

AN ARTIFACT-FREE WAVELET MODEL FOR PERCEPTUAL CONTRAST ENHANCEMENT OF COLOR IMAGES

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Abstract: Contrast enhancement of color images can prove to be a difficult task because artifacts and unnatural colors can appear after the process. In this paper we propose a wavelet-based variational framework in which contrast enhancement is obtained through the minimization of a suitable energy functional of wavelet coefficients. We will show that this new approach has certain advantages with respect to the usual spatial techniques sustained by the fact that the wavelet representation is intrinsically local, multiscale and sparse. The Euler-Lagrange equations of the model are implicit equations involving the detail wavelet coefficients of the image. These equations can be quickly solved by Newton's method, so that the algorithm can rapidly compute the enhanced detail coefficients. We will discuss the influence of the parameters tests on natural images to show that the method is artifact free within an ample range of variability of its parameters.

1 INTRODUCTION

Digital images can present poor contrast, globally or locally, due to many factors: wrong camera exposition or aperture settings, back-light conditions, high dynamic range of the scene, and so on. Contrast enhancement can help improving detail visibility and, in general, the overall look of the image. When we deal with color images, the issue of contrast enhancement is quite complex because artifact and unnatural colors can appear after the contrast modification.

Since humans are capable of a high-quality color vision, it is quite natural to design algorithms that try to mimic the Human Visual System (HVS) features in order to reach an efficient enhancement. The algorithms built in this way are usually called perceptually-inspired and their use can be found in research fields as computational photography, image quality, interior design and robotic vision to cite but a few.

In this paper we analyze the problem of perceptual contrast enhancement with variational techniques from the point of view of wavelet theory. For this purpose we propose a functional of detail coefficients whose minimization induces a local and multiscale improvement of contrast. We will show that the Euler-Lagrange equations of the functional are implicit non-linear equations which enhance the wavelet detail coefficients of the image. By using Newton's method those equations can be quickly solved, ensuring a

global computational complexity of $O(N)$, N being the total number of image pixels. Moreover, the sparsity of the wavelet representation allows the algorithm to be fast.

For the sake of clarity, it is worthwhile to introduce here the notation that we are going to use throughout the paper. Given a discrete RGB image, we will denote by $\mathcal{J} \subset \mathbb{Z}^2$ its spatial domain and by $x \equiv (x_1, x_2)$ and $y \equiv (y_1, y_2)$ the coordinates of two arbitrary pixels in \mathcal{J} . We will always consider a normalized dynamic range in $[0, 1]$, so that a color image function will be $\vec{I} : \mathcal{J} \rightarrow [0, 1]^3$, $\vec{I}(x) = (I_R(x), I_G(x), I_B(x))$, where $I_k(x)$ is the intensity level of the pixel $x \in \mathcal{J}$ in the chromatic channel $k \in \{R, G, B\}$. We stress that every computation will be performed on the scalar components of the image, thus treating independently each chromatic component as in Retinex-like algorithms (Land and McCann, 1971). Therefore, we will avoid the subscript k and write simply $I(x)$ to denote the intensity of the pixel x in a given chromatic channel.

2 A PERCEPTUAL CONTRAST FUNCTIONAL IN THE WAVELET DOMAIN

In this section we shall motivate our choice for the

contrast functional to be minimized in order to obtain a perceptually-inspired contrast enhancement in the wavelet domain.

In (Palma-Amestoy et al., 2009), the authors proved that there exists only a type of contrast functional that comply with a set of basic phenomenological HVS properties: *color constancy*, i.e. the ability to perceive colors as (almost) the same independently on the illumination conditions, *locality of contrast enhancement*, exhibited by well-known phenomena as e.g. Mach bands or simultaneous contrast, and *Weber-Fechner's law of contrast perception*, i.e. the logarithmic response of the HVS to changes of spot light intensity. This functional is the following¹:

$$C_w(I) = \sum_{x \in \mathcal{J}} \sum_{y \in \mathcal{J}} w(x,y) \frac{\min\{I(x), I(y)\}}{\max\{I(x), I(y)\}}, \quad (1)$$

