):

$$TMS_{a,b} = \lambda_a / \lambda_b$$

Often the criteria are not expressed on the same scale, which in real terms is quite likely, therefore it is necessary to standardize each scale applying the following equations:

In the case of increasing preference

$$x_{ij} = \frac{X_{ij} - \min_i(X_{ij})}{\max_i(X_{ij}) - \min_i(X_{ij})}$$

In the case of decreasing preference

$$x_{ij} = \frac{\max_i(X_{ij}) - X_{ij}}{\max_i(X_{ij}) - \min_i(X_{ij})}$$

Another possibility is to use the following formulas:

In case of increasing preference: $x_{ij} = \frac{X_{ij} - \mu_j}{\sigma_j}$

In case of decreasing preference: $x_{ij} = \frac{\mu_j - X_{ij}}{\sigma_j}$

Symbols μj and σj are the mean and standard deviation respectively; $\min_i (X_{ij})$ and $\max_i(X_{ij})$ the minimum and maximum for each criteria.

We perform the calculation for each alternative, getting a weighted average. The weights are given by the indicators of preference for each criterion and the values considered are the measurements assigned to each alternative (for each criteria),

$$X_i = \sum_{j=1}^{m} (\lambda_j X_{ij}) \text{ and } \sum_{j=1}^{m} \lambda_j = 1$$

In the simple additive model the decision is made by maximizing the values obtained.

In any decision-making process there are risks and all methods present limitations, the limitations of this model are presented in (Valadares Tavares *et al.,* 1996).

The best way to minimize risk is to reduce the number of parameters considering only the most relevant to decision making. And still must make the application of the same scale for all criteria, or to proceed to its standardization.

If there are two or three criteria they can be represented graphically and make a sensitivity analysis. This analysis is always important because it allows us to identify the ranges for each λj in order to remain the same solution; the stability of the solution reduces uncertainty because it makes it so relevant to the choice of the alternative to implement the weighting given to each criterion or aggregation of criteria.

Graphing is fairly simple if the number of criteria we have is just two: $x_i = \lambda_1 C_{i1} + \lambda_2 C_{i2}$.

If there are three criteria begins with the following transformation λ_3=1-λ_1+λ_2 then for each alternative is identified $x_i = \lambda_1 C_{i1} + \lambda_2 C_{i2} + (1 - \lambda_1 - \lambda_2) C_{i3}$ then to represent each pair of alternatives a line called the indifference which give rise to different areas that will allow an analysis.

In this case the graphical representation ceases to be simple but it will be easier to use the MacModel software (Coelho, 2001).

The number of lines of indifference is given by $\frac{m(m-1)}{2}$ where m is the number of alternatives, (Valadares Tavares *et al.*, 1996).

4 NON-COMPENSATORY MODELS

This method was developed by Bernard Roy in 1968 which to identify relationships of dominance between two alternatives.

The comparison between alternatives is made for the values j (all criteria) and results in a clash between any two alternatives can be observed two situations:

• condition of agreement, defined by the average order of preference;

• condition of disagreement, a sense of "veto" the decision maker can use when the average direction of the disagreement is very strong in one criterion.

They also defined weights for different criteria. The sum of the weights is unity.

The notion of integration remains of criteria, i.e. a criterion can result from the integration of sub-criteria, as in the previous process and weights are also assigned to the sub criteria. But the analysis is done using binary comparisons.

We use a relational system of preferences by comparing the two alternatives.

Considering a practical application to three criteria we will calculate binary comparisons between any two alternatives thus obtaining R1, R2, R3.

When comparing the two alternatives we can conclude that there is: indifference, equivalence or dominance.

This method is applied on one hand to the average order of preference and on the other to a sense of veto in the case of the average direction of disagreement to be very strong.

Note that this method can be applied even if the quantifiers are attributes, xij is a qualitative variable.

When comparing two alternatives by applying the condition of agreement it is necessary to establish a value α (0 ≤ "α" ≤ 1) representing the minimum amount required to be accepted that a prevails over b:

$$C_{ab} = \left[\sum_{i=1}^{n} \lambda_j \left(para\ j: x_{aj} \geq x_{bj}\right)\right] \geq \alpha$$

The decision maker may also evaluate the disagreement between two alternatives, calculating the difference for each quantifier of the two alternatives under study.