where $w : \mathcal{J} \times \mathcal{J} \rightarrow (0, +\infty)$ is a weight function that induces locality. The full details about why this functional complies with the basic HVS features listed above can be found in the quoted paper, here we briefly report why the minimization of C_w induces contrast enhancement and how it is related to color constancy. Regarding contrast enhancement, observe that the function $c(I(x), I(y)) = \frac{\min\{I(x), I(y)\}}{\max\{I(x), I(y)\}}$ is minimized when the minimum intensity value decreases and the maximum increases, which of course corresponds to a contrast intensification. The relation with color constancy comes from the observation that c is a homogeneous function of degree zero, i.e. $c(\lambda I(x), \lambda I(y)) = c(I(x), I(y))$ for all $\lambda \neq 0$; in image formation models λ is interpreted as the 'color temperature' of the light source that illuminates a scene, thus the homogeneity property implies that the functional C_w is able to automatically discard the color cast induced by a global illuminant, coherently with the HVS property of color constancy.

In this framework the function c plays the role of basic perceptual contrast variable, a concept that interested also E. Peli in (Peli, 1990). Peli generalized the pioneering work (Hess et al., 1983) and studied the perceptual contrast through a multiscale approach in which, at each given scale j , he defined the *perceptual contrast of x with respect to a neighborhood $U(x)$* as a ratio, precisely

$$c_{j,U(x)}^{\text{Peli}} = \frac{g_j * I(x)}{h_j * I(x)} \quad (2)$$

where g and h are a band-pass and a low-pass filter, respectively, of a filter bank with support in $U(x)$ and $*$ denotes the convolution as usual. Peli's ideas were then embedded in the wavelet framework and used by (Bradley, 1999) to build a wavelet-based visible difference predictor and by (Vandergheynst et al., 2000) to implement digital watermarking. The details on the considerations that led Peli to this definition can be found in the previously quoted paper.

We are now going to show that the similarity between the two approaches to perceptual contrast just described becomes even stronger if we recast the variational framework of (Palma-Amestoy et al., 2009) into the wavelet domain.

For this purpose, let us start recalling that, following the classical reference book (Mallat, 2008), an orthogonal wavelet multi-resolution analysis of an image between two scales 2^L and 2^J , $L, J \in \mathbb{Z}$, $L < J$, is given by three sets of detail coefficients $\{d_{j,k}^H, d_{j,k}^V, d_{j,k}^D\}_{k \in \mathcal{J}, j=L, \dots, J}$, which correspond to the *horizontal, vertical and diagonal detail coefficients*, respectively, completed by $\{a_{j,k}\}_{k \in \mathcal{J}}$, the approximation coefficients at the coarser scale. If the image is in color, then each chromatic channel has its own set of detail and approximation coefficients. The set $\{a_{j,k}\}_{k \in \mathcal{J}}$ gives a coarse description of the image at the scale J and it is obtained by convolution between the image and a low pass filter. The set $\{d_{j,k}\}_{k \in \mathcal{J}}$ is obtained by convolution between the image and a spatially localized band pass filter, so that the $\{d_{j,k}\}_{k \in \mathcal{J}}$ give a measure of *local contrast* in the image at the scale 2^j .

Our proposal for a perceptual contrast functional in the wavelet domain is

$$C_{p_j, \{a_{j,k}\}}(\{d_{j,k}\}) = \sum_{k \in \mathcal{J}} p_j \frac{a_{j,k}}{d_{j,k}}, \quad (3)$$

where p_j are positive coefficients that permits to modulate the strength of contrast enhancement. This definition makes sense if the detail coefficients are different from zero, for this reason we fix a threshold $T_j > 0$ for each scale and consider only those $d_{j,k}$ satisfying $|d_{j,k}| > T_j$; the other coefficients will be left unchanged.

Thanks to the locality of the wavelet representation, the functional $C_{p_j, \{a_{j,k}\}}$ is intrinsically local and does not need the introduction of any further weighting function, which it is instead essential in the spatial variational framework to localize the computation.

If we keep the approximation coefficients fixed and let the other free to vary, then the minimization of

¹In the quoted paper the definition of C_w allows an increasing diffeomorphism ϕ to act on the fraction inside the integral and the case $\phi \equiv \text{id}$, id being the identity map used here, is studied as a subcase. Since ϕ will not have any prominent role in the present paper, we have omitted its presence since the beginning to simplify the notation as much as possible.

$\frac{a_{j,k}}{d_{j,k}}$ corresponds to the intensification of the detail coefficients and thus of local contrast. Observe also that here the basic contrast variable is $\frac{a_{j,k}}{d_{j,k}}$, which is still a homogeneous function of degree 0 as in the variational framework recalled above and that, at the same time, it is in line with the multiscale contrast interpretation of Peli, since the coefficients a and d come from low and band pass filters, respectively.

We cannot determine the enhanced detail coefficients solely by minimizing the functional $C_{p_j, \{a_{j,k}\}}$ because that could lead to an uncontrollable over-enhancement of contrast, thus we have to introduce a dispersion control term, $\mathcal{D}_{d_{j,k}^0}$, that balances the effect of C_{p_j} with a conservative action that tends to maintain the detail coefficients to their original values $\{d_{j,k}^0\}$. In (Palma-Amestoy et al., 2009) it has been proven that a suitable choice for the dispersion term to preserve dimensional coherence when the contrast is a homogeneous functional of degree 0 is the *entropic dispersion*, which in the present problem can be written as:

$$\mathcal{D}_{d_{j,k}^0}(\{d_{j,k}\}) = \sum_{k \in \mathcal{J}} \left[d_{j,k}^0 \log \frac{d_{j,k}^0}{d_{j,k}} - (d_{j,k}^0 - d_{j,k}) \right]. \quad (4)$$

We then define the wavelet-based perceptually-inspired contrast-enhancement energy as the sum of the two previous functionals, i.e.

$$\mathcal{E}_{p_j, \{a_{j,k}\}, d_{j,k}^0}(\{d_{j,k}\}) = \left(C_{p_j, \{a_{j,k}\}} + \mathcal{D}_{d_{j,k}^0} \right) (\{d_{j,k}\}). \quad (5)$$

By setting to zero the first variation of this energy we find its Euler-Lagrange equations, as we show in the following proposition. Its proof is postponed to the Appendix for the sake of a better readability of the paper.

Proposition 2.1. *The minimization of $\mathcal{E}_{p_j, \{a_{j,k}\}, d_{j,k}^0}$ gives rise to the following Euler-Lagrange equations for the detail coefficients:*

$$\frac{\partial \mathcal{E}_{p_j, \{a_{j,k}\}, d_{j,k}^0}}{\partial \{d_{j,k}\}} = 0 \implies d_{j,k} = d_{j,k}^0 + p_j \frac{a_{j,k}}{d_{j,k}}. \quad (6)$$

We can now summarize the steps of the variational wavelet-based algorithm for perceptual contrast enhancement as follows: 1) Consider the three chromatic components of an image independently² and use the discrete wavelet transform to obtain a multiresolution analysis of each component over a certain number of scales; 2) Compute, for each scale,

²Process color images by performing operations separately on the three chromatic channels is common in all Retinex-like algorithms.

the new detail coefficients (horizontal, vertical and diagonal) as prescribed by eq. (6) and substitute the original with these new ones; 3) Apply the inverse wavelet transform to obtain the filtered image. In addition to these steps, we operate a linear stretching of the coarser approximation coefficients $a_{J,k}$ in order to maximize the dynamic range reproduced.

The wavelet algorithm previously described has computational complexity $O(N)$, N being the number of image pixels, and we implemented it in MATLAB using the ‘wavelet toolbox’.

Besides the direct and inverse wavelet transformations, the operation that requires more time is the iterative computation of the enhanced detail coefficients, i.e. the resolution of the implicit equation (6). An efficient way to do that is using Newton’s method (Fausett, 2007), initialized with the original values $d_{j,k}^0$. Our algorithm stops when the relative error between two subsequent iterations is smaller than 10^{-3} and typically convergence is reached in just two, or at maximum three, iterations. Thanks to the quadratic convergence of Newton’s algorithm and to the low computational cost of the discrete wavelet transform, the wavelet algorithm is considerably faster than the spatial variational algorithm of (Palma-Amestoy et al., 2009). To have an idea about the speed up, we report that it took only 4.98 seconds with the MATLAB code to filter a quite large image of dimension 922×691 over five scales, while it took 391.35 seconds to filter it with the C++ implementation of the algorithm presented in (Palma-Amestoy et al., 2009) on the same computer. We also stress that MATLAB is an interpreted language, so that an optimized code for graphic card can further speed up the wavelet algorithm in order to reach real-time performances.