We will get j results in a problem with j criteria. If the objective is to maximize, the greater of the calculated values will be chosen. Getting just the disagreement between any two alternatives, β defines the maximum permissible level of disagreement:

$$D_{ab} = \max_j \left[x_{bj} - x_{aj}\right] \leq \beta$$

When the quantifiers are qualitative a correspondence should be performed to a scale so that the agreement can be calculated Similarly in the case of disagreement the decision maker must decide how many levels are considered severe enough to apply the "veto". For example if the match is made with mediocre, poor, fair, good and very good condition and the disagreement is over 2 levels when compared with the good will only be applied to the mediocre.

The prevalence among alternatives is the more difficult the higher the value assigned to α and lower the value assigned to β.

The prevalence relation is not transitive. It is likely that an alternative to prevail over another but is dominated by another by analyzing three alternatives.

In this case the decision process may not have finished and be more advisable to collect information, analyze more fully each of the alternatives still under possible selection.

In all cases it will carry out sensitivity analysis which is performed by changing the values of α and β checking for intervals remain the same solutions that will strengthen the choice of a particular alternative.

5 APPLICATION TO A PRACTICAL CASE

Consider six different locations to install a landfill. All decision making is based on three criteria time, cost and environmental impact. The latter is considered as the aggregation of three sub criteria pollution, aesthetic and Agricultural Land Unusable (ALU).

The collection of information and different measurements has been performed and is presented in the following tables. Starting with the 3rd criteria Environmental Impact, sub criteria are aggregated using the following system of weights: 20%, 10% and 70%, respectively, as shown below:

Table 1: Quantifiers linking each alternative to sub criteria for Environmental Impact, values entered in MacModel.

		Subcriteria			Criteria
		Pollution	Aesthetic	ALU	Environmental
Locations	1	10	8	4	5.6
	2	6	10	8	7.8
	3	6	6	10	8.8
	4	0	5	9	6.8
	5	5	8	0	1.8
	6	8	0	3	3.7

It is obvious that the aggregation weights for this are debatable, and lend themselves to many other possible choices. You can now submit all the values for the three criteria and its value, the weights are considered 10%, 25% and 65%, respectively, Environmental Impact (EI), time and cost.

Table 2: Quantifiers linking each alternative to the different values placed on MacModel were only related to Cost and Time criteria.

		Criteria			
		Cost	Time	EI	Value
Locations	1	6	10	5.6	6.96
	2	10	5	7.8	8.53
	3	6	6	8.8	6.28
	4	9	4	6.8	7.53
	5	9	5	1.8	7.28
	6	5	8	3.7	5.62

The scale used is 0-10 in the three criteria, it is not necessary to standardize. Note that the alternatives 4, 5 and 6 are dominated respectively by the two alternatives, 2 and 1. From what these alternatives might already be taken. The analysis that follows through Software MacModel not only exclude the alternative 6 as will be dominated by analyzing the results obtained with the alternatives 1, 2 and 3 criteria based on cost, time and environmental impact.

The criteria and sub criteria can be grouped as illustrated in the figure below.

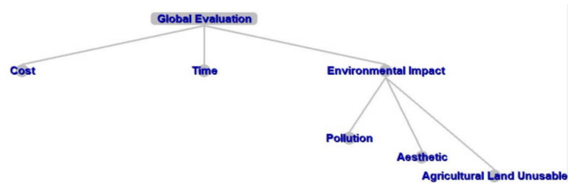
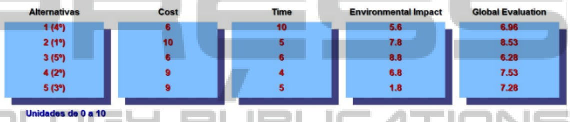


Figure 2: Representation of the criteria in tree, output MacModel.

The table below shows the values entered in the Software that proceeded immediately to the ordering of the alternatives according to the weights above.



Alternativas	Cost	Time	Environmental Impact	Global Evaluation
1 (4º)	6	10	5.6	6.96
2 (1º)	10	5	7.8	8.53
3 (5º)	6	6	8.8	6.28
4 (2º)	9	4	6.8	7.53
5 (3º)	9	5	1.8	7.28

Unidades de 0 a 10

Figure 3: Output of MacModel already with the Global Assessment for each alternative.

To define the lines of indifference we examine:

$$\begin{cases} 6\lambda_1 + 10\lambda_2 + 5.6(1-\lambda_1-\lambda_2) = 0.4\lambda_1 + 4.4\lambda_2 + 5.6 \\ 10\lambda_1 + 5\lambda_2 + 7.8(1-\lambda_1-\lambda_2) = 2.2\lambda_1 - 2.8\lambda_2 + 7.8 \\ 6\lambda_1 + 6\lambda_2 + 8.8(1-\lambda_1-\lambda_2) = -2.8\lambda_1 - 2.8\lambda_2 + 8.8 \end{cases}$$

$$\Leftrightarrow \begin{cases} -1.8\lambda_1 + 7.2\lambda_2 = 2.2 \\ 3.2\lambda_1 + 7.2\lambda_2 = 3.2 \\ \lambda_1 = 1/5 \end{cases}$$

The 1st equality refers to the tie between locations 1 and 2.

The 2nd equality refers to the tie between locations 1 and 3; finally the 3rd equality refers to the tie between locations 2 and 3.

The sensitivity analysis of the weights can be done using the Trident method (Valadares Tavares, 1984).

The decision should be made between the first three locations. The location has a rating of 10 in a time criterion while the second location has the highest rating in the criteria cost. If greater weight is given to the environmental impact criteria the appropriate location is the 3.

Note that dominated alternatives, disappear in the Trident analysis, since with any system of weights they would never be in the first place.

Analyzing five alternatives (in which none is dominated) analysis Trident shows five polygons.

At the other end if there is one that dominates all others we will have a single region: the whole triangle.

According to the weights assigned earlier 65%, 25% and 10% the decision is location 2. One can also consider several decision makers and get to the centroid of the most balanced solution as illustrated in the following figure.

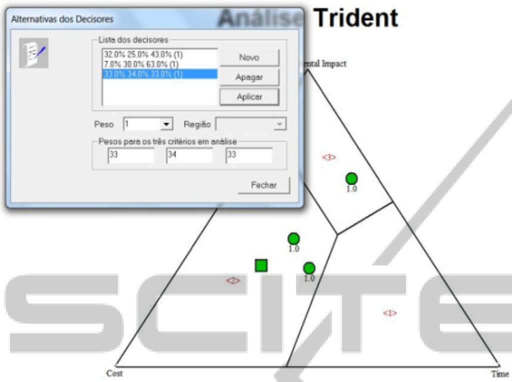


Figure 4: Output Analysis of MacModel with Trident, when applied to multiple decision makers with different weights assigned to the same criteria.

Continuing to analyze the same problem now we apply the non-compensatory process Electre.

Electre is a non-compensatory method because it is based on dichotomous comparisons, based on the comparison between pairs of alternatives.

Infinitesimal changes of their values do not change the final decision (provided they do not alter the meaning of the order relation) as opposed to compensatory model in which any change in measurement changes the value.

1 - Matrix of agreement on what is considered the same weights 65%, 25%, 10% and that means how much better alternative is superior to the line of the column.

Table 3: Matrix of Agreement.

		Locations				
		1	2	3	4	5
Locations	1		0.25	0.9	0.25	0.35
	2	0.75		0.65	1	1
	3	0.75	0.35		0.35	0.35
	4	0.75	0	0.65		0.75
	5	0.65	0.25	0.65	0.9	

The sum for each alternative in the matrix of agreement is respectively 1.75, 3.4, 1.8, 2.15 and 2.45.

There is no doubt that the second alternative has a higher value, such as in the compensatory model.

Alternative 5 has a high value because the criterion with a big weight has a high value.

2 - Disagreement Matrix

It identifies all the alternatives now that the Disagreement Matrix presents values greater than or equal to 5. This is because we decided to veto all alternatives that have a value greater than 5.

Note that a value of 5 in Matrix Disagreement is equivalent to an increase of 5 on certain criteria.

Consequently we eliminate the alternatives 4 and 5.

Table 4: Matrix of Disagreement.

		Locations				
		1	2	3	4	5
Locations	1		4	3.2	3	3
	2	5		1	0	0
	3	4	4		3	3
	4	6	1	2		1
	5	5	6	7	5	

In all cases the matrix of disagreement has two zeros at the same alternative so our present decision is the best regardless of whether we want to apply a method or the other (keeping the same weights).

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