In the next section we shall discuss the effect of parameters on the wavelet algorithm and its performances on natural images.

3 TESTS

As it was presented above, the wavelet algorithm has 4 different types of parameters: 1) the threshold T_j beyond which the wavelet coefficients are considered significantly above 0 at the scale 2^j ; 2) the number of scales over which the computation is performed; 3) the coefficients p_j , $2^L \leq 2^j \leq 2^J$, that express how much we permit to change the original wavelet detail coefficients in each scale; 4) the mother wavelet ψ . In the next subsections we shall discuss how the algorithm performances are influenced by these parameters, but before that we would like to show the

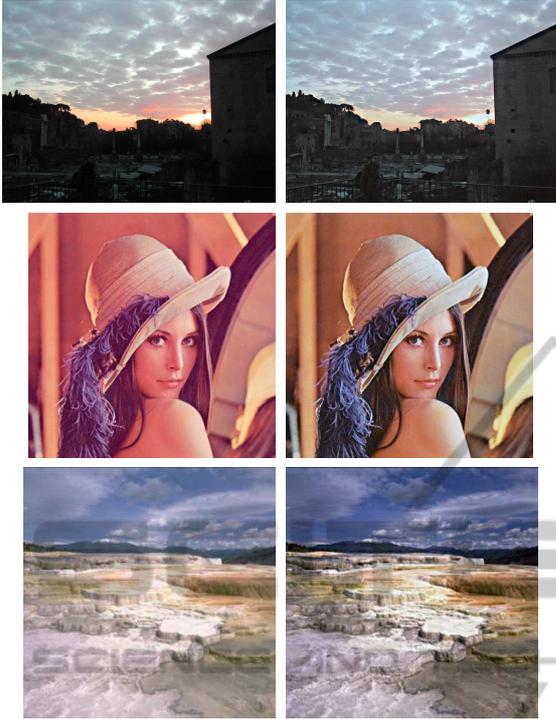


Figure 1: *Images on the left:* Original ones. *Images on the right:* enhanced versions after the wavelet algorithm: details appear in originally underexposed and overexposed areas, and the pink color cast in the ‘Lena’ image is removed. The filtering parameters are the following: the mother wavelet is the Daubechies wavelet with two vanishing moments, the computation is performed over the maximum number of scales allowed for each image (see Subsection 3.2 for more details), $p_j = 0.5$, and $T_j = \frac{\max_{k \in \mathcal{J}} \{d_{j,k}\}}{10}$ for each scale 2^j .

efficiency of the wavelet algorithm on three images affected by distinct problems: under-exposure, color cast and over-exposure; as can be seen in Figure 1 the wavelet algorithm is able to perform a radiometric adjustment of the non-optimally exposed pictures and to strongly reduce the color cast.

3.1 The Threshold Parameter T_j

In the computational algorithm we have set the threshold parameter to be $T_j \equiv \frac{\max_{k \in \mathcal{J}} \{d_{j,k}\}}{K}$, $K > 1$, for all the scales 2^j . Of course selecting $K \simeq 1$ we deal only with the largest detail coefficients, while if we set $K \gg 1$ we introduce in the computation also the smaller ones. Our tests have shown that an optimal value for K is 10 for every scale, in fact, selecting values of K bigger than 10 the algorithm does not introduce significant improvement in detail rendition but it may have the unwanted effect to intensify the noise corresponding to small detail coefficients. Thus, we have set once



Figure 2: *From left to right:* ‘Lena’ image filtered with the wavelet algorithm with decreasing values of the threshold $T_j = \frac{\max_{k \in \mathcal{J}} \{d_{j,k}\}}{K}$ corresponding to $K = 10$ and 50 , respectively. We can see that when $K = 50$ the resulting image is affected by noise due to the intensification of small detail coefficients corresponding to noise. The other parameters are maintained fixed: the computation is performed over the maximum number of scales allowed for each image (see Subsection 3.2 for more details), $p_j = 0.5$ for each scale, and the mother wavelet is ‘Sym8’, the Symlet with support a width of 15 pixels (arbitrary chosen).

and for all $T_j = \frac{\max_{k \in \mathcal{J}} \{d_{j,k}\}}{10}$ for all the scales 2^j , which means that we only deal with the detail coefficients that lie in the same decimal order of magnitude of the biggest ones. In Figure 2 we show the effect of decreasing too much the threshold T_j .

3.2 The Number of Scales

The *number of scales* $J - L$ that can be used depends on the image dimension and the width \mathcal{W}_ψ of the mother wavelet support. In fact, the maximum number of meaningful scales is the highest value of $J - L$ such that the following inequality holds: $2^{J-L} \mathcal{W}_\psi \leq \min\{\text{width}(\mathcal{J}), \text{height}(\mathcal{J})\}$. This value can be automatically computed with the command ‘wmaxlev’ in the MATLAB wavelet toolbox. Our tests have shown that the best contrast enhancement performances of the wavelet algorithm in terms of detail rendition and elimination of color cast corresponds to the highest number of scales allowed. For this reason we have used the command ‘wmaxlev’ to automatically set the number of scales over which carry on the computation of the enhanced detail coefficients, thus eliminating the variability of this parameter.

3.3 The Contrast Enhancement Coefficients p_j

From eq. (6) it follows that, if we increase the value of the coefficients p_j , the effect of contrast enhancement becomes more intense. However, if we increase them too much, contrast can be over-enhanced, resulting in unpleasant images with unnaturally high contrast.



Figure 3: From left to right: effect of increasing the coefficients p_j from 0.5 to 5, respectively. The images filtered with $p_j = 10$ have unnatural high contrast. In all the computations the other parameters are maintained fixed: the computation is performed over the maximum number of scales allowed, $T_j = \frac{\max_{k \in \mathcal{J}} \{d_{j,k}\}}{10}$ for each scale and the mother wavelet is the ‘Sym8’ (arbitrarily chosen).

This effect is shown in Figure 3, where we show the difference produced by increasing the coefficients p_j of one order of magnitude.

In general, setting $p_j = 0.5$ corresponds to overall good performances of the wavelet algorithm, thus 0.5 can be considered as a ‘reference value’ for the coefficients p_j . However, since their setting is very intuitive, they can also be easily tuned around this reference value by a user that may want more or less contrast enhancement.

3.4 The Mother Wavelet ψ

Different mother wavelets ψ have, in general, different support and symmetry properties³. As a consequence, different mother wavelets induce different local contrast enhancement, as can be seen in Figure 4. How to properly choose the family of wavelet is still an open problem that we would like to address in the future.

Let us suppose that a give wavelet class is chosen, then one has a further degree of freedom given by the number of vanishing moments. These ones have a strong relation with local contrast: it can be proved (see (Mallat, 2008)) that the bigger is the number of vanishing moments of ψ , the higher must be the image contrast detected in the support of ψ to get detail coefficients appreciably different from zero. So, the rationale for choosing the number of moments of a mother wavelet within the wavelet-based algorithm discussed in this paper is the following: if a user is interested in highlighting only high contrast regions, then a wavelet with a high number of vanishing mo-

³For an extensive discussion about mother wavelet properties, the interested reader is referred to chapter 7 of the standard book (Daubechies, 1992). Moreover, MATLAB provides information about symmetry, size of the support and number of vanishing moments of every wavelet family with the command ‘waveinfo’.



Figure 4: From left to right: output of the wavelet algorithm obtained with the Daubechies and Coiflet wavelet, respectively, with 4 vanishing moments. The other parameters are maintained fixed: the computation is performed over the maximum number of scales allowed, $T_j = \frac{\max_{k \in \mathcal{J}} \{d_{j,k}\}}{10}$ and $p_j = 0.5$ for each scale.

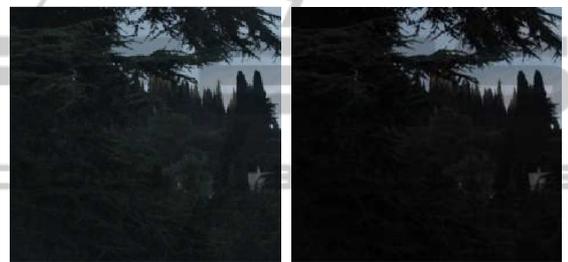


Figure 5: From left to right: output of the wavelet algorithm obtained with the Daubechies wavelet with 3 and 8 vanishing moments, respectively. The other parameters are maintained fixed: the computation is performed over the maximum number of scales allowed, $T_j = \frac{\max_{k \in \mathcal{J}} \{d_{j,k}\}}{10}$ and $p_j = 0.5$ for each scale.

ments should be selected; viceversa, if one is also interested in enhancing lower contrast regions, then a smaller number of vanishing moments must be preferred. This fact is best shown in dark image zones, as in Figure 5, where we show the effect of changing the number of vanishing moments of the Daubechies wavelet from 3 to 8. Coherently with what stated above, it can be seen that the contrast enhancement on low contrast areas provided by a wavelet with a smaller number of vanishing moments is better since a greater number of detail coefficients appreciably greater than zero can be enhanced.

4 CONCLUSIONS AND PERSPECTIVES

We have proposed a variational model of perceptually-inspired contrast enhancement of color images based on the wavelet representation. The wavelet framework underlines the similarities between the interpretation of perceptual contrast

given by (Palma-Amestoy et al., 2009) and by (Peli, 1990). The new definition of perceptual contrast in the wavelet domain proposed here permits to construct a fast algorithm that can be used to intensify contrast in color images without introducing artifacts or unnatural colors.

The wavelet algorithm is intrinsically local and has computational complexity $O(N)$, N being the number of image pixels, and it can be parallelized in order to achieve real-time performances even for large images, thus it could be also used to efficiently process video sequences (e.g. to reduce flickering or remove color cast due to film ageing). This improvement with respect to the variational algorithm presented in (Palma-Amestoy et al., 2009) is provided by the sparsity of the wavelet representation and by quadratic convergence of the Newton algorithm, which is used to solve the implicit equations that give the enhanced detail coefficients.

Qualitative and quantitative tests about the wavelet-based algorithm shows that it is able to enhance both under and over exposed images and to remove color cast, as the spatial variational method of (Palma-Amestoy et al., 2009).

The wavelet framework points out new issues whose discussion is beyond the scope of this paper, but that we consider interesting for future investigation: 1) What is the relation between the intrinsic features of the mother wavelet ψ , i.e. shape, support width and symmetry, and the color normalization abilities of the wavelet algorithm? 2) Can we devise an analogue model by suitably apply the windowed Fourier transform to the spatial variational algorithm presented in (Palma-Amestoy et al., 2009)? If so, how does that model relates to the one described in this paper? 3) Which is the optimal selection of the coefficients p_j for contrast enhancement? 4) Can neuroscience models of vision provide insights to properly choose the mother wavelet ψ and the coefficients p_j or to guide towards a more complete model?

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APPENDIX

The computation of the first variation of $\mathcal{D}_{d_{j,k}}^0$ with respect to $\{d_{j,k}\}$ gives:

$$\frac{\partial \mathcal{D}_{d_{j,k}}^0}{\partial \{d_{j,k}\}} = 1 - \frac{d_{j,k}^0}{d_{j,k}}. \quad (7)$$

The first variation of $\mathcal{C}_{p_j, \{a_{j,k}\}}$ with respect to $\{d_{j,k}\}$ gives

$$\frac{\partial \mathcal{C}_{p_j, \{a_{j,k}\}}}{\partial \{d_{j,k}\}} = -p_j \frac{a_{j,k}}{(d_{j,k})^2}. \quad (8)$$

Summing (7) and (8) we get

$$1 - \frac{d_{j,k}^0}{d_{j,k}} - p_j \frac{a_{j,k}}{(d_{j,k})^2} = 0, \quad (9)$$

multiplying by $d_{j,k}$ and simplifying the algebraic expression, we arrive to the result stated in Proposition 2.1. \